UNIT 17:
PRACTICAL GEOMETRY TRIANGLES

Definition:
The branch of mathematics in which we study the practical construction of various figures like triangle, rectangle, parallelogram etc is known as practical geometry. Here we discuss the practical construction of triangles.

Elements of a Triangle:
A triangle has six elements, three sides and three angles.
i.e. $\overline{AB}, \overline{AC}, \overline{BC}, \angle A, \angle B, \angle C$

Construction of a triangle in the following three cases:
1. When two sides and one included angle are given.
2. When one side and two of the angles are given.
3. When two sides and one opposite angle are given.

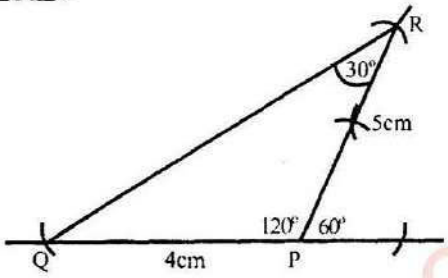
We use the following geometrical instruments:
1. A ruler for measurement of lines in cm or mm.
2. A protractor for measurement of angles.
3. A compass with pencil to draw arcs and circles etc.
4. A set square to draw parallel and perpendicular lines.
5. An eraser to omit wrong, points.

Note: First label a diagram with the help of given data and then draw the steps through pen.

EXAMPLE 1

Construct a triangle PQR given that $\overline{PQ} = 4cm, \overline{PR} = 5cm$ and $m\angle P = 120°$.

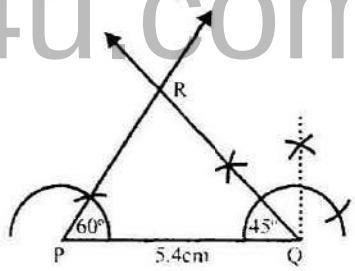
Solution:



Steps of Construction:
1. Draw $\overline{PQ}$ of measure 4cm.
2. At P, construct an angle of 120° with the help of a compass.
3. With P as center, draw an arc $\overline{PR}$ of radius 5cm i.e. $\overline{PR} = 5cm$.
4. Join R to Q.
   PQR is the required triangle.

EXAMPLE 2

Construct ΔPQR such that, $m\overline{PQ} = 5.4$ cm, $m\angle PQR = 45°$ and $m\angle RPQ = 60°$.

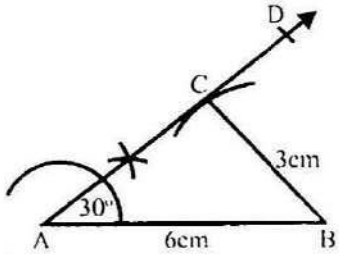


Steps of Construction:
1. Draw $\overline{PQ}$ measuring 5.4cm.
2. At P, draw an angle of 60° and at Q, draw an angle of 45°.
3. Let the two arms meet at R.
4. PQR is the required triangle.

EXAMPLE 3

Construct ΔABC such that $m\overline{BC} = 3cm$, $m\overline{AB} = 6cm$ and $m\angle A = 30°$.

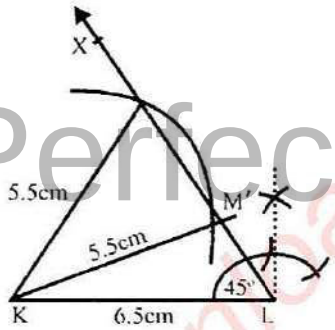
Steps of Construction:
1. Draw $\overline{AB}$ measuring 6cm.
2. At A, construct $\angle BAD$ such that its measure is 30°.



3. With B as center, draw an arc of radius 3cm.
4. The arc intersects $\overline{AD}$ at only one-point C.
5. Join C to B.
6. ABC is the required triangle.

EXAMPLE 4

Construct ΔKLM, such that, $m\overline{KL} = 6.5$ cm, $m\overline{KM} = 5.5$ cm and $m\angle L = 45°$.



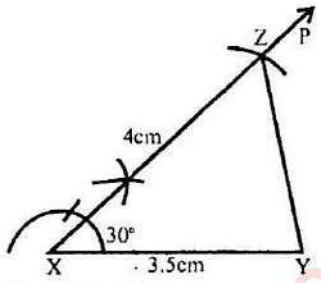
Steps of Construction:

1. Draw $\overline{KL}$ such that $m\overline{KL} = 6.5cm$.
2. At L construct an $\angle KLX = 45°$.
3. With K as centre, draw an arc of radius 5.5cm to intersect $\overrightarrow{LX}$ at M and M'.
4. Join K to M and M'.
5. KLM and KLM' are the required triangles.

EXERCISE 17.1

Q1: Construct a ΔXYZ for the following assumptions:

i) $m\angle X = 30°$, $m\overline{XY} = 3.5cm$ and $m\overline{XZ} = 4cm$.

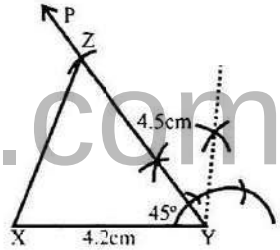


Steps of Construction:

1. Draw $\overline{XY}$ of length 3.5cm.
2. Taking X as centre and construct an angle of 30° with compass.
3. From point X draw an arc of 4cm which cuts $\overrightarrow{XP}$ at point Z.
4. Join Z to Y.

Hence ΔXYZ is the required triangle.

ii) $m\angle Y = 45°$, $m\overline{XY} = 4.2cm$ $m\overline{YZ} = 4.5cm$.

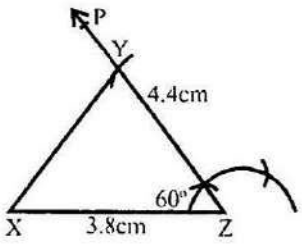


Steps of Construction:

1. Draw $\overline{XY}$ of length 4.2cm.
2. At point Y construct an angle of 45° with compass.
3. Draw an arc of 4.5cm from point Y cut off $\overrightarrow{YP}$ at point Z.
4. Join Z to X.

Hence XYZ is the required triangle.

iii) $m\angle Z = 60°$, $mXZ = 3.8cm$, $m\overline{YZ} = 4.4cm$.

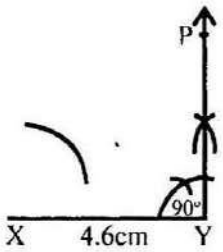


Steps of Construction:

1. Draw $\overline{XZ} = 3.8cm$
2. Construct an angle of 60° at point Z with compass.
3. Draw an arc of 4.4cm from Z, which cuts $\overrightarrow{ZP}$ at point Y.
4. Join Y to X.

Hence ΔXYZ is the required triangle.

iv) $m\angle Y = 90^\circ$, $m\overline{XY} = 4.6cm$, $m\overline{YZ} = 2.9cm$.



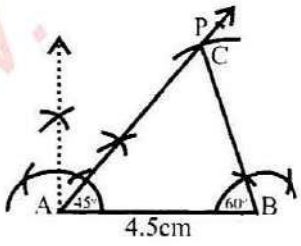
Steps of Construction:

1. Draw $\overline{XY}$ of length 4.6cm.
2. Construct an angle of 90° at point Y with compass.
3. With X as centre draw an arc of 2.9cm.
4. This arc does not meet the arm $\overrightarrow{YP}$ of $\angle Y$.

Hence, in this case ΔXYZ is not possible.

Q2: Construct a ΔABC for the following assumptions:

i) $mAB = 4.5cm, m\angle A = 45^\circ$ and $m\angle B = 60^\circ$

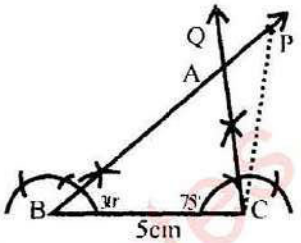


Steps of Construction:

1. Draw $\overline{AB} = 4.5cm$.
2. At point A and B construct angles of 45° and 60° with compass respectively.
3. Draw a ray $\overrightarrow{AP}$ from A.
4. Draw arc from B cut off $\overrightarrow{AP}$ at point C.
5. Join B to C.

Hence ΔABC is required triangle.

ii) $m\overline{BC} = 5cm$, $m\angle B = 30^\circ$, $m\angle C = 75^\circ$

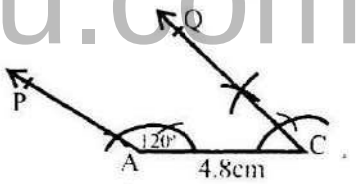


Steps of Construction:

1. Draw $\overline{BC} = 5cm$.
2. Construct angle, 30° and 75° at points B and C respectively with compass.
3. Draw rays at B and C as $\overrightarrow{BP}$ & $\overrightarrow{CQ}$.
4. Both rays meet at point A.

Hence ΔABC is the required triangle.

iii) $m\overline{AC} = 4.8cm, m\angle A = 120^\circ, m\angle C = 30^\circ$

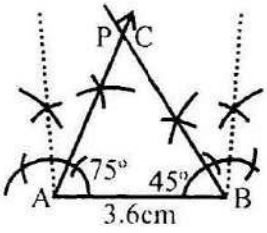


Steps of Construction:

1. Draw $\overline{AC} = 4.8cm$.
2. Construct angles 120° and 30° at points A and C respectively.
3. Draw rays $\overrightarrow{AP}$ and $\overrightarrow{CQ}$.
4. The two rays do not meet.

Hence ΔABC is not possible.

iv) $m\overline{AB} = 3.6cm$, $m\angle A = 75^\circ$, $m\angle B = 45^\circ$.



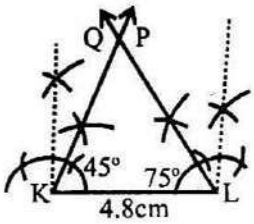
Steps of Construction:

1. Draw $\overline{AB} = 3.6cm$.
2. Construct angles of 75° and 45° at points A and B with compass.
3. Draw rays from A and B.
4. Both rays meet each other at point C.

Hence ΔABC is the required triangle.

Q3: Construct a ΔKLM for the following assumptions:

i) $m\overline{KL} = 4.8cm$, $m\angle K = 45^\circ$, and $m\angle M = 60^\circ$



Steps of Construction:

1. Draw $\overline{KL} = 4.8cm$.
2. Construct angles of 45° and 75° at points K and L with compass.
3. Draw rays $\overrightarrow{KP}$ and $\overrightarrow{LQ}$ which intersects at M.
4. ΔKLM is the required triangle.

ii) $m\overline{LM} = 3.8cm, m\angle K = 30^\circ, \angle M = 75^\circ$

Solution:

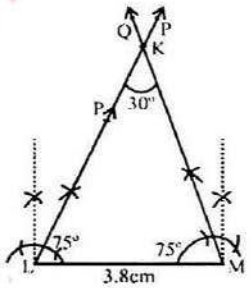
Since $\angle K + \angle L + \angle M = 180^\circ$

$30^\circ + \angle L + 75^\circ = 180^\circ$

$m\angle L + 105^\circ = 180^\circ$

$\Rightarrow m\angle L = 180^\circ - 105^\circ$

$\Rightarrow m\angle L = 75^\circ$



Steps of Construction:

1. Draw $\overline{LM} = 3.8cm$.
2. Construct angles of 75° each on L and M with compass.
3. Draw rays $\overrightarrow{LP}$ and $\overrightarrow{MQ}$ which intersects each other at point K.

Hence ΔKLM is the required triangle.

iii) $m\overline{KM} = 5cm, m\angle K = 105^\circ, m\angle L = 45^\circ$

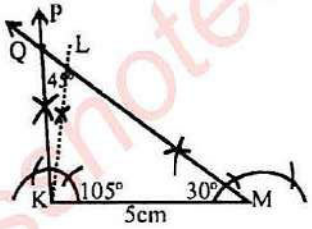
Solution:

Since $\angle K + \angle M + \angle L = 180^\circ$

$105^\circ + \angle M + 45^\circ = 180^\circ$

$\angle M + 150^\circ = 180^\circ$

$\angle M = 180^\circ - 150^\circ = 30^\circ$



Steps of Construction:

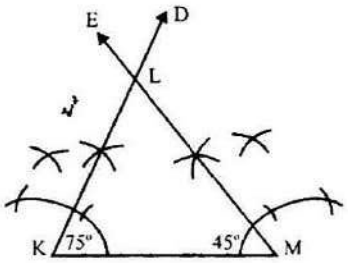
1. Draw $\overline{KM} = 5cm$.
2. Construct angles of 105° and 30° on points K and M with the help of compass.
3. From K and M draw rays $\overrightarrow{KP}$ and $\overrightarrow{MQ}$.
4. Both rays meet at point L.

Hence ΔKLM is the required triangle.

iv) $m\overline{KM} = 5.4cm, m\angle K = 75^\circ, m\angle M = 45^\circ$

Solution:

Given $m\overline{KM} = 5.4cm$, $m\angle K = 75^\circ$ and $m\angle M = 45^\circ$



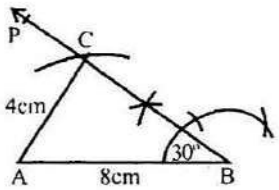
Steps of Construction:

1. Draw $m\overline{KM} = 5.4cm$.
2. At point K make $\angle MKD = 75^\circ$.

3. At point M make $\angle KME = 45°$.
4. Cuts $\overline{KD}$ and $\overline{ME}$ at point L. Thus ΔKLM is required triangle.

Q4: Construct a ΔABC (whenever possible), for the following assumptions:

i) $m\angle B = 30°$, $m\overline{AB} = 8cm$, & $m\overline{AC} = 4cm$



Steps of Construction:

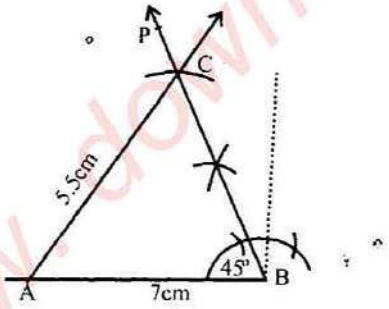
1. Draw $\overline{AB} = 8cm$.
2. At point B construct an angle of $30°$ with compass.
3. At point A draw an arc of 4cm cutting $\overrightarrow{BP}$ at point C.
4. Join C to A.

Hence ΔABC is the required triangle.

ii) $m\overline{AB} = 7cm, m\overline{AC} = 5.5cm$ & $m\angle B = 45°$

Steps of Construction:

1. Draw $\overline{AB} = 7cm$.
2. Construct an angle of $45°$ at point B with compass.



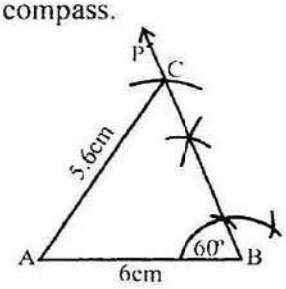
3. At point A draw an arc of 5.5cm cutting $\overrightarrow{BP}$ at point C.
4. Join C to A.

Hence ΔABC is the required triangle.

iii) $m\overline{AB} = 6cm, m\overline{AC} = 5.6cm, m\angle B = 60°$

Steps of Construction:

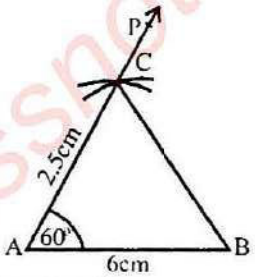
1. Draw $\overline{AB} = 6cm$.
2. At point B construct an angle of $60°$ with compass.



3. At point A draw an arc of 5.6cm with compass cutting $\overrightarrow{BP}$ at point C.
4. Join C to A.

Hence ΔABC is the required triangle.

iv) $m\overline{AB} = 6cm, m\overline{AC} = 2.5cm, m\angle A = 60°$



Steps of Construction:

1. Draw $\overline{AB} = 6cm$.
2. Construct an angle $60°$ on A with compass.
3. Draw an arc of 2.5cm from A cutting $\overline{AP}$ at C.
4. Join C to B.

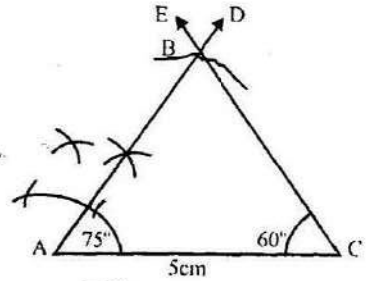
Hence ΔABC is the required triangle.

v) $m\overline{AC} = 5cm, m\angle A = 75°$ & $m\angle C = 60°$

Solution:

Given: $m\overline{AC} = 5cm$, $m\angle A = 75°$ and $m\angle C = 60°$

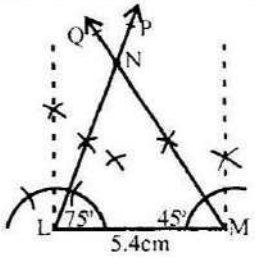
Steps of Construction:



1. Draw $m\overline{AC} = 5cm$

2. At point A make $\angle CAD = 75^o$.
3. At point C make $\angle ACE = 60^o$.
4. Cuts $\overline{AD}$ and $\overline{CE}$ at point B. Thus ΔABC is required triangle.

Q5: Construct ΔLMN such that,
$m\overline{LM} = 5.4cm, m\angle L = 75^o$ & $m\angle M = 45^o$.



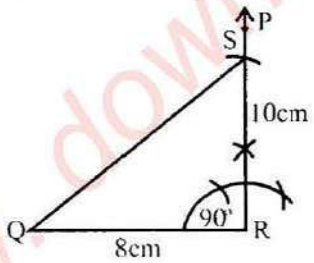
Steps of Construction:

1. Draw $\overline{LM} = 5.4cm$.
2. Construct angles of 75^o and 45^o at point L and M with compass.
3. Draw rays $\overrightarrow{LP}$ and $\overrightarrow{MQ}$ from points L and M.
4. Both rays intersect at N.

Hence ΔLMN is the required triangle.

Q6: Construct ΔPQR such that,
$m\overline{QR} = 8cm, m\overline{RS} = 12cm$ & $m\angle R = 90^o$.

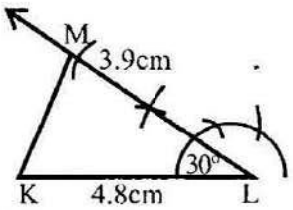
Steps of Construction:



1. Draw $\overline{QR} = 8cm$.
2. Construct an angle of 90^o at point R with compass.
3. Draw an arc at point R of radius 10cm.
4. Join S to Q.

Hence PQR is a right-angled triangle.

Q7: Construct ΔKLM such that,
$m\overline{KL} = 4.8cm$, $m\overline{LM} = 3.9cm$ & $m\angle L = 30^o$.

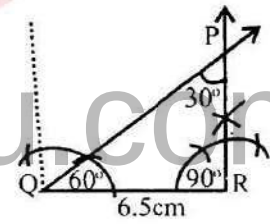


Steps of Construction:

1. Draw $\overline{KL} = 4.8cm$.
2. At L draw an angle of 30^o with compass.
3. Draw an arc of 3.9cm from L cutting $\overrightarrow{LP}$ at M.
4. Join M to K.

Hence ΔKLM is the required triangle.

Q8: Construct ΔPQR such that,
$m\overline{QR} = 6.5cm, m\angle P = 30^o$ & $m\angle Q = 60^o$.



Steps of Construction:

1. Draw $\overline{QR} = 6.5cm$.
2. Construct an angle of 60^o at Q and 90^o at R.
3. Draw the rays from Q and R meeting at P such that $\angle P = 30^o$.

Hence ΔPQR is the required triangle.

Concurrent Lines and Point of Concurrency:

Remember that:

a) The two coplanar lines are either parallel or intersecting.
b) The two lines intersect each other at one point which is called their point of intersection.

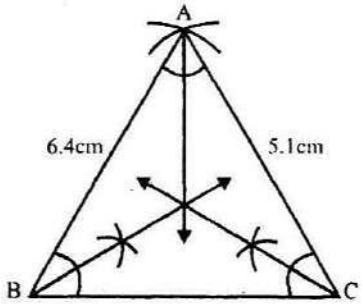
Definition:

The three lines are said to be concurrent if they pass through one point. The point at

which they intersect is called point of intersection or point of concurrency.

EXAMPLE 6

Construct ΔABC whose $m\overline{AB} = 4.6cm$, $m\overline{BC} = 5cm$ and $m\overline{CA} = 5.1cm$. Draw angle bisectors of the triangle, and verify that these are concurrent.

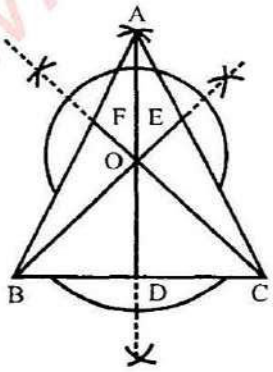


Steps of Construction:

1. Draw $\overline{BC}$ measuring 5cm.
2. With B and C as centers and radii respectively equal to 4.6 and 5.1, draw arcs intersecting at A.
3. Draw $\overline{AB}$ and $\overline{AC}$.
4. Draw bisectors pass through the same point I. Thus, angle bisectors of a triangle are concurrent.

EXAMPLE 7

Construct ΔABC such that $m\overline{AB} = 5.6$ cm, $m\overline{BC} = 6cm$ & $m\overline{CA} = 5cm$. Draw its altitudes and verify that these are concurrent.



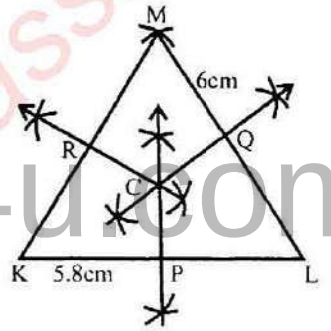
Steps of Construction:

1. Construct ΔABC with the help of given data.
2. Draw perpendiculars from A to $\overline{BC}$. B to $\overline{CA}$ and C to $\overline{AB}$.
3. $\overline{AD}, \overline{BE}$ and $\overline{CF}$ are the required altitudes.

We see that the three altitudes pass through the same points O. i.e. altitudes of a triangle are concurrent.

EXAMPLE 8

Construct ΔKLM such that $m\overline{KL} = 5.8$ cm, $m\overline{LM} = 6cm$ and $m\angle L = 60°$. Draw its perpendicular bisectors and verify their concurrency.



Steps of Construction:

1. Construct ΔKLM according to the given data.
2. Draw the right bisectors of $\overline{KL}, \overline{LM}$ and $\overline{MK}$. .
3. $\overline{CP}, \overline{CQ}$ and $\overline{CR}$ are the three perpendicular bisectors of $\overline{KL}, \overline{LM}$ and $\overline{MK}$ respectively.

All the three perpendicular bisectors pass through C i.e. they are concurrent.

EXAMPLE 9

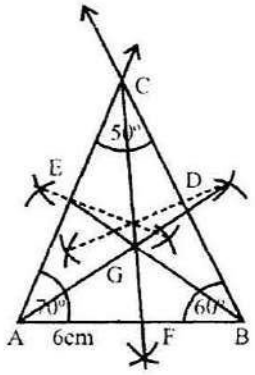
Construct ΔABC such that $m\overline{AB} = 6$ cm, $m\angle A = 70°$ and $m\angle C = 50°$. Draw its medians and verify their concurrency.

$\because$ $m\angle A = 70°$ and $m\angle C = 50°$

$\because \quad m\angle B = 180^\circ - (70^\circ + 50^\circ)$

$= 180^\circ - 120^\circ$

$= 60^\circ$



Remember: *Line segment joining the mid-point of one side of a triangle to its opposite vertex is called median.*

Steps of Construction:

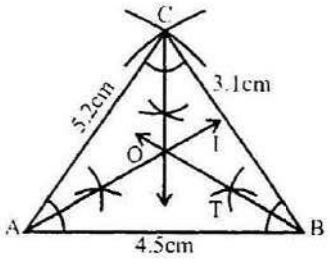
1. Construct ΔABC with the help of given data.
2. Find the midpoint D, E and F of $\overline{BC}, \overline{AC}$ and $\overline{AB}$ respectively.
3. Draw the medians $\overline{AD}, \overline{BE}$ and $\overline{CF}$.
4. All the three medians pass through G.

Therefore the medians of a triangle are concurrent.

EXERCISE 17.2

Q1: Construct ΔABC. Draw their angle bisectors and verify their concurrency.

i) $m\overline{AB} = 4.5cm$, $m\overline{BC} = 3.1cm$ and $m\overline{CA} = 5.2cm$.

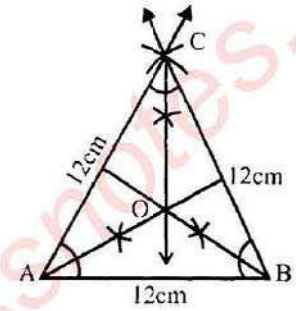


Steps of Construction:

1. Draw $\overline{AB} = 4.5cm$.
2. At points A and B draw arcs of radius 5.2cm and 3.1cm meeting at point C.
3. Draw bisector of angles A, B and C with compass.
4. These bisectors intersect at point O.

Hence these bisectors are concurrent.

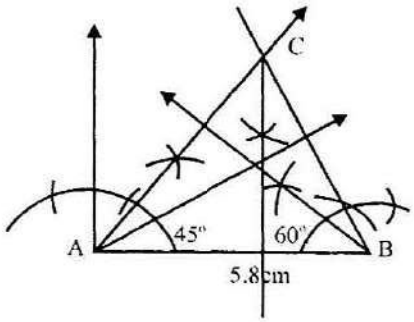
ii) $m\overline{AB} = m\overline{BC} = m\overline{CA} = 12.$



Steps of Construction:

1. Draw $\overline{AB} = 12cm$.
2. With A and B draw arcs of 12cm intersecting at point C.
3. Draw bisectors of angle A, B, C with compass.
4. These bisectors intersects at one point O. Hence they are concurrent.

iii) $m\overline{CA} = 5.8cm$, $m\angle A = 45^\circ$ & $m\angle C = 75^\circ$.



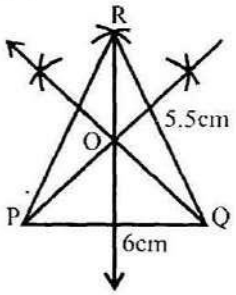
Steps of Construction:

1. Draw $\overline{AB} = 5.8cm$.
2. Construct angles 45° and 60° at points A and B.
3. Draw arcs from A and B which meet at C.

4. Draw angle bisectors of A, B, C meeting at O. Hence the angle bisectors of ΔABC are concurrent.

Q2: Construct ΔPQR, draw their altitudes and verify their concurrency.

i) $m\overline{PQ} = 6cm$, $m\overline{QR} = 4.5cm$ and $m\overline{PR} = 5.5cm$.



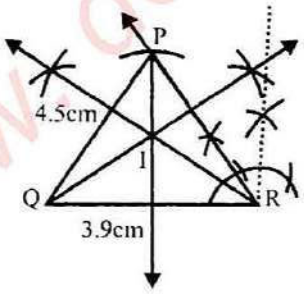
Steps of Construction:

1. Construct a triangle PQR with the help of data.
2. Draw perpendiculars from P to $\overline{QR}$, from Q to $\overline{PR}$ and from R to $\overline{PQ}$.
3. These altitudes intersects at point O. Hence they are concurrent.

ii) $m\overline{PQ} = 4.5cm$, $m\overline{QR} = 3.9cm$ and $m\angle R = 45^o$.

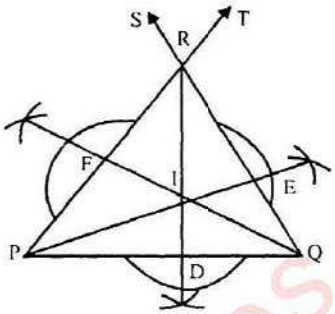
Steps of Construction:

1. Construct a triangle PQR with the help of given data.



2. Draw perpendiculars from point P, Q, R on their opposite side of ΔPQR.
3. These perpendiculars intersect at point I. Hence they are concurrent.

iii) $m\overline{PQ} = 6cm$, $m\angle P = 70^o$ and $m\angle Q = 65^o$.



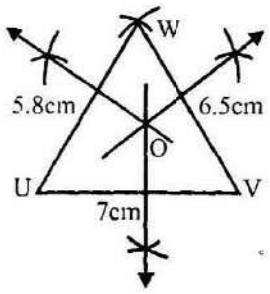
Steps of Construction:

1. Draw $m\overline{PQ} = 6cm$.
2. Take Q is the center and make $m\angle PQS = 65^o$.
3. Take P is a center and make $\angle QPT = 70^o$.
4. $\overline{QS}$ and $\overline{PT}$ intersect each other at point R.
5. ΔPQR is constructed.
6. Draw the perpendiculars from P to $\overline{QR}$, Q to $\overline{PR}$ and R to $\overline{QP}$.
7. Thus $\overline{RD}, \overline{PE}$ and $\overline{QF}$ are the required altitudes which passes through the point I.

Thus altitude of ΔPQR are concurrent.

Q3: Construct ΔUVW, draw their perpendicular bisectors and verify their concurrency.

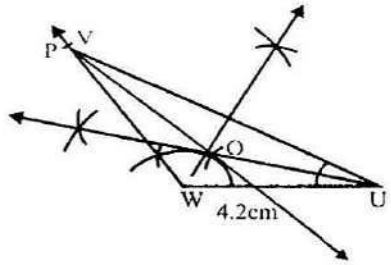
i) $m\overline{UV} = 7cm$, $m\overline{VW} = 6.5cm$ and $m\overline{WU} = 5.8cm$.



Steps of Construction:

1. Draw $UV = 7cm$.
2. Construct ΔUVW according to given data.
3. Draw their perpendicular bisectors to the side $\overline{UV}, \overline{VW}, \overline{UW}$. They meet at one point O. Hence they are concurrent.

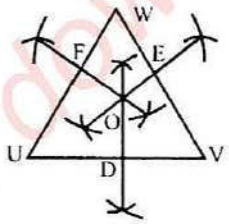
ii) $m\overline{VW} = 10cm$, $m\overline{WU} = 4.2cm$ **and** $m\angle W = 120°$.



Steps of Construction:

1. Draw $\overline{WU} = 4.2cm$.
2. Construct ΔUVW with the help of given data.
3. Draw the perpendicular bisectors to their side where they meet at point O. Hence perpendicular bisectors are concurrent.

iii) $m\overline{UV} = m\overline{VW} = m\overline{WU} = 0.8m$.



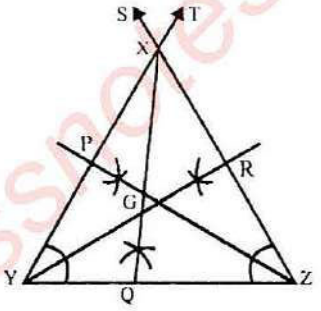
Steps of Construction:

1. Draw $m\overline{UV} = 0.8m$.
2. At U, draw an arc of radius 0.8m.
3. At V, draw an arc of radius 0.8m.
4. Let both arcs meet at point W.
5. Join W to U and V. Thus UVW is constructed.
6. Draw the right bisectors of the three sides $\overline{UV}, \overline{VW}, \overline{UW}$.
7. $\overline{OD}, \overline{OE}, \overline{OF}$ are the three perpendicular bisectors of $\overline{UV}, \overline{VW}, \overline{UW}$ respectively.

Which are passes through the point "O". Hence the perpendicular bisectors of ΔUVW are concurrent.

Q4: Construct ΔXYZ, draw their medians and verify their concurrency.

i) $m\overline{YZ} = 4.1cm$, $m\angle Y = 60°$ **and** $m\angle X = 45°$.



Steps of Construction:

1. Draw $m\overline{YZ} = 4.1cm$.
2. Make $\angle ZYT = 60°$

   As $\angle X + \angle Y + \angle Z = 180°$

   $\Rightarrow 75° + 60° + \angle Z = 180°$

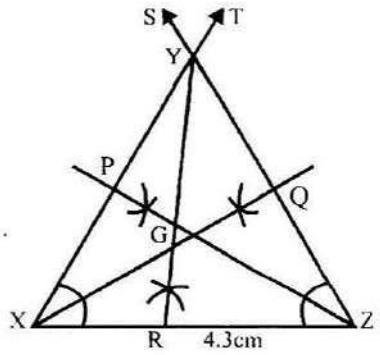
   $\Rightarrow \angle Z = 180° - 75° - 60°$

   $\Rightarrow \angle Z = 45°$
3. Take Z is a center and make angle $\angle YZS = 45°$.
4. $\overline{YT}$ and $\overline{ZS}$ intersect each other at point X. Thus ΔXYZ is constructed.
5. Find the midpoints P, Q, R of sides $\overline{XY}, \overline{YZ}, \overline{ZX}$.
6. Draw the medians $\overline{ZP}, \overline{XQ}, \overline{YR}$.
7. All the medians passes through the point G. Hence the medians are concurrent.

ii) $m\overline{ZX} = 4.3cm$, $m\angle X = 75°$ **and** $m\angle Y = 45°$.

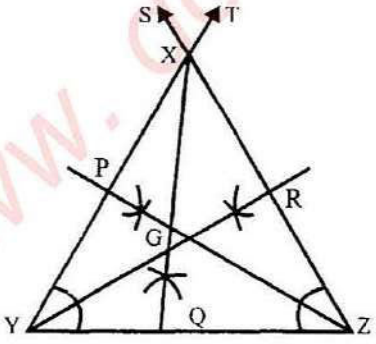
Steps of Construction:

1. Draw $m\overline{ZX} = 4.3cm$.



2. Make $\angle ZXT = 75^o$
   As $\angle X + \angle Y + \angle Z = 180^o$
   $\Rightarrow 75^o + 45^o + \angle Z = 180^o$
   $\Rightarrow \angle Z = 180^o - 75^o - 45^o$
   $\Rightarrow \angle Z = 60^o$
3. Take Z is a center and make angle $\angle YZS = 60^o$.
4. $\overline{YT}$ and $\overline{ZS}$ at point X. Thus ΔXYZ is constructed.
5. Find the midpoints P, Q, R of sides $\overline{XY}, \overline{YZ}, \overline{ZX}$ respectively.
6. Draw the medians $\overline{ZP}, \overline{XQ}, \overline{YR}$.
7. All the medians passes through the point G. Hence the medians are concurrent.

iii) $m\overline{XY} = 4.5cm$, $m\overline{YZ} = 3.4cm$, and $m\overline{ZX} = 5.6cm$.



Steps of Construction:

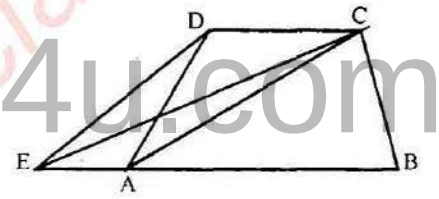
1. Draw $m\overline{YZ} = 3.4cm$.
2. Take Y as a center and draw an arc of length 4.5cm.
3. Take Z as a center and draw an arc of length 5.6cm.
4. Both arcs intersect each other at point X. Thus ΔXYZ is constructed.
5. Find the midpoints P, Q, R of sides $\overline{XY}, \overline{YZ}, \overline{ZX}$ respectively.
6. Draw the medians $\overline{ZP}, \overline{XQ}, \overline{YR}$.
7. All the medians passes through the point G. Hence the medians are concurrent.

Figures with equal areas:

a) Construct a triangle equal in area to a given quadrilateral.

Given: A quadrilateral $ABCD$

Required: To construct a triangle equal in area to the quadrilateral $ABCD$.



Steps of Construction:

1. Let $ABCD$ is given quadrilateral.
2. Join A to C.
3. Draw $\overline{DE} \parallel \overline{CA}$ which intersect $\overline{BA}$ at point E.
4. Join E to C.
5. Thus EBC is required triangle.

b) Construct a rectangle equal in area to a given triangle.

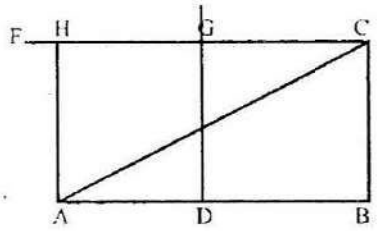
Given: ΔABC

Required: To construct a rectangle equal in area to ΔABC.

Steps of Construction:

1. Let ABC be the given triangle.
2. Bisect $\overline{AB}$ at point D.
3. At point D, draw an angle $\angle ADE = 90^o$.
4. Th ough C, draw $\overline{CF} \parallel \overline{BA}$, meeting

$\overline{DE}$ at point G.

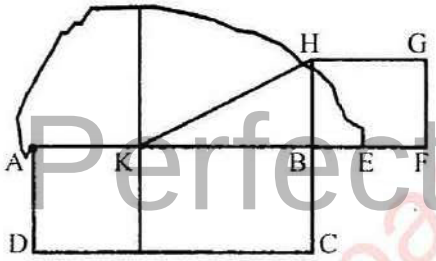


5. At point A draw $\overline{AH} \parallel \overline{DG}$, meeting $\overline{CF}$ at H.
6. Thus $ADGH$ is the required rectangle.

c) Construct a square in area to a given rectangle.

Given: A rectangle $ABCD$.

Required: To construct a square equal in area to rectangle $ABCD$.



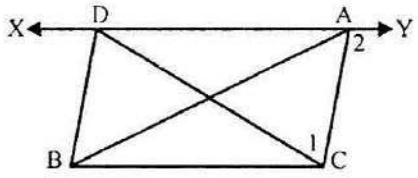
Steps of Construction:

1. Let $ABCD$ is the given rectangle.
2. On $\overline{AB}$ take point E such that $\overline{BE} \cong \overline{DC}$.
3. Bisect $\overline{AE}$ at point K.
4. Take K is a center and draw a semi circle of radius KE.
5. Let the circle meet $\overline{CB}$ at point H.
6. Take $\overline{BH}$ is one side of the square and draw other sides.
7. Thus $BHGF$ is the required square.

d) Construct a triangle to equivalent area on a base of given data.

Given: A ΔABC with $\overline{BC}$ as its base.

Required: To construct a triangle of equivalent area with base $\overline{BC}$.



Steps of Construction:

1. Let ABC be the given triangle.
2. Through the point A draw $\overline{XYZ} \parallel BC$ by constructing $\angle 2 \cong \angle 1$.
3. Take any point D on $\overline{XY}$ and joint D to B and C.
4. ΔDBC is the required triangle.

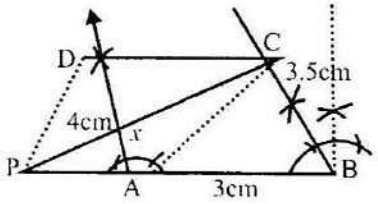
EXERCISE 17.3

Q1: Draw a quadrilateral $ABCD$, such that $m\overline{AB} = 3cm$, $m\angle B = 60°$, $m\angle A = 110°$. $m\overline{BC} = 3.5cm$ and $m\overline{AD} = 4cm$. Construct a triangle equal in area to the quadrilateral $ABCD$.

Given: A quadrilateral $ABCD$.

Required: To construct a triangle equal in area to the quadrilateral $ABCD$.

Steps of Construction:



1. Join C to P.
2. Through D draw $\overline{DP} \parallel \overline{CA}$ which meet $\overline{AB}$ which produced to P.
3. $\because \Delta APC$ and ΔADC stand on the same base AB and between same parallel lines $\overline{AC}$ and $\overline{PD}$.
   $\therefore$ Area of ΔADC + Area of ΔABC
4. Area of ΔAPC + Area of ΔABC

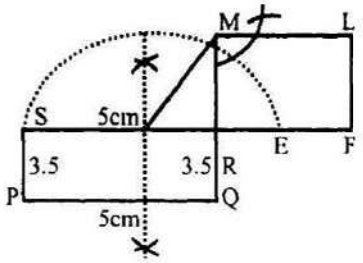
= Area of ΔADC + Area of ΔABC

5. Area of ΔPBC = Area of quadrilateral $ABCD$. Hence ΔPBC is the required triangle.

Q2: Draw a rectangle $PQRS$ such that $m\overline{PQ} = 5cm$ and $m\overline{QR} = 3.5cm$. Construct a square equal in area to rectangle $PQRS$.

Given: A rectangle $PQRS$

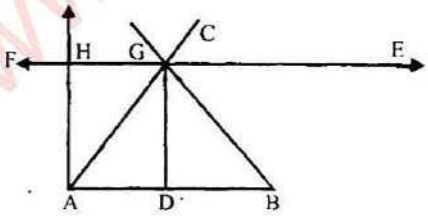
Required: Construct a square equal in area to rectangle $PQRS$.



Steps of Construction:

1. Produce $\overline{SR}$ to F such that $\overline{SR} = \overline{RF}$.
2. Bisect $\overline{SE}$ at O.
3. With centre as O and radius $\overline{SO}$ draw semi-circle.
4. Produce $\overline{QR}$ to meet the semi-circle at M.
5. On $\overline{RM}$ as a side complete a square $RFLM$. Which is the required square.

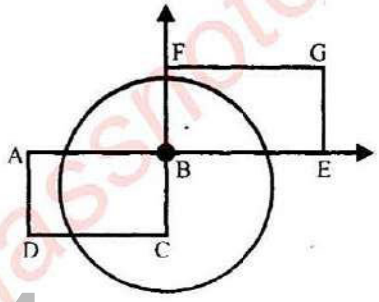
Q3: Draw a triangle ABC such that $m\overline{AB} = 5cm$, $m\overline{BC} = 4cm$ and $m\overline{CA} = 4.5cm$. Construct a rectangle equal in area to the given triangle.



Steps of Construction:

1. Draw $\overline{AB} = 5cm$.
2. With A as centre draw an arc of length $4.5cm$.
3. With B as centre draw an arc of $4cm$.
4. Join A with C and C with B respectively.
5. $\therefore \Delta ABC$ is the required triangle.
6. Bisect AB at point such that $m\angle ADC = 90°$.
7. Draw $\overline{EF} \parallel \overline{AB}$ through point C.
8. From point A draw $\overline{AH} \parallel \overline{DG}$.

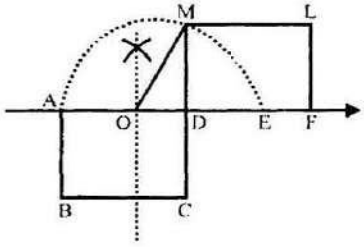
Q4: Construct a square having area equal to the given rectangle.



Steps of Construction:

1. Draw a rectangle $ABCD$.
2. At B draw an arc of radius B.
3. Produce $\overline{AB}$ to point E such that $\overline{AB} = \overline{BE}$.
4. Construct a circle with centre "B" and segment $\overline{AE}$.
5. Produce $\overline{CB}$ to F such that $\overline{CB} = \overline{BF}$.
6. $\overline{BF}$ is the side of the required square.

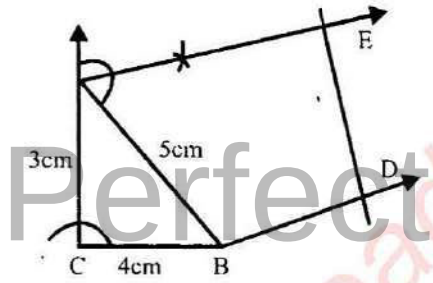
Q5: Construct a square equal in area to a rectangle whose adjacent sides are 4.5 cm and 2.2cm respectively. Measure the sides of the square and find its area and compare with the area of the rectangle.



Steps of Construction:

1. Draw a rectangle $ABCD$ with $m\overline{AB} = 2.2cm$, $m\overline{BC} = 4.5cm$.
2. Extend $\overline{AD}$ to F such that $AD = DF$.
3. From point O draw an arc of radius $\overline{MD}$.
4. Extend $\overline{CD}$ to M such that $\overline{CD} = \overline{DM}$.
5. At point D, we construct a square FLM.

Q6: Construct a square equal in area to the sum of two squares having sides 3*cm* and 4*cm* respectively.



Steps of Construction:

1. Construct a right angle triangle ABC with $\overline{BC} = 4cm$, $\overline{AC} = 3cm$.
2. From Pythagoras theorem,

$$(AB)^2 = (AC)^2 + (BC)^2$$

$$\overline{AB} = \sqrt{(AC)^2 + (BC)^2}$$

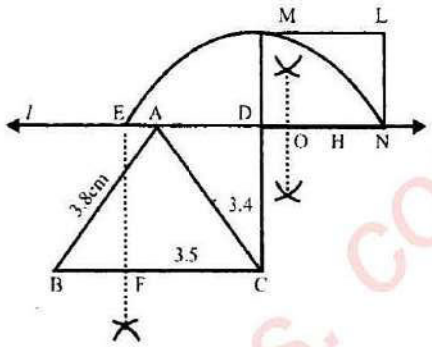
$$= \sqrt{3^2 + 4^2} = \sqrt{25}$$

$$AB = 5cm$$

3. From side AB, make a square.
4. $ABDE$ is the required square.

Q7: Construct a triangle having base 3.5 *cm* and other two sides equal to 3.4*cm* and 3.8*cm* respectively. Transform it into an equal square.

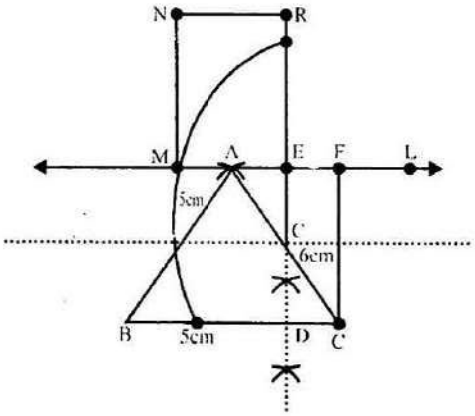
Steps of Construction:



1. Construct a ΔABC with the given data.
2. Take a bisector of $\overline{AB}$ and a line ℓ passing through $C \parallel AB$ cutting at point E.
3. Draw $\overline{AB}$ perpendicular to L.
4. Here $BDEF$ is a rectangle.
5. Bisect $\overline{EH}$ at point O.
6. From point O draw a semi-circle of radius $\overline{OE}$.
7. Hence $DNLM$ is the required square.

Q8: Construct a triangle having base 5 *cm* and other sides equal to 5*cm* and 6*cm*. Also construct a square equal in area to the given triangle.

Steps of Construction:



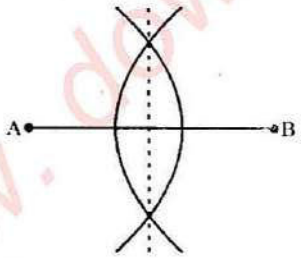
1. Draw $\overline{BC}$ of length $5cm$.
2. From point C draw an arc of $6cm$ and $5cm$ each from point B, which meets at A.
3. Draw a line $\ell \parallel \overline{BC}$ through point A.
4. Draw perpendicular bisector of $\overline{BC}$ which cuts the line at E.
5. Draw $\overline{CF} \perp$ to the line ℓ. So $DEFC$ is a rectangle.
6. Extend $\overline{DE}$ to R.
7. Draw a square $EMNR$ which is the required square having equal area with ΔABC.

REVIEW EXERCISE 13

Q1: Circle the correct answers:

i) What is the first step in constructing an angle bisector?
(a) Draw a ray
(b) Label the points of intersection
(c) Measure the line
✓(d) Place the compass point on the vertex

ii) What geometric construction is shown in the diagram below?



(a) A line parallel to a given line
(b) An angle bisector
(c) An angle congruent to a given angle
✓(d) A perpendicular

iii) A line segment joining the midpoint of one side of a triangle to its opposite vertex is called:
(a) Perpendicular bisector
✓(b) Median
(c) Altitude
(d) Angle bisector

iv) You are looking at a triangle whose orthocenter, centroid and circumcenter are all the same point. What type of triangle are you looking at?
(a) Scalene
(b) Isosceles
✓(c) Equilateral
(d) Right

v) The centroid of a triangle divides the medians into the ratio of:
✓(a) 2 : 1 (b) 3 : 1
(c) 4 : 1 (d) 5 : 1

vi) A line which is perpendicular to a line segment at its midpoint is called a / an:
✓(a) Perpendicular bisector
(b) Median
(c) Altitude
(d) Angle bisector

vii) The point of intersection of the bisectors of the angles of a triangle is equidistant from the….of the triangle.
✓(a) Vertices
(b) Sides
(c) Altitudes
(d) Medians

viii) Altitudes of a triangle are…….
(a) Equal in length
(b) Equidistant from the vertices
✓(c) Concurrent
(d) Perpendicular bisectors

ix) If measures of three angles of a triangle are known, how many triangles can be constructed?
✓(a) Only one triangle
(b) Two triangles
(c) No triangle

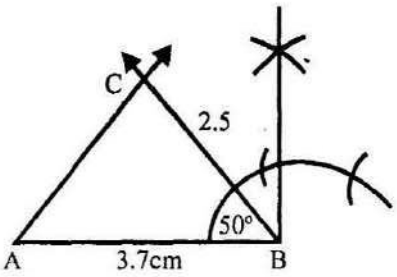
(d) Infinite triangles

x) The point of intersection of the perpendicular bisectors of the sides of a triangle is equidistant from the..... of the triangle.

(a) Altitudes (b) Medians

✓(c) **Sides** (d) Vertices

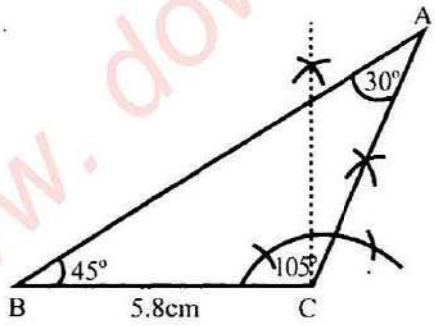
Q2: Construct ΔABC such that, $m\overline{AB} = 3.7cm$, $m\overline{BC} = 2.5cm$ and $m\angle B = 50^\circ$.



Steps of Construction:

1. Take $\overline{AB}$ of length 3.7*cm*.
2. Make an angle of 50° at point B.
3. Taking *B* as centre draw an arc of 2.5 *cm*.
4. Join *C* with *A* and *B* respectively.
5. Hence *ABC* is the required triangle.

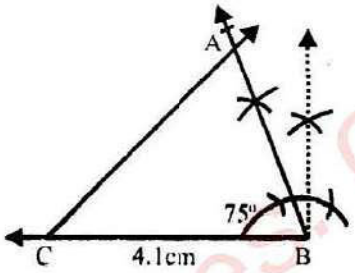
Q3: Construct a ΔABC such that, $m\overline{BC} = 5.8cm$, $m\angle A = 30^\circ$ and $m\angle B = 45^\circ$.



Steps of Construction:

1. Take $\overline{BC}$ of length 5.8*cm*.
2. Make an angle of 105° at point *C* with compass.
3. Make an angle of 45° at *B*.
4. Join *A* with *B* and *C*.
5. Hence ΔABC is the required triangle.

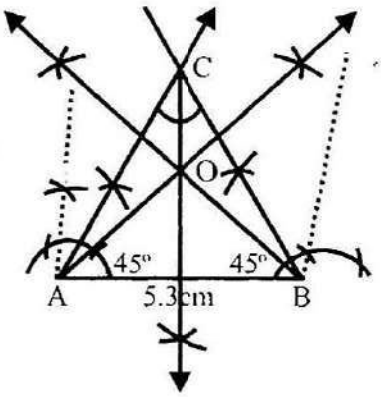
Q4: Construct ΔABC, such that, $m\overline{AC} = 4.5cm$, $m\overline{BC} = 4.1cm$ and $m\angle B = 75^\circ$.



Steps of Construction:

1. Draw $\overline{BC}$ of length 4.1*cm*.
2. Construct an angle of 75° with compass at *B*.
3. Draw rays from *B* and *C* meet at point *A*.
4. ΔABC is the required triangle.

Q5: Construct ΔABC, draw their angle bisectors and verify their concurrency. $m\overline{AB} = 5.3cm$, $m\angle A = 45^\circ$ & $m\angle B = 45^\circ$.



Steps of Construction:

1. Take $m\overline{AB} = 5.3cm$.
2. Make angles of 45° at point *A* and *B* with compass.
3. The two rays meet at point *C*.
4. ΔABC is formed.

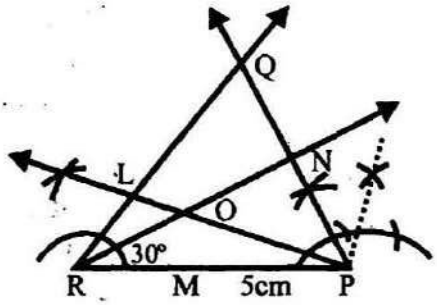
Q6: Construct ΔPQR, draw their altitudes and verify their concurrency. $m\overline{PR} = 5.8cm$, $m\angle P = 45°$ and $m\angle Q = 105°$.

Solution:

As $m\angle P + \angle Q + \angle R = 180°$

$45° + 105° + \angle R = 180°$

$m\angle R = 180° - 150° = 30°$



Steps of Construction:

1. Take $\overline{PR} = 5cm$.
2. Construct an angle 30° at point R with compass.
3. Make an angle 45° at P.
4. The two rays join at point Q.
5. ΔPQR is the triangle.
6. Draw perpendicular $\overline{PM}$ and $\overline{RL}$ from point Q to $\overline{QN}$.
7. Hence required altitude $\overline{QN}, \overline{PM}$ & $\overline{RL}$.

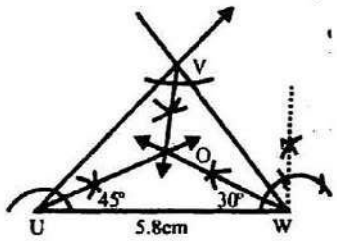
Q7: Construct ΔUVW, draw their perpendicular bisectors and verify their concurrency. $m\overline{UW} = 5.8cm$, $m\angle U = 45°$ and $m\angle V = 105°$.

Solution:

As $m\angle U + m\angle V + m\angle W = 180°$

$45° + 105° + m\angle W = 180°$

$m\angle W = 180° - 150° = 30°$



Steps of Construction:

1. Draw $m\overline{UW}$ of length 5.8cm.
2. Make an angle 45° at point U.
3. Make an angle 30° at point W.
4. The two rays meet at point V.
5. So ΔUVW is the triangle.
6. Draw angle bisectors of $\angle U, \angle W$ and $\angle V$.
7. These bisectors pass through point O.
8. Hence these bisectors are concurrent.

Q8: Construct ΔXYZ, draw their medians and verify their concurrency. $m\overline{ZX} = 6cm, m\angle Y = 60°$ & $m\angle Z = 75°$

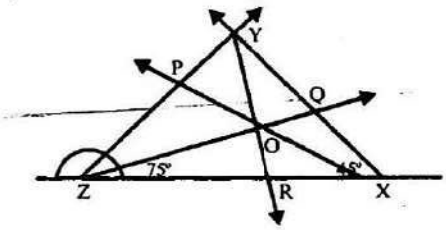
Solution:

$m\angle X + m\angle Y + m\angle Z = 180°$

$m\angle X + 60° + 75° = 180°$

$m\angle X = 180° - 135° = 45°$

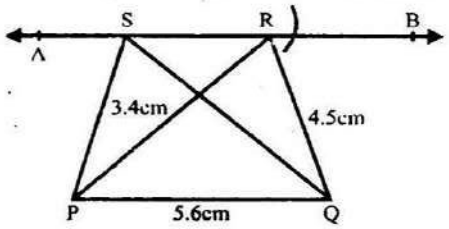
Steps of Construction:



1. Draw $m\overline{XZ} = 6cm$.
2. Make an angle 75° at point Z with the help of compass.
3. Make an angle 45° at point X.
4. The two rays meet at point Y.
5. Required a triangle XYZ.

6. P, Q, R are the midpoints of $\overline{XZ}$, $\overline{XY}$ and $\overline{ZY}$.
7. Draw median $\overline{OR}, \overline{OP}, \overline{OQ}$.
8. Medians pass through point O. Hence these medians of a triangle are concurrent.

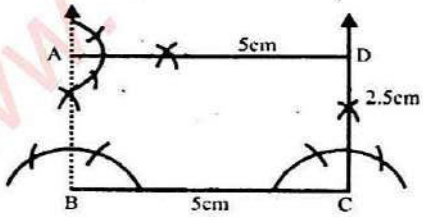
Q9: Draw a triangle PQR such that $m\overline{PQ} = 5.6cm$, $m\overline{QR} = 4.5cm$ and $m\overline{RP} = 3.4cm$. Construct a triangle SPQ equivalent in area to the triangle PQR.



Steps of Construction:

1. Taking $\overline{PQ} = 5.6cm$.
2. At point Q draw an arc of $4.5cm$.
3. At point P draw an arc of $3.4cm$.
4. These two arcs will meet at point R.
5. $\therefore \Delta PQR$ is the required triangle.
6. Draw $\overleftrightarrow{ARB}$ through point R.
7. Take point S on $\overleftrightarrow{AB}$ and joint with P.
8. Hence ΔPQS is equivalent to ΔPQR.

Q10: Construct a rectangle whose adjacent sides are $2.5cm$ & $5cm$ respectively.



Steps of Construction:

1. Draw $\overline{BC} = 5cm$.
2. At point B and C construct an angle of 90° with compass.
3. Cut off the rays $\overline{CD}$ in $2.5cm$ and $\overline{BA}$ in $2.5cm$.
4. Make an angle of 90° at point A with compass.
5. Join point A with D.
6. Hence $ABCD$ is the required triangle.

Additional MCQs of Unit 17:
Practical Geometry Triangles

1. The relationship between theory and practice is called.....
   (a) Analytic geometry (b) Practical geometry
   (c) Trigonometry (d) none
   ✓ **Ans. (b) Practical geometry**

2. Before the construction of a building practical geometry is used by.....
   (a) Engineers (b) Architects (c) Builders (d) All
   ✓ **Ans. (d) All**

3. To design different projects which of the following are used.....
   (a) Shapes (b) Lines (c) Figures (d) All
   ✓ **Ans. (d) All**

4. Construction in practical geometry is performed with the straight edge and.....
   (a) Ruler (b) Protector (c) Pencil (d) Compass
   ✓ **Ans. (d) Compass**

5. What is a powerful learning technique which force you to think about properties of geometric objects?
   (a) Diagram (b) Figure (c) Construction (d) none
   ✓ Ans. (c) Construction

6. The three sides and three angles in a triangle are called.....of triangle
   (a) Components (b) Elements (c) Parts (d) none
   ✓ Ans. (b) Elements

7. The construction of a triangle is possible if two sides and.....are given.
   (a) Included angle (b) 3rd side (c) Median (d) none
   ✓ Ans. (a) Included angle

8. When two sides and an angle opposite to one of the sides is given then which can be true?
   (a) One triangle (b) Two triangles (c) Non triangle (d) All
   ✓ Ans. (d) All

9. When the two lines are either parallel or intersect each other then they are..... lines?
   (a) Vertical (b) Horizontal (c) Coplanar (d) none
   ✓ **Ans. (c) Coplanar**

10. When the three lines pass through the same point then the lines are called……
(a) Intersecting (b) Concurrent (c) Parallel (d) none
✓ **Ans. (b) Concurrent**

11. A line which is only perpendicular to a given line segment is called……
(a) Altitude (b) Right bisector (c) Median (d) none
✓ **Ans. (a) Altitude**

12. A line which is perpendicular to a line segment and divides it into two equal points is called……
(a) Altitude (b) Right bisector (c) Median (d) none
✓ **Ans. (b) Right bisector**

13. Line segment joining the midpoint of one side of a triangle to its opposite vertex is called……
(a) Median (b) Altitude (c) Right bisector (d) none
✓ **Ans. (a) Median**

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