UNIT 15:
PYTHAGORAS THEOREM

Definition:

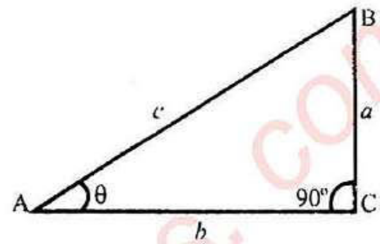
That triangle in which one angle is of 90^o is called right angle triangle.

In ΔABC, $m\angle C = 90^o$

$\overline{AC}$ is hypotenuse, because the side opposite to right angle is hypotenuse.

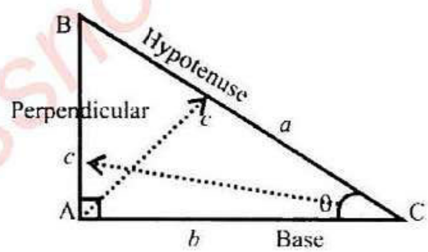
$\overline{BC}$ is perpendicular, because the side opposite to angle θ is perpendicular.

$\overline{AC}$ is the base of this triangle ABC.



One of the most famous theorem in mathematics is the Pythagoras theorem, named for the ancient Greek mathematician Pythagoras (around 500 BC). This theorem can be used to find information about the lengths of the sides of a right angle triangle.

The following figure shows to right – angle triangle with $BAC = 90^o$. The side opposite the right angle A is called the hypotenuse it is the longest side of a right angled triangle.



THEOREM 15.1
PYTHAGORAS' THEOREM

Statement: In a right – angled triangle, the square of the length of hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

Given:

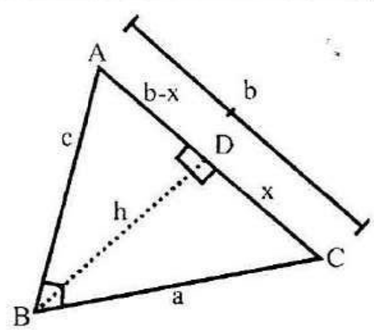
ABC is a right – angled triangle, having right angle at B, a, b and c are the measure of the sides $\overline{BC}$, and $\overline{CA}$ and $\overline{AB}$ respectively.

To Prove:

$b^2 = c^2 + a^2$

Construction:

Draw $\overline{BD} \perp \overline{AC}$. Let $\overline{BD} = h$ and $m\overline{CD} = x$. Therefore $m\overline{AD} = b - x$.



Proof:

Statement	Reasons
In $\Delta ABC \longleftrightarrow \Delta EDC$	
$\angle ABC \cong \Delta BDC$	Right angle,
$\angle C \cong \angle C$	Common (Self – congruent)
And $\angle CAB \cong \angle CBD$	Complement of $\angle C$

Statements	Reasons
$\therefore \quad \Delta ABC \sim \Delta BDC$	By definition
Hence $\frac{b}{a} = \frac{a}{x}$	Corresponding sides of similar triangles
i.e. $\boxed{bx = a^2}$ 63	
Again in $\Delta ABC \longleftrightarrow \Delta ADB$	
$\angle ABC \cong \angle ADB$	Right angles
$\angle A \cong \angle A$	Common / self – congruent
And $\angle BCA \cong \angle DBA$	Complement of $\angle A$
$\therefore \quad \Delta ABC \sim \Delta ADB$	By definition
Hence $\frac{b}{c} = \frac{c}{b-x}$	Corresponding sides of similar triangles
i.e. $b(b-x) = c^2$	Or $b^2 - bx = c^2$ ....(ii)
Adding equation (i) and (ii), we get	
$b^2 - bx + bx = c^2 + a^1$	
or $b^2 = c^2 + a^2$	

EXAMPLE 1

Find the length of the hypotenuse of the right-angle triangle whose one length is 8cm and other length is 6cm.

Solution:

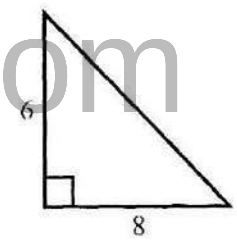
$(Hypotenuse)^2 = (Perpendicular)^2 + (Base)^2$

$x^2 = (8cm)^2 + (6cm)^2$

$x^2 = 64cm^2 + 36cm^2$

$x^2 = 100cm^2$

$x = 10cm$



EXAMPLE 2

The top of the ladder rests against wall 23 feet above the ground. The base of the ladder is 6 feet away from the wall. What is length of the ladder?

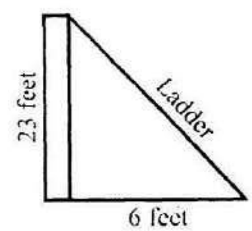
Solution:"

(Length of ladder)^2 = (23 feet)^2 + (6 feet)^2

$x^2 + 529$ feet^2 +36 feet^2

$x^2 = 565$ feet^2

$x = 23.76$ feet



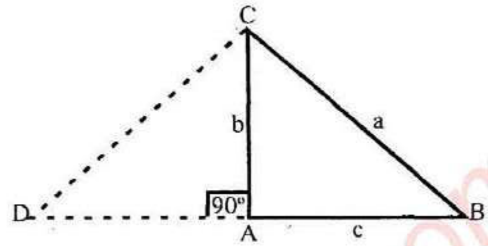
THEOREM 15.2

Converse of Pythagoras' theorem:

If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the triangle is a right-angled triangle.

Given:

In ΔABC, measure of $\overline{BC}, \overline{CX}$ and $\overline{AB}$ are a, b and c respectively. Also $a^2 = b^2 + c^2$.



To Prove:

ΔABC is a right triangle i.e. $\angle CAB = 90^o$

Construction:

Draw $\overline{DA} \perp \overline{CA}$ see with at $DA \cong BA$. Join D to C.

Proof:

Statement	Reasons
ACD is a right – angled triangle	$\because (m\angle CAD = 90^o)$
$\therefore \quad (m\overline{CD})^2 = (m\overline{AD})^2 + (m\overline{AC})^2$	By Pythagoras' theorem
or $\quad m(\overline{CD})^2 = c^2 + b^2 \rightarrow (1)$	$(\because m\overline{DA} = m\overline{BA} = C)$
But $\quad a^2 = b^2 + c^2 \rightarrow (2)$	Given
$\therefore \quad (m\overline{CD})^2 = a^2$	From (1) and (2)
or $\quad m\overline{CD} = a = m\overline{CD}$	
Now in $\Delta DAC \longleftrightarrow \Delta BAC$	
$\overline{CD} = \overline{CB}$	Given
$\overline{CA} = \overline{CA}$	Common
$\overline{DA} = \overline{BA}$	Construction
$\therefore \quad \Delta DAC \cong \Delta BAC$	(S.S.S ≅ S.S.S)
Hence $\angle CAD = \angle CAB$	Corresponding angles
But $\quad m\angle CAD = 90^o$	Construction
$\therefore \quad m\angle CAB = 90^o$	
or ΔABC is a right – triangle.	

EXAMPLE 3

Is the triangle whose sides are 3cm, 4cm and 5cm is a right-angle triangle?

Solution:

A triangle is a right angled if the square of the longest side is equal to the sum of the squares of the other two sides. (Converse of Pythagoras theorem)

$$(5cm)^2 = (3cm)^2 + (4cm)^2$$
$$25cm^2 = 9cm^2 + 16cm^2$$
$$25cm^2 = 25cm^2$$

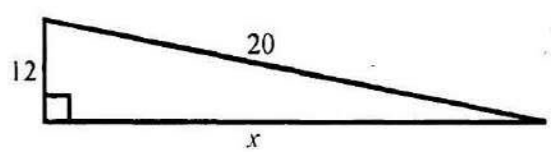
Therefore, the given triangle is a right-angled triangle.

EXAMPLE 4

Calculate the value of x.

Solution:

$$12^2 + x^2 = 20^2 \qquad \Rightarrow x^2 = 20^2 - 12^2$$
$$= 400 - 144 = 256$$
$$x = \sqrt{256} \qquad \Rightarrow x = 16$$



EXERCISE 15.1

Q1: Find the hypotenuse of right – angled triangle when the sides containing the right angle are:

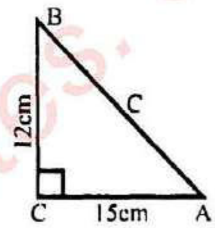
i) 12cm and 5cm **ii) 6cm and 2.5cm**
iii) 15cm and 8cm

Solution:

i) 12cm and 5cm

Let the hypotenuse is c and the sides containing the right angle are a and b respectively

$a = 12cm,\ b = 15cm,\ c = ?$



By Pythagoras theorem,

$$c^2 = a^2 + b^2 \Rightarrow c^2 = (12)^2 + (5)^2$$

$$c^2 = 144 + 25 \Rightarrow c^2 = 169$$

$$\sqrt{c^2} = \sqrt{169} \Rightarrow \boxed{c = 13cm} \quad \text{Ans.}$$

ii) 6cm and 2.5cm

Let the hypotenuse is c and the sides containing the right angle are a and b respectively

$a = 6cm,\ b = 2.5cm,\ c = ?$

By Pythagoras theorem,

$$c^2 = a^2 + b^2 \Rightarrow c^2 = (6)^2 + (2.5)^2$$

$$c^2 = 36 + 6.25 \Rightarrow c^2 = 42.25$$

$$\sqrt{c^2} = \sqrt{42.25} \Rightarrow \boxed{c = 6.5cm} \quad \text{Ans.}$$

iii) 15cm and 8cm

Let $a = 15cm,\ b = 8cm,\ c = ?$

By Pythagoras theorem,

$$c^2 + a^2 + b^2 \Rightarrow c^2 = (15)^2 + (8)^2$$

$$c^2 = 225 + 64 \Rightarrow c^2 = 289$$

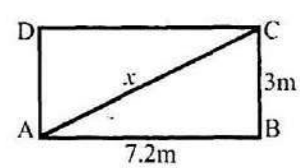
$$\sqrt{c^2} = \sqrt{289} \Rightarrow \boxed{c = 17cm} \quad \text{Ans.}$$

Q2: Find the diagonal of a rectangular field whose sides are:

i) 7.2m, 3m **ii) 15.6m and 6.5m**
iii) 6.72m, 5.04m

Solution:

i) 7.2m, 3m



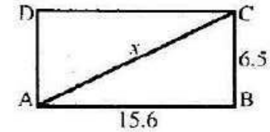
ΔABC is right triangle,

So $x^2 = (3)^2 + (7.2)^2 = 9 + 51.84$

$$x^2 = 60.84 \Rightarrow \sqrt{x^2} = \sqrt{60.84}$$

$$x^2 = \sqrt{60.84m} = 7.8m \quad \text{Ans.}$$

ii) 15.6m and 6.5m

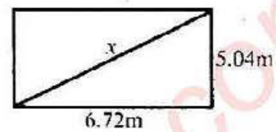


In ΔABC,

$$x^2 = (15.6)^2 + (6.5)^2 = 285.61$$

$$x = \sqrt{285.61} = 16.9m \quad \text{Ans.}$$

iii) 6.72m, 5.04m

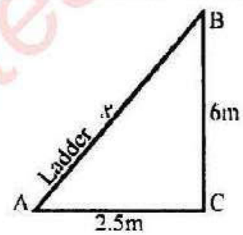


$$x^2 = (6.72)^2 + (5.64)^2$$

$$x^2 = 45.158 + 25.4016 = 70.5596$$

$$x = \sqrt{70.5596} = 8.399 = 8.40m \quad \text{Ans.}$$

Q3: A ladder whose foot is 2.5m from the front of a house reaches a window 6m above the ground. Calculate the length of the ladder.



<u>Solution</u>: By Pythagoras theorem

$$(AB)^2 = (AC)^2 + (BC)^2$$

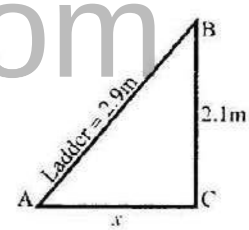
$$x^2 = (2.5)^2 + (6)^2$$

$$x^2 = 6.25 + 36 = 42.25$$

$$x = \sqrt{42.25} = 6.5m$$

Hence length of ladder $= 6.5m$ Ans.

Q4: A ladder 2.9m long just reaches the top of a wall 2.1m high. How far from the foot of the wall is the foot of the ladder.



<u>Solution</u>: By Pythagoras theorem

$$(AB)^2 = (AC)^2 + (BC)^2$$

$$(2.9)^2 = x^2 + (2.1)^2$$

$$x^2 = (2.9)^2 - (2.1)^2$$

$$x^2 = 8.41 - 4.41 = 4$$

$$\sqrt{x^2} = \sqrt{4} \quad \Rightarrow x = 2m \quad \text{Ans.}$$

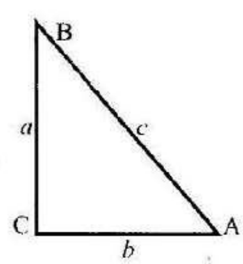
Q5: Find the length of the unknown side.

i) $a = 4cm, 6 = 7m$ **ii)** $a = 7cm, c = 25cm$

iii) $b = 6cm, \ c = 9cm$

<u>Solution</u>:

i) $a = 4cm, 6 = 7m$



By Pythagoras theorem,

$$c^2 = a^2 + b^2 = (4)^2 + (7)^2$$

$$c^2 = 16 + 49 = 65 \quad \Rightarrow \sqrt{c^2} = \sqrt{65}$$

$$\Rightarrow c = 8.062cm \quad \text{Ans.}$$

ii) $a = 7cm, c = 25cm$

$$c^2 = a^2 + b^2 \quad \Rightarrow b^2 = c^2 - a^2$$

$$b^2 = (25)^2 - (7)^2 = 625 - 49 = 576$$

$$\sqrt{b^2} = \sqrt{576} \Rightarrow \boxed{b = 24cm} \quad \text{Ans.}$$

iii) $b = 6cm,\ c = 9cm$

$c^2 = a^2 + b^2 \quad \Rightarrow a^2 = c^2 - b^2$

$a^2 = (9)^2 - (6)^2 = 81 - 36$

$a^2 = 45 \quad \Rightarrow \sqrt{a^2} = \sqrt{45} \Rightarrow \boxed{a = 6.7cm}$ Ans.

Q6: Verify whether a triangle with the given side lengths is a right-angled triangle?

i) 9, 12 and 15 **ii) 9, 10 and 15**

Solution:

i) 9, 12 and 15

Let $a = 9,\ b = 12,\ c = 15$

As $c^2 = a^2 + b^2 \quad \Rightarrow \quad (15)^2 = (9)^2 + (12)^2$

$225 = 81 + 144 = 225$

L.H.S = R.H.S so 9, 12, 15

Verified the lengths of right-angle triangle.

ii) 9, 10 and 15

Let $a = 9,\ b = 10,\ c = 15$

As $c^2 = a^2 + b^2 \quad \Rightarrow \quad (15)^2 = (9)^2 + (10)^2$

$225 = 81 + 100 = 181$

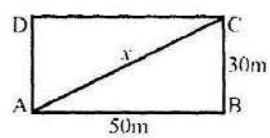
L.H.S $\neq$ R.H.S

Hence 9, 10, 15 are not the sides of the right-angle triangle.

Q7: The sides of a rectangular swimming pool are 50m and 30m. What is the length between the opposite corners?

Solution: In ΔABC,

By Pythagoras theorem,



$(AC)^2 = (AB)^2 + (BC)^2$

$\Rightarrow x^2 = 3400$ (Taking square root)

$x^2 = (50)^2 + (30)^2 = 2500 + 900 = 3400$

$\sqrt{x^2} = \sqrt{3400} \quad \Rightarrow \quad x = \sqrt{34 \times 100} = 10\sqrt{34}m$

REVIEW EXERCISE 15

Q1: Circle the correct answers:

i) Diagonal of a rectangle 6.5cm. If its width is 2.5cm, its length is:

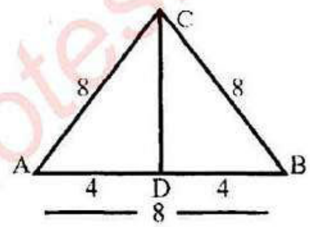
(a) 4cm (b) 9cm ✓**(c) 6cm** (d) 3cm

ii) Which of the following are the sides of a right-angled triangle?

(a) 2, 3, 4 ✓**(b) 3, 4, 5** (c) 4, 5, 6 (d) 5, 6, 7

Q2: Measure of each of the sides of an equilateral triangle is 8cm. Find the length of the altitude.

Solution: In ΔACD,



$AC = 8,\ AD = 4$

$(AC)^2 = (AD)^2 + (CD)^2$

$(8)^2 = (4)^2 + (CD)^2$

$64 = 16 + (CD)^2$

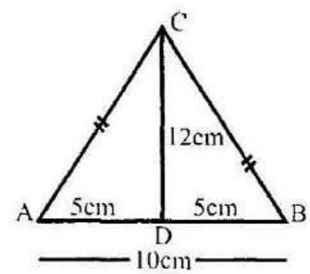
$64 - 16 = (CD)^2 \quad \Rightarrow (CD)^2 = 48$

$\sqrt{(CD)^2} = \sqrt{48} \quad \Rightarrow CD = \sqrt{16 \times 3}$

$\overline{CD} = 4\sqrt{3}cm$ Ans.

Q3: Measure of the base of an isosceles triangle is 10cm. If the perpendicular drawn from the vertex to the base is 12cm long, find the measures of the congruent sides of the triangle.

Solution: In ΔACD,



$AD = 5,\ CD = 12$

$(AC)^2 = (AD)^2 + (CD)^2$

$(AC)^2 = (5)^2 + (12)^2 = 25 + 144 = 169$

$(AC)^2 = 169$

$\sqrt{(AC)^2} = \sqrt{169} \Rightarrow \overline{AC} = 13cm$

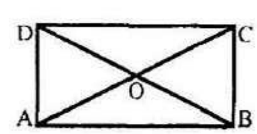
Hence $\overline{AC} = \overline{BC} = 13cm$ Ans.

Q4: Diagonals of a quadrilateral *ABCD* are perpendicular to each other. Prove that $(m\overline{AB})^2 + (m\overline{CD})^2 = (m\overline{AD})^2 + (m\overline{BC})^2$.

Solution:

Given: Quadrilateral $ABCD$ in which $\overline{AC}$ and $\overline{BD}$ are diagonals such that $\overline{AC} \perp \overline{BD}$. So ABC and ACD are the two triangles.

To Prove:



$(mAB)^2 + (mCD)^2 = (mAD)^2 + (mBC)^2$

Proof: $m\angle AOB = m\angle BOC = m\angle COD = m\angle AOD = 90^o$

Because $\overline{AC} \perp \overline{BD}$ (Given)

So ΔAOB becomes right angle triangle from Pythagoras' theorem.

$$(mAB)^2 = (mAO)^2 + (mOC)^2 \longrightarrow (1)$$

In ΔCOD, the Pythagoras' theorem is,

$$(mDC)^2 = (mDO)^2 + (mOC)^2 \longrightarrow (2)$$

Add equation (1) and equation (2), we get,

$$m(AB)^2 + m(DC)^2 = m(AO)^2 + m(OB)^2 + m(DO)^2 + (mOC)^2 \longrightarrow (3)$$

Similarly in ΔBOC,

$$(mBC)^2 = (mBO)^2 + (mOC)^2 \longrightarrow (4)$$

In ΔAOD, $(mAD)^2 = (mAO)^2 + (mOD)^2 \longrightarrow (5)$

Add equation (4) and equation (5)

$$(mAD)^2 + (mBC)^2 = (mAO)^2 + (mOD)^2 + (mBO)^2 + (mOC)^2 \longrightarrow (6)$$

From equation (5) and equation (6), we have

$$(mAB)^2 + (mDC)^2 = (mAD)^2 + (BC)^2$$

By transitive property

Q5: The sides of a triangle have length *x*, *x* + 4, 20. If the length of the largest side is 20. What value of *x* make the right triangle?

Solution: From the figure, $BC = x + 4$, $AB = x$ and $AC = 20$

Using Pythagoras' theorem on ΔABC,

$$(BC)^2 + (AB)^2 = (AC)^2$$

$$(x+4)^2 + (x)^2 = (20)^2 \quad \Rightarrow \quad x^2 + 8x + 16 + x^2 = 400$$

$$2x^2 + 8x + 16 - 400 = 0 \quad \Rightarrow \quad 2x^2 + 8x - 384 = 0$$

$$a = 2,\ b = 8,\ c = -384$$

Use quadratic formula

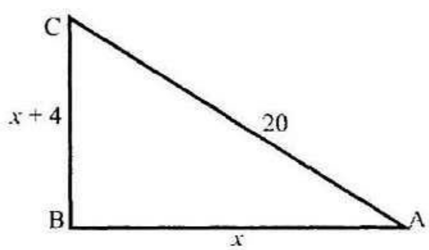
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-8 \pm \sqrt{64 - 4(2)(-384)}}{4}$$

$$x = \frac{-8 \pm \sqrt{3136}}{4} = \frac{-8 \pm 56}{4}$$

$$x = \frac{-8 + 56}{4} = 12 \text{ and } x = \frac{-8 - 56}{4} = -16$$

$\therefore$ $\boxed{x = 12, -16}$ Ans.



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Additional MCQs of Unit 15:
Pythagoras' Theorem

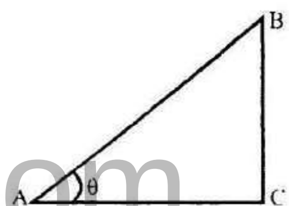
1. The square of hypotenuse is equal to the sum of the square of base and perpendicular is..........theorem.
(a) Pythagoras (b) Factor (c) Remainder (d) none
✓ **Ans. (a) Pythagoras**

2. Pythagoras was..........mathematician in 500 B.C.
(a) American (b) German (c) Greek (d) none
✓ **Ans. (c) Greek**

3. The side opposite to the right angle of ΔABC is..........
(a) Base (b) Hypotenuse (c) Perpendicular (d) none
✓ **Ans. (c) Greek**

4. The longest side of a right angled triangle is..........
(a) Base (b) Perpendicular (c) Hypotenuse (d) none
✓ **Ans. (c) Hypotenuse**

5. In the figure the side opposite to angle θ is called..........
(a) Perpendicular (b) Base
(c) Hypotenuse (d) none
✓ **Ans. (a) Perpendicular**



6. If one length is 8cm and other m then length of hypotenuse is..........
(a) 15cm (b) 20cm (c) 30cm (d) 10cm
✓ **Ans. (d) 10cm**

7. The height of wall is 4cm and the base of ladder 3cm then length of the ladders..........
(a) 6cm (b) 4cm (c) 5cm (d) 10cm
✓ **Ans. (c) 5cm**

8. If $c^2 = a^2 + b^2$ and $m\angle C = 90^o$ then ΔABC is..........triangle.
(a) Right (b) Acute (c) Abtuse (d) none
✓ **Ans. (a) Right**

9. If the sides of triangle are 3cm, 4cm, 5cm then the triangle..........
(a) Acute (b) Abtuse (c) Right angle
✓ **Ans. (c) Right angle**

10. In the figure the value of x is..........
(a) 25 (b) 5
(c) 16 (d) 20
✓ **Ans. (c) 16**