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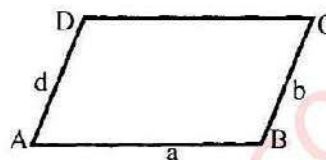
UNIT 16:
THEOREMS RELATED WITH AREA

Definition:
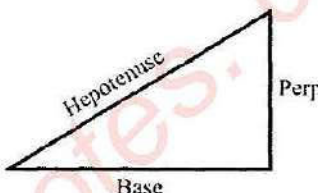Sum of the four sides of a parallelogram is called the perimeter of parallelogram.
- Perimeter of parallelogram
  $= (a + b + c + d)$

Definition:
The area of right angle triangle is given as;

$$Area = \frac{1}{2}[Base \times Perpendicular]$$

Definition:
If any side of a triangle is taken as its base then the perpendicular to that side from the opposite vertex is called altitude of the triangle.

Note:
1. The perpendicular distance between two parallel lines remains the same and does not change.
2. If two figures are of equal area, it is not necessary that the two figures will also be congruent.

THEOREM 16.1

Parallelograms on the same base lying between the same parallel lines are equal in area.
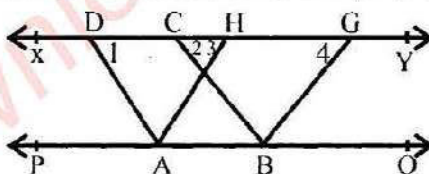
Given:
$ABCD$ and $ABHG$ are two parallelograms having the same base $\overline{AB}$ and lying between two parallel lines $\overleftrightarrow{XY}$ and $\overleftrightarrow{PQ}$ (i.e. both have the same altitude).

To Prove:
Parallelograms $ABCD$ and $ABGH$ are equal in area.

Proof:

Statements	Reasons
In $\triangle ADH \longleftrightarrow \triangle BCG$	
$AH \cong \overline{BG}$	Opposite sides of parallelogram
$\angle 1 \cong \angle 2$	Corresponding angles
$\angle 3 \cong \angle 4$	Corresponding angles
$\therefore \triangle ADH \cong \triangle BCG$	(A.A.S $\cong$ A.A.S)

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or $\triangle ADH$ and $\triangle BCG$ have equal area

Now $ABCH -$

$\triangle ADH \cong ABCH - \triangle BCG$

Parallelogram ABCD $\cong$ parallelogram ABGH

or both parallelograms have same area.

THEOREM 16.2

Parallelograms on equal bases and having the same altitude are equal in area.
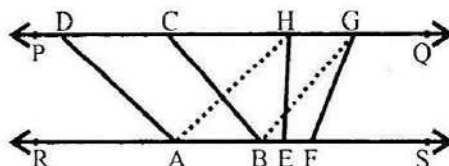
Given:
$ABCD$ and $EFGH$ are two parallelograms having the same altitude and $\overline{AB} \cong \overline{EF}$.

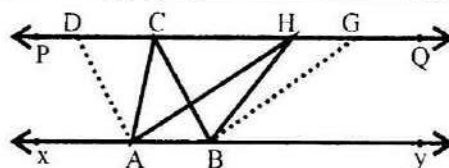### To Prove:
The parallelograms are equal in area.

Construction:
Join A to H and B to G.

Proof:

	Statements	Reasons
$\therefore$	$\overline{HG} \cong \overline{EF}$	Opposite sides of parallelogram
And	$\overline{EF} \cong \overline{AB}$	Given
$\therefore$	$\overline{AB} \cong \overline{HG}$	Transitive property
But	$\overline{AB} \parallel \overline{HG}$	Lying on two parallel lines
$\therefore$	$ABGH$ is a parallelogram	Opposite sides are parallel and congruent.
Hence	Area of parallelogram $ABCD$ = Area of parallelogram $ABGH$	
Also	Area of parallelogram $ABGH$ = Area of parallelogram $AFGH$	
$\therefore$	Area of parallelogram $ABCD$ = Area of parallelogram $EFGH$	By transitive property

THEOREM 16.3

Triangles on the same bas and of the same altitude are equal in area.

Given:
$\triangle ABC$ and $\triangle ABH$ have the same base $\overline{AB}$ and area of the same altitude.

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To Prove:

Area of $\triangle ABC$ = Area of $\triangle ABH$
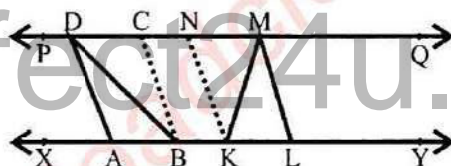
Construction:

Draw $\overline{AD} \parallel \overline{BC}$ and $\overline{BG} \parallel \overline{AH}$ to cut $\overline{PQ}$ at D and G.

Proof:

Statements	Reasons
$ABCD$ & $ABGH$ are parallelograms	Opposite sides are parallel
$\therefore$ Area of parallelogram $ABCD$ = Area of parallelogram $ABGH$	
But Area of $\triangle ABC = \dfrac{1}{2}$ (Area of parallelogram $ABCD$)	Diagonal $\overline{AC}$ bisects parallelogram
Similarly	
Area of $\triangle ABH = \dfrac{1}{2}$ (Area of parallelogram $ABGH$)	Diagonal $\overline{BH}$ bisects parallelogram
Hence Area of $\triangle ABC$ = Area of $\triangle ABH$	

THEOREM 16.4

Triangles on equal bases and of the same altitude area equal in area.

Given:

$\triangle ABD$ and $\triangle KLM$ are between the same parallel lines $\overleftrightarrow{PQ}$ and $\overleftrightarrow{XY}$. Also $\overline{AB} \cong \overline{KL}$

To Prove: Area of $\triangle ABD$ = Area of $\triangle KLM$

Construction: Draw $\overline{BC} \parallel \overline{AD}$ and $\overline{KN} \parallel \overline{LM}$ to intersect $\overleftrightarrow{PQ}$ at C and N.

Proof:

Statements	Reasons
$ABCD$ & $KLMN$ are parallelograms	Opposite sides are parallel
$\because \overline{AB} \cong \overline{KL}$	Given
Both the parallelogram are having the same altitude	Given
$\therefore$ Area of parallelogram $ABCD$ = Area of parallelogram $KLMN$	Diagonal bisects the parallelogram
But Area of $\triangle ABD = \dfrac{1}{2}$ Area of parallelogram $ABCD$	Diagonal bisects the parallelogram
Area of $\triangle KLM = \dfrac{1}{2}$ Area of parallelogram $KLMN$	
Hence Area of $\triangle ABD$ = Area of $\triangle KLM$	

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EXERCISE 16.1

Q1: In $\triangle ABC$, $\overline{PQ}$ is drawn parallel to $\overline{BC}$, cutting $\overline{AB}$ and $\overline{AC}$ at point P and Q respectively. $\overline{BQ}$ and $\overline{CP}$ are drawn to meet at R. Prove that the following are equal in areas.

i)	$\triangle PBC$ and $\triangle QBC$	ii)	$\triangle BRP$ and $\triangle CRQ$
iii)	$\triangle PQB$ and $\triangle PQC$	iv)	$\triangle ABQ$ and $\triangle ACP$

Solution:
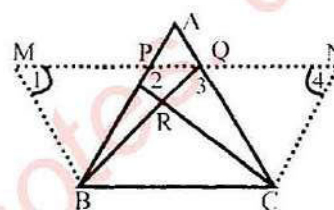
i) $\triangle PBC$ and $\triangle QBC$

Given:

In $\triangle ABC$, $\overline{PQ} \parallel \overline{BC}$

Construction:

Draw $\overline{BM} \parallel \overline{CP}$ and $\overline{CN} \parallel \overline{BQ}$

Now draw $\overline{MP} \parallel \overline{BC}$ and $\overline{QN} \parallel \overline{BC}$

Proof:

Statements	Reasons
$BCMP$ is $\parallel$gm	Construction
$BCNQ$ is $\parallel$gm	Construction
In $\parallel$gm $BCPM \leftrightarrow \parallel$gm $BCNQ$	
$\quad BCPM \cong BCNQ$	Base $\overline{BC}$ and altitude of both $\parallel$gm have same
In $\triangle MBP \longleftrightarrow \triangle NCQ$	
$\quad \overline{BM} \cong \overline{CN}$	Corresponding sides of $\parallel$gm
$\quad \angle 1 \cong \angle 4$	Corresponding sides of $\parallel$gm
$\quad \overline{MP} \cong \overline{QN}$	Both $\cong$ to the same base
$\therefore \quad \triangle MBP \cong \triangle NCQ$	(S.A.S $\cong$ S.A.S)
$\parallel$gm $BCPM - \triangle MBP \cong \triangle NCQ$	Subtracting $\triangle s$ from $\parallel$gms
i) $\quad \triangle PBC \cong \triangle QBC$	
Area of $\triangle PBC \cong$ Area of $\triangle QBC$	
$\quad \triangle BRC \cong \triangle BRC$	Self congruent
$\quad \triangle BRC \cong \triangle CRQ$	
ii) $\therefore \triangle BRP \cong \triangle CRQ$	
In $\triangle PQB \longleftrightarrow \triangle QPC$	
$\quad \overline{PQ} \cong \overline{PQ}$	Corresponding sides of triangles
$\quad \overline{PB} \cong \overline{QC}$	Corresponding sides of $\parallel$gms
iii) $\quad \triangle PQB \cong \triangle QPC$	S.S.S $\cong$ S.S.S
$\quad \triangle ABC - \triangle PBC \cong \triangle ABC - \triangle QBC$	
iv) $\therefore \triangle ABQ \cong \triangle APC$	$\triangle ABQ \cong \triangle APC$

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Q2: *PQRS* is a parallelogram. *A* and *B* are the midpoints of $\overline{PQ}$ and $\overline{PS}$ respectively. **What fraction of the area of the parallelogram is:**
i) ΔPAB ii) ΔQBR iii) ΔBAR

Given:

PQRS is a parallelogram.

A and *B* are the midpoints of $\overline{PQ}$ and $\overline{PS}$ respectively.

Construction: Join *B* with midpoint of $\overline{QR}$.
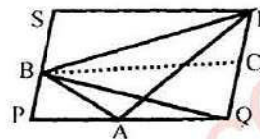
Proof:

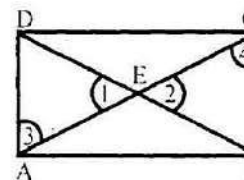| Statements	Reasons
In $\quad \Delta PQB \longleftrightarrow \Delta SRB$	
$\overline{PB} \cong \overline{SB}$	*B* is midpoint of $\overline{PS}$
$\overline{PQ} \cong \overline{SR}$	Corresponding sides of parallelogram
$\overline{QB} \cong \overline{RB}$	Having same altitude
$\Delta PQB \cong \Delta SRB$	S.S.S $\cong$ S.S.S
$\Delta PQB \cong \Delta SRB \cong \Delta BQC \cong \Delta BCR$	Both have equal base
$\Delta PAB = \dfrac{1}{2}\Delta PQB$	
$\Delta PAB = \dfrac{1}{2} \times \dfrac{1}{4} \parallel \text{gm}$	Product of fraction
$= \dfrac{1}{8} \text{ area of } \parallel \text{gm } PQRS$	
$\Delta BQR = \Delta BQC + \Delta BCR$	
$= \dfrac{1}{4} \text{ area of } \parallel \text{gm } PQRS +$	
$\dfrac{1}{4} \text{ area of } \parallel \text{gm } PQRS$	
$= \dfrac{1}{2} \text{ area of } \parallel \text{gm } PQRS$	

Q3: *ABCD* is a quadrilateral. The diagonals $\overline{AC}$ and $\overline{BD}$ meet at *E*. If ΔABE and ΔCDE are equal in area then prove that $\overline{AD} \parallel \overline{BC}$.

Given:

$\overline{AC}$ and $\overline{BD}$ are diagonals of a quadrilateral *ABCD* meet at *E*.

Construction:

$\overline{AD} \parallel \overline{BC}$

Proof:

Statements	Reasons
ΔABE and ΔCDE have equal area	Given
$\Delta ABE \cong \Delta CDE$	Both have equal area

$\overline{AB} \cong \overline{CE}$	Corresponding sides of $\cong \triangle$
$\overline{BE} \cong \overline{DE}$	Corresponding sides of $\cong \triangle$
In $\triangle AED \longleftrightarrow \triangle CEB$	
$\overline{AE} \cong \overline{CE}$	Proved
$\angle 1 \cong \angle 2$	Vertical angles
$\overline{BE} \cong \overline{DE}$	Proved
$\therefore \quad \triangle AED \cong \triangle CEB$	(S.A.S $\cong$ S.A.S)
So $\quad \angle 3 \cong \angle 4$	Corresponding angles of $\cong \triangle s$
Hence $\overline{AD} \parallel \overline{BC}$	($\because \angle 3, \angle 4$ are alternate angles)

Q4: How many tiles, each 8 inches square, will be required to pave a rectangular space 18×30 feet?

Solution:

Area of each tile $= 8$ square inch

or $\qquad = \dfrac{8}{12}$ square foot

As $\quad 12$ inch $= 1$ foot

So $\quad 1$ inch $= \dfrac{1}{12}$ foot

Rectangular space $= 18 \times 30$ feet

Hence total number of tiles $= \dfrac{\overset{2}{\cancel{8}}}{\underset{\cancel{2}}{\cancel{12}}} \times 18 \times \overset{10}{\cancel{30}} = 360$ Ans.

REVIEW EXERCISE 16

Q1: Circle the correct answers:

i) Perpendicular distance between two lines is the same. The lines are:
   (a) Perpendicular to each other   ✓ **(b) Parallel to each other**
   (c) Intersecting   (d) None of the above

ii) If two triangles have equal area then they will……be congruent as well.
   ✓ **(a) Not necessarily**   (b) Necessarily
   (c) Definitely   (d) None of the above

iii) Perpendicular from a vertex of a triangle to its opposite side is called…
   (a) Median   (b) Perpendicular bisector
   ✓ **(c) Altitude**   (d) Angle bisector

iv) Parallelogram having same base and same altitude are:
   (a) Congruent   (b) Equal in area
   (c) Similar   ✓ **(d) All of the above**

v) Two parallelograms have equal bases. They will be having the same area if……
   (a) Their altitudes are equal   (b) Their altitude is the same
   (c) They lie between the same parallel lines
   ✓ **(d) All of the above**

vi) If two triangles have equal bases and equal altitudes, what else will they have equal?

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✓ (a) Area   (b) Perimeter   (c) Size   (d) Angles

vii) Suppose a triangle has a base length of 4 feet and a height of 4 feet. Its interior area is:

(a) 4 square feet   ✓ (b) 8 square feet

(c) 16 square feet   (d) Impossible to determine without more information

viii) Suppose a square has a diagonal measure of 10 units. The area of the square is:

(a) 25 square feet   ✓ (b) 50 square feet

(c) 100 square feet   (d) Impossible to determine without more information

ix) Find the area of a triangle with base $\frac{15}{4}$ inches and altitude $\frac{8}{5}$ inches.

(a) 2 sq. inches   (b) 6 sq. inches

✓ (c) 3 sq. inches   (d) 4.35 sq. inches

Q2: Prove that a median of a triangle divides it into two parts of equal area.

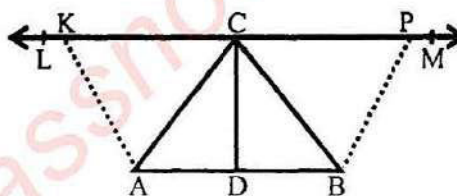**Given:** In $\triangle ABC$, $\overline{CD}$ is a median.

To Prove: $\triangle ADC$ and $\triangle DBC$ have equal area.

Construction:

Draw $\overleftrightarrow{LM} \parallel \overline{AB}$. Join A to K and B to P such that

$\overline{AK} \parallel \overline{BC}$ and $\overline{BP} \parallel \overline{AC}$.

Proof:

Statements	Reasons
In $\triangle DPC \longleftrightarrow \triangle DCK$	
$\overline{PC} \cong \overline{KC}$	Each is equal to base $\overline{AB}$
$\overline{CD} \cong \overline{CD}$	Self congruent
$\overline{BD} \cong \overline{AD}$	Given
$\overline{BP} \cong \overline{AK}$	Corresponding sides
$\therefore \quad DBPC \cong ADCK$	All corresponding sides are equal.
$\therefore$ Area $DBPC \cong$ Area $ADCK$	

Hence $\triangle ADC$ and $\triangle DBC$ have equal area.

Q3: P is any point on the median $\overline{AD}$ of $\triangle ABC$. Prove that $\triangle ABP$ and $\triangle ACP$ are equal in area.

Given: P is any point on the median $\overline{AD}$ of $\triangle ABC$.

To Prove: Area of $\triangle ACP$ = Area of $\triangle ABP$

Proof:

Statements	Reasons
$m\overline{BD} = m\overline{CD}$	Given
Area of $\triangle ACD$ = Area of $\triangle ABD \rightarrow$ (1)	
Similarly,	
Area of $\triangle PDC$ = Area of $\triangle PBD \rightarrow$ (2)	
Area of $\triangle ACP$ = Area of $\triangle ABD$	From (1) and (2)
	By transitive property

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Additional MCQs of Unit 16:
Theorems Related with Area

1. The area of two parallelograms will be...........if they have same base and parallel lines.
   (a) Maximum   (b) Equal   (c) Minimum   (d) none
   ✓ Ans. (b) Equal

2. The two parallelograms have equal area if they have equal bases and same.........
   (a) Altitude   (b) Angle   (c) Sides   (d) none
   ✓ Ans. (a) Altitude

3. The two lines will be parallel if the perpendicular distance between is...........
   (a) Less   (b) Greater   (c) Same   (d) none
   ✓ Ans. (c) Same

4. A line from the opposite vertex which is perpendicular is called...........
   (a) Altitude   (b) Median   (c) Bisector   (d) none
   ✓ Ans. (a) Altitude

5. If base = 4cm and altitude = 5 then area of $\triangle ABC$ = ...
   (a) 15   (b) 25   (c) 10   (d) 20
   ✓ Ans. (c) 10

6. If the diagonal of a square is 6cm then area of a square is...........
   (a) $18 cm^2$ (b) $20 cm^2$ (c) $30 cm^2$ (d) none
   ✓ Ans. (a) $18 cm^2$

7. Area of rectangle is...........if length is 4cm and width 5cm.
   (a) $15 cm^2$ (b) $25 cm^2$ (c) $20 cm^2$ (d) $30 cm^2$
   ✓ Ans. (c) $20 cm^2$

8. Perimeter of rectangle having length 6cm and width 4cm is...........
   (a) $20 cm$ (b) $30 cm$ (c) $15 cm$ (d) $25 cm$
   ✓ Ans. (a) $20 cm$

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