UNIT 14:
RATIO & PROPORTION

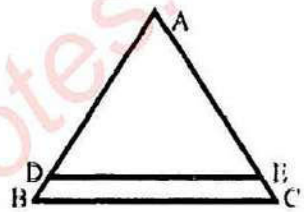
When we divide two similar quantities on each other then it is called ratio of the quantities. For example,

$$\frac{\overline{AB}}{\overline{CD}} = \overline{AB}:\overline{CD} \text{ and } \frac{\overline{CD}}{\overline{AC}} = \overline{CD}:\overline{AC}$$

The quantity between the two ratios of the sides of a triangle is called proportion i.e.

$$\frac{\overline{AB}}{\overline{CD}} = \frac{\overline{CD}}{\overline{AC}}$$

This represents proportion.



If in ΔABC

$$\frac{m\overline{AD}}{m\overline{DB}} = \frac{m\overline{AE}}{m\overline{EC}},$$

Then $\frac{m\overline{AB}}{m\overline{DB}} = \frac{m\overline{AC}}{m\overline{EC}}$ (Componendo property)

And $\frac{m\overline{AD}}{m\overline{AB}} = \frac{m\overline{AE}}{m\overline{AC}}$ (Componendo and invertendo property)

If in a correspondence $\Delta ABC \longleftrightarrow \Delta DEF$, the correspondence angles are congruent then the triangles are similar i.e. $\Delta ABC \sim \Delta DEF$.
Correspondence in this case is called *similarity*.

THEOREM 14.1

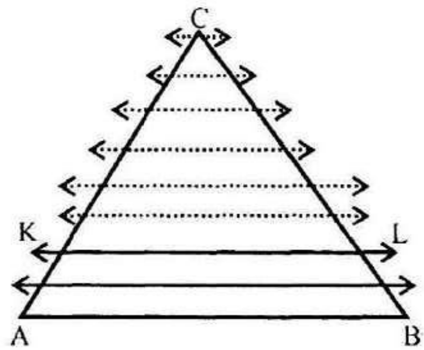
A line parallel to one side of a triangle, intersecting the other two sides, divides them proportionally.

Given:

A triangle ABC in which $\overline{KL} \parallel \overline{AB}$. $\overline{KL}$ meets $\overline{CA}$ and $\overline{CB}$ at distinct points K and L respectively.

To Prove:

$$\frac{m\overline{CK}}{m\overline{KA}} = \frac{m\overline{CL}}{m\overline{LB}}$$



Construction:

Suppose the unit of length is taken such that $m\overline{CK} = p$ and $m\overline{KA} = q$ units (where p and q are whole numbers, $q \neq 0$)

i.e. $\frac{m\overline{CK}}{m\overline{KA}} = \frac{p}{q}$

divide $\overline{CK}$ into p congruent segments and $\overline{KA}$ into q congruent segments. From each dividing point, draw lines parallel to $\overline{KL}$.

Proof:

Statement	Reasons
The parallel lines divides $\overline{CA}$ into $(p+q)$ congruent segments	Construction
$\overline{CB}$ is another transverse which cuts these parallel lines.	
$\therefore$ These parallel lines divide $\overline{CB}$ also into $(p+q)$ congruent segments.	Same number of congruent segments are intercepted on any other transversal.
i.e. $\overline{CL}$ is divided into p congruent and $\overline{LB}$ is divided into q congruent segments.	
Suppose the length of each congruent segments is 'a' unit.	
$\therefore \quad \dfrac{m\overline{CL}}{m\overline{LB}} = \dfrac{a \times p}{a \times q}$	
Or $\quad \dfrac{m\overline{CL}}{m\overline{LB}} = \dfrac{p}{q} \longrightarrow (i)$	Construction
But $\quad \dfrac{m\overline{CK}}{m\overline{KA}} = \dfrac{p}{q} \longrightarrow (ii)$	
$\therefore \quad \dfrac{m\overline{CK}}{m\overline{KA}} = \dfrac{m\overline{CL}}{m\overline{CB}}$ (from (i) & (ii))	From (i) and (ii)

THEOREM 14.2

If a line segment intersects the two sides of a triangle in the same ratio then it is parallel to the third side.

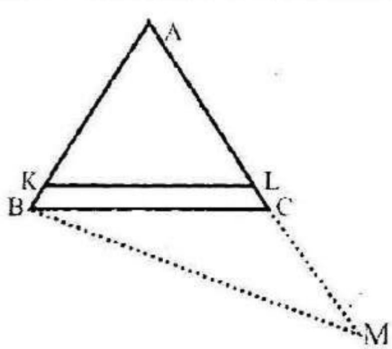
Given:

$\overline{KL}$ intersects two sides $\overline{AB}$

And $\overline{AC}$ of the triangle ΔABC

At K and L such that

$$\frac{m\overline{AK}}{m\overline{KB}} = \frac{m\overline{AL}}{m\overline{LC}}$$



To Prove:

$\overline{KL} \parallel \overline{BC}$

Proof:

Statement	Reasons
Suppose $\overline{KL}$ is not parallel to $\overline{BC}$, then let $\overline{BM}$ be drawn through B, parallel to $\overline{KL}$.	

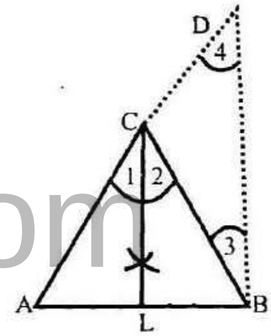
meeting $\overline{AC}$ at M.	
$\therefore \quad \frac{m\overline{AK}}{m\overline{KB}} = \frac{m\overline{AL}}{m\overline{LM}} \longrightarrow (i)$	Proportional segments are cut by line || to one side of the triangle.
But $\frac{m\overline{AK}}{m\overline{KB}} = \frac{m\overline{AL}}{m\overline{LC}} \longrightarrow (ii)$	Given
Thus $\frac{m\overline{AL}}{m\overline{LM}} = \frac{m\overline{AL}}{m\overline{LC}}$	From (i) and (ii)
Or $m\overline{LM} = m\overline{LC}$	Denominators of equal fraction
This is possible if C and M coincide.	

THEOREM 14.3

The internal bisector of an angle of a triangle divides the side opposite to it in the ratio of the lengths of the sides containing the anles.

Given:

In ΔABC, $\overline{CL}$ is the bisector of $\angle C$, which meets the opposite side $\overline{AB}$ at L.



To Prove:

$$\frac{m\overline{AL}}{m\overline{BL}} = \frac{m\overline{AC}}{m\overline{BC}}$$

Construction:

Through B, draw $\overline{BD} \parallel \overline{LC}$ which intersects $\overline{AC}$ at D.

Proof:

Statement	Reasons
$\overline{LC} \parallel \overline{BD}$	Construction
$\therefore \quad \angle 2 \cong \angle 3 \longrightarrow (i)$	Alternate angle
And $\angle 1 \cong \angle 4 \longrightarrow (ii)$	Corresponding angles
But $\angle 1 \cong \angle 2$	Given
$\therefore \quad \angle 3 \cong \angle 4$	Transitive property
Hence $\overline{BC} \cong \overline{DC}$	Opposite sides of congruent angles
Now in ΔABD	Construction
$\overline{CL} \parallel \overline{DB}$	
$\therefore \quad \frac{m\overline{AL}}{m\overline{BL}} = \frac{m\overline{AC}}{m\overline{DC}}$	Proportional segments are cut by line || to one side of the triangles.
Or $\frac{m\overline{AL}}{m\overline{BL}} = \frac{m\overline{AC}}{m\overline{BC}}$	

THEOREM 14.4

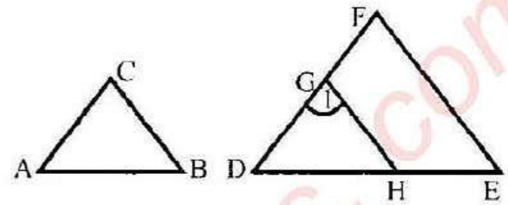
If two triangles are similar, then the measures of their corresponding sides are proportional.

Given:

$\Delta ABC \sim \Delta DEF$

i.e. $\angle A \cong \angle D. \angle B \cong \angle E$

And $\angle C \cong \angle F$



To Prove:

$$\frac{m\overline{AB}}{m\overline{DE}} = \frac{m\overline{BC}}{m\overline{EF}} = \frac{m\overline{AC}}{m\overline{DF}}$$

Construction:

Take point G and H on $\overline{DF}$ and $\overline{DE}$ such that $\overline{AC} \cong \overline{DG}$ and $\overline{AB} \cong \overline{DH}$. Join G to H.

Proof:

Statement	Reasons
In $\Delta ABC \longleftrightarrow \Delta DHG$ GD	
$\overline{AB} \cong \overline{DH}$	Construction
$\overline{AC} \cong \overline{DG}$	Construction
$\angle A \cong \angle D$	Given
$\therefore$ $\Delta ABC \cong \Delta DHG$	(S.A.S ≅ S.A.S)
Hence $\angle C \cong \angle 1$	Corresponding ($\angle$) of congruent triangle
But $\angle C \cong \angle F$	Given
$\therefore$ $\angle 1 \cong \angle F$	Transitive property
But corresponding angles	
$\therefore$ $\overline{GH} \parallel \overline{FE}$	
Hence $\dfrac{m\overline{DG}}{m\overline{DF}} = \dfrac{m\overline{OH}}{m\overline{DE}}$	Remaining sides proportional
Or $\dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{AB}}{m\overline{DE}} \longrightarrow (i)$	$\because$ $\overline{AC} \cong DG$ and $\overline{AB} = \overline{DH}$
Similarly it can be proved that	
$\dfrac{m\overline{AC}}{m\overline{DF}} = \dfrac{m\overline{BC}}{m\overline{EF}} \longrightarrow (ii)$	
Hence $\dfrac{m\overline{AB}}{m\overline{DE}} = \dfrac{m\overline{BC}}{m\overline{EF}} = \dfrac{m\overline{AC}}{m\overline{DF}}$	From (i) and (ii)

EXERCISE 14.1

Q1: The three angles of a triangle are in the ratio 1:2:6. What is the measure of the smallest angle?

Solution:

Let a be the smallest angle then other two angles are $2a$ and $6a$.

$\therefore \quad a + 2a + 6a = 180''$

$$\Rightarrow 9a = 180'' \quad \Rightarrow a = \frac{180''}{9} = 20''$$

Hence $a = 20''$ is the smallest angle.

Q2: The measure of the three angles of a triangle are in the ratio 2:3:4. Find the measure of the largest angle of the triangle.

Solution:

Let the largest angle $= 4a$ then other two angles are $3a$ and $2a$

Since $2a + 3a + 4a = 180''$

$$\Rightarrow 9a = 180'' \quad \Rightarrow a = 20''$$

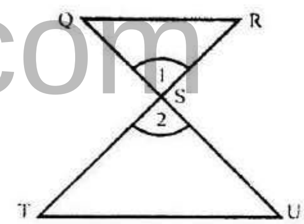
Hence measure of the largest angle is

$$4a = 4(20'') = 80''$$

Q3: In the given figure, show that ΔQRS and ΔTUS are similar.

Solution:

Given ΔQRS and ΔTUS



To Prove:

ΔQRS and ΔTUS are similar.

Proof:

Statement	Reasons
$m\angle 1 = m\angle 2$	Vertical angles
$TU \longleftrightarrow QR$	One – one correspondence
$TS \longleftrightarrow SR$	One – one correspondence
$SU \longleftrightarrow SQ$	One – one correspondence
$m\angle U = m\angle Q$	Opposite angles
$m\angle T = m\angle R$	
Hence ΔQRS and ΔTUS are similar triangles.	

Q4: In the given figure show that $\Delta MNO \sim \Delta PQR$.

Solution:

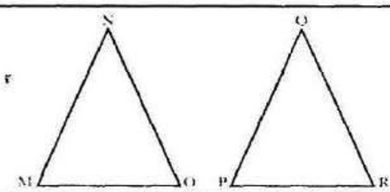
Statement	Reasons
$\overline{MO} \longleftrightarrow \overline{PR}$ $\overline{MN} \longleftrightarrow \overline{PQ}$	Corresponding sides

$\overline{QR} \longleftrightarrow \overline{ON}$

$\angle M \longleftrightarrow \angle P$

$\angle O \longleftrightarrow \angle R$

Hence $\Delta MNO \longleftrightarrow \Delta PQR$



Q5: In ΔABC, DE is parallel to BC. If mAD = 1.5cm, mBD = 3cm, mAE = 1.3cm, then find mCE.

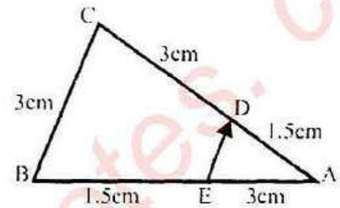
Solution:

Given $\overline{DE}$ is parallel to $\overline{BC}$

$m\overline{AD} = 1.5cm$

$m\overline{DC} = 3cm$

$m\overline{BE} = 1.5cm$



Required: Length of $\overline{AC}$

Statement	Reasons
$\overline{DE} \parallel \overline{BC}$	Given
In ΔABC	
$\dfrac{m\overline{AC}}{m\overline{DC}} = \dfrac{m\overline{AB}}{m\overline{EA}}$	
$\dfrac{m\overline{AC}}{m\overline{DC}} = \dfrac{m\overline{BE} + m\overline{EA}}{m\overline{EA}}$	$\because m\overline{AB} = \overline{BE} + \overline{EA}$
$\dfrac{m\overline{AC}}{3} = \dfrac{1.5+3}{3}$	$m\overline{EA} = m\overline{DC} = 30m$
$m\overline{AC} = \left(\dfrac{1.5+3}{\cancel{3}}\right) \times \cancel{3}$	
$\boxed{m\overline{AC} = 4.5cm}$	

Q6: In the given figure, find the value of x.

Solution:

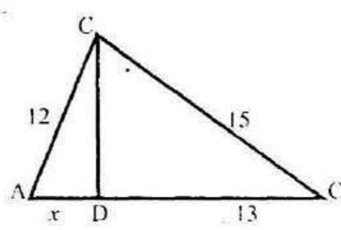
As ΔBCD is a right triangle so by Pythagoras theorem,

$(BC)^2 + (DC)^2 + (BD)^2$

$(15)^2 + (13)^2 + (BD)^2$

$225 - 169 = (BD)^2$

$(BD)^2 = 56$



Now in ΔABD, again

$(AB)^2 = (AD)^2 + (BD)^2$

$(12)^2 = \propto^2 + 56$

$144 - 56 = \propto^2 = \propto^2 = 88 \Rightarrow \sqrt{\propto^2} = \sqrt{88} \quad \Rightarrow \boxed{\propto = 9.4}$

Q7: In $\Delta ABC, \overline{AD}$ is the bisector of $\angle A$. If $m\overline{AB} = 9cm$, $m\overline{AC} = 10\ cm$ and $m\overline{BC} = 12cm$. Find the measure of $\overline{DC}$ and $\overline{BD}$.

Solution:

Given in ΔABC $\angle 1 \cong \angle 2$, $AB = 9cm$

$\overline{AC} = 10cm$ and $\overline{BC} = 12cm$

To find: $\overline{DC} = ?$ and $\overline{BD} = ?$

Proof: In $\Delta ABC, \overline{AD}$ is the bisector of $\angle A$.

$$\therefore \frac{\overline{BD}}{\overline{DC}} = \frac{\overline{AB}}{\overline{AC}}$$

$$\overline{BC} = 12cm$$

$$\therefore \frac{\overline{BD}}{\overline{DC}} = \frac{9}{10} \qquad \text{(Dividing } 12cm \text{ in } 9:10)$$

$$\frac{5.68}{6.32} = \frac{9}{10}$$

$\overline{BD} = 5.68cm$ and $\overline{DC} = 6.32cm$

Q8: $\overline{PS}$ is the bisector of $\angle P$ in ΔPQR. If $m\overline{QS} = 3cm, m\overline{SR} = 7$ and $m\overline{PQ} + m\overline{PR} = 20cm$, find the measure of $\overline{PQ}$ and $\overline{PR}$.

Solution:

Given in ΔPQR, $\overline{PS}$ is the bisector of $\angle P$ and $\overline{QS} = 3cm$ and $\overline{SR} = 7cm$, $PQ + PR = 20cm$.

To Find: PQ and PR

Proof:

In ΔPQR, $\overline{RS}$ is the bisector of $\angle P$.

Therefore $\frac{QS}{SR} = \frac{PQ}{PR} 3:7 = PQ:PR$

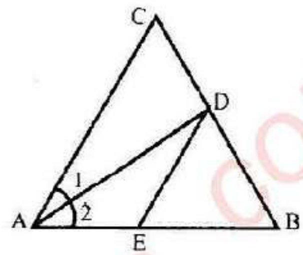
$$\frac{3}{7} = \frac{PQ}{PR}\ \frac{3}{\cancel{10}} \times \cancel{20}^{2} = 6$$

$$\frac{QS}{SR} = \frac{PQ}{PR} = \frac{7}{\cancel{10}} \times \cancel{20}^{2} = 14$$

$\frac{3}{7} = \frac{6}{14}$ hence $\overline{PQ} = 6cm$ and $\overline{PR} = 14cm$.

Q9: ABC is a triangle in which $m\overline{BC} = m\overline{CA} = 2m\overline{AB}$. Internal bisector of A meets $\overline{BC}$ at D and DE is drawn parallel to $\overline{CA}$ meeting $\overline{AB}$ at E. Find $m\overline{DE} : m\overline{AB}$.

Solution:



Given: In ΔABC

$m\overline{BC} = m\overline{CA} = 2m\overline{AB}$

$m\angle 1 = m\angle 2$ and $\overline{DC} \parallel \overline{AC}$

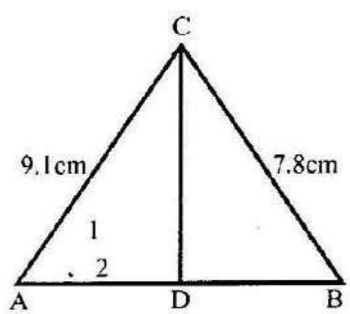
To Prove: $m\overline{OE} : m\overline{AB}$

Proof:

Statement	Reasons
As $\overline{AD}$ is the internal bisector of A then,	
$\frac{m\overline{CD}}{m\overline{DC}} = \frac{m\overline{CA}}{2m\overline{AB}} \longrightarrow (i)$	As $DEHCA$
Also $\frac{m\overline{CD}}{m\overline{OB}} = \frac{m\overline{AE}}{m\overline{EB}} \longrightarrow (ii)$	
From equation (i) and (ii)	
$\frac{mCA}{2mAB} = \frac{mAE}{mEB}$	Transitive property
$\frac{2mOF}{2m\overline{AB}} = \frac{m\overline{AE}}{mEB}$	$\because \overline{DC} \parallel \overline{CA} \Rightarrow AC = 2\overline{DE}$
$\frac{m\overline{DE}}{m\overline{AB}} = \frac{\cancel{m\overline{AE}}}{\cancel{m\overline{AE}}} = \frac{1}{2}$	Because E is the midpoint of AB
$\Rightarrow \quad m\overline{DE} : m\overline{AB} = 1:1$	

Q10: Measures of the sides of a triangle are 6.5cm, 7.8cm and 9.1cm. Find the lengths of the segments into which the smallest side is divided by the internal bisector of the opposite angle.

Solution:



Given $\overline{AB} = 6.5cm$

$\overline{AC} = 9.1cm$

$\overline{BC} = 7.8cm$

We find $\overline{AD}$ and $\overline{BD}$

$\frac{m\overline{BC}}{m\overline{AC}} = \frac{m\overline{AD}}{m\overline{BD}} \longrightarrow (1)$

Since $\overline{AB} = \overline{AD} + \overline{BD}$ or $\overline{AD} = \overline{AB} - \overline{AD} \longrightarrow (2)$

Put equation (2) in equation (1), we gets

$$\frac{m\overline{BC}}{m\overline{AC}} = \frac{m\overline{AB} - m\overline{BD}}{m\overline{BD}} = \frac{m\overline{AB}}{m\overline{BD}} - 1$$

Putting the values

$$\frac{7.8}{9.1} = \frac{6.5}{m\overline{BD}} - 1 \qquad \Rightarrow \frac{7.8}{9.1} + 1 = m\overline{BD}$$

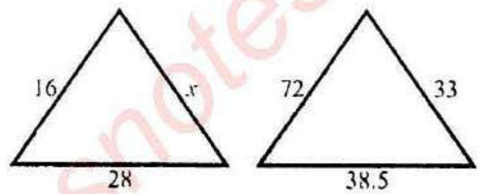
or $\boxed{m\overline{BD} = 3.5cm}$ Put this value in equation (2)

$$\overline{AD} = \overline{AB} - \overline{BD} = 6.5 - 3.5 = 3cm$$

$\therefore$ $\boxed{mAD = 3cm}$ Ans.

Q11: The given triangles are similar, find x.

Solution:



From two similar triangle their ratio is

$$16 : x = 22 : 33$$

$$\Rightarrow \frac{16}{x} = \frac{22}{3} \ \Rightarrow x = \frac{\cancel{16}^{2} \times \cancel{33}^{3}}{\cancel{22}\ \cancel{11}_{1}}$$

$\Rightarrow \boxed{x = 24}$ Ans.

Q12: If $\Delta ABC \sim \Delta DEF$ and $m\overline{AC} = 12cm, m\overline{AD} = 4cm, m\overline{BC} = 10cm$ and $m\overline{DF} = 9cm$, then find the measure of $\overline{DE}$ and $\overline{EF}$.

Solution:

$$\Delta ABC \sim \Delta DEF$$

To Find: $m\overline{DE} = ?$ and $m\overline{EF} = ?$

Proof: We know that,

$$\Delta ABC \sim \Delta DEF$$

$$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

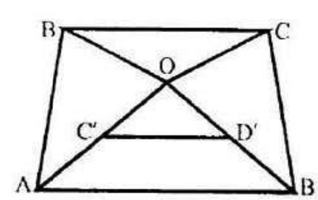
Let $\frac{4}{DE} = \frac{10}{EF} = \frac{12}{9} \qquad \Rightarrow \frac{\cancel{4}}{DE} = \frac{\cancel{12}^{3}}{\cancel{9}_{3}} \Rightarrow DE = 3cm$

$$\frac{10}{EF} = \frac{12}{9}\ \frac{\cancel{10}^{3}}{EF} = \frac{\cancel{12}^{2}}{\cancel{9}_{3}} \quad \Rightarrow EF = \frac{15}{2} \quad \Rightarrow EF = 9.5cm$$

So $DE = 3cm$ and $EF = 7.5cm$

Q13: Diagonals of a trapezoid $ABCD$ intersect each other at O. If $\Delta AOB \sim \Delta COD$, then prove that $\overline{AB} \parallel \overline{CD}$.

Solution:



Taking C' and D' on $\overline{AC}$ and $\overline{BD}$ diagonals such that

$$\overline{OC'} = \overline{OC} \text{ and } \overline{OD'} = \overline{OD}$$

Proof:

Statement	Reasons
In $\Delta AOB \sim \Delta OCD$	
$\frac{\overline{AB}}{\overline{CD}} = \frac{\overline{OA}}{\overline{OC}} = \frac{\overline{OB}}{\overline{OD}}$	Find the property of similar triangles
$\frac{\overline{OA}}{\overline{OC}} = \frac{\overline{OB}}{\overline{OD}}$	
$\frac{\overline{AC'}}{\overline{C'O}} + \frac{\overline{C'O}}{\overline{C'O}} = \frac{\overline{BD'}}{\overline{OD'}} + \frac{\overline{OD'}}{\overline{OD'}}$	
$\frac{\overline{AC''}}{\overline{C'O}} + \cancel{1} = \frac{\overline{BD'}}{\overline{OD'}} + \cancel{1}$	
$\Rightarrow \frac{\overline{AC''}}{\overline{C'O}} = \frac{\overline{BD'}}{\overline{OD'}}$	
$\therefore \; C'D' \parallel \overline{AD}$ and $\overline{CD} \parallel \overline{AB}$	

Q14: In the accompanying figure, the line segment, $\overline{KL}$, is drawn parallel to $\overline{ST}$, intersecting $\overline{RS}$ at K and $\overline{RT}$ at L in RST. If $PK = 5$, $KS = 10$, and $RT = 18$, then find RL.

Solution:

Using ratio theorem,

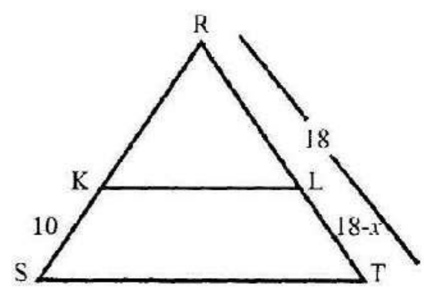
$$\frac{m\overline{LR}}{m\overline{KR}} = \frac{m\overline{LT}}{m\overline{LR}}$$

$$\frac{x}{5} = \frac{18-x}{10}$$

$$\Rightarrow \cancel{10}^2 \times \frac{x}{\cancel{5}} = \cancel{10}\left(\frac{18-x}{\cancel{10}}\right)$$

$$2x = 18 - x \quad \Rightarrow 2x + x = 18$$

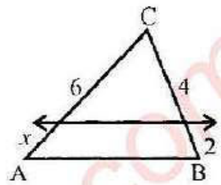
$$\Rightarrow 3x = 18 \quad \Rightarrow \boxed{x = 6} \quad \text{Ans.}$$



REVIEW EXERCISE 14

Q1: Select the correct answers:

i) In the given figure compute x:



(a) 7
(b) 9
(c) 2
✓(d) 3

ii) Which of the following is not valid for proving triangles similarity?

(a) SSS (b) AA **✓(c) SSA** (d) SAS

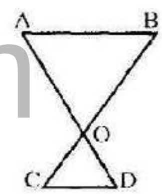
iii) In a mathematics class with 32 students, the ratio of girls to boys is 5 to 3. How many more girls are there than boys?

(a) 2 (b) 12 **✓(c) 8** (d) 20

iv) The measure of a line segment joining the midpoints of $\overline{AB}$ and $\overline{AC}$ of ΔABC is 3.5cm. Find $m\overline{BC}$ = …..

(a) 4.5cm (b) 5.5cm (c) 6cm **✓(d) 7cm**

v) In the figure $\overline{AB}||\overline{CD}$. If $\overline{AD}$ and $\overline{BC}$ intersects at O. Then ΔAOB and ΔDOC are:

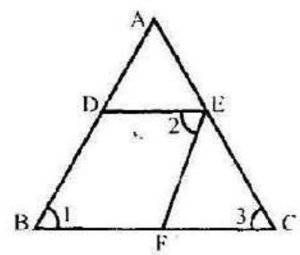


(a) Congruent
✓(b) Similar
(c) Not similar
(d) None of these

Q2: Prove that a line passing through the midpoint of one side of a triangle and parallel to a second side bisects the third side of the triangle.

Solution:

Given:



In ΔABC, $\overline{DE} \parallel \overline{BC}$ bisect $\overline{AB}$ at point D

And $\overline{AC}$ at point E respectively

To Prove:

$\overline{AE} = \overline{EC}$

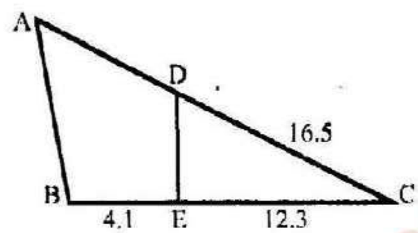
Proof:

Statement	Reasons
Since BFED is a parallelogram	Given
$\overline{EF} \cong \overline{BD} \cong \overline{AD}$	Opposite sides of parallelogram alternate angles
$m\angle 1 \cong m\angle 2$	
$m\angle 3 \cong m\angle 2$	
$\therefore$ $m\angle 1 \cong m\angle 3$	Transitive property

In $\Delta ADE \longleftrightarrow \Delta EFC$	
$AD \cong EF$	$m\angle 1 \cong m\angle 3$ proved
$\therefore$ $\Delta ADE \cong \Delta EFC$	$m\angle 1 \cong m\angle 2$ proved
Hence $\overline{AE} \cong \overline{EC}$	

Q3: In the figure $\overline{AB} \parallel \overline{DE}$. If $m\overline{BE} = 4.1cm$, $m\overline{EC} = 12.3cm$ and $m\overline{DC} = 16.5cm$, find the measure of $\overline{AC}$.

Solution:



Proof:

Statement	Reasons
$\overline{AB} \parallel \overline{DE}$	Given
In ΔABC	
$\dfrac{m\overline{AC}}{m\overline{DC}} = \dfrac{m\overline{BC}}{m\overline{EC}}$	As $\overline{AB} \parallel \overline{DE}$ (Given)
$\dfrac{m\overline{AC}}{m\overline{DC}} = \dfrac{m\overline{BE} + m\overline{EC}}{m\overline{EC}}$	$\because m\overline{BC} = m\overline{BE} + m\overline{EC}$
$\dfrac{m\overline{AC}}{16.5} = \dfrac{4.1 + 12.3}{12.3}$	Putting values
$= \dfrac{16.4}{12.3}$	
$m\overline{AC} = \dfrac{16.4 \times 16.5}{12.3} = 22$ Ans.	

Q4: Let $\overline{PQ}$ and $\overline{RS}$ intersect O and $\dfrac{m\overline{PQ}}{m\overline{OQ}} = \dfrac{m\overline{RO}}{m\overline{OS}}$. Prove that $\Delta OPR \sim \Delta OQS$.

Solution:

Given: $\overline{PQ}$ and $\overline{RS}$ intersect each other at O

And $\dfrac{m\overline{PO}}{m\overline{OQ}} = \dfrac{m\overline{RO}}{m\overline{OS}}$

To Prove:

$\Delta OPR \sim \Delta OQS$

Proof:

Statement	Reasons
In $\Delta POS \longleftrightarrow \Delta ROQ$	
$\dfrac{m\overline{PO}}{m\overline{OQ}} = \dfrac{m\overline{RO}}{m\overline{OS}}$	Given
$m\angle 1 \cong m\angle 2$	Vertical angles
$\overline{PS} \parallel \overline{RQ}$	
Hence $\dfrac{m\overline{OP}}{m\overline{OQ}} = \dfrac{m\overline{OR}}{m\overline{OS}} = \dfrac{m\overline{RS}}{m\overline{RQ}}$	
$\therefore \quad \Delta POS \sim \Delta ROQ$	

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Additional MCQs of Unit 14:
Ratio and Proportion

1. A line parallel to one side of a triangle, intersecting the other two sides divides them ....
(a) Equally (b) Proportionally (c) Linearly (d) none
✓ **Ans. (b) Proportionally**

2. If a line segment intersects the two sides of a triangle in the same ratio then it is ....to third side.
(a) Equal (b) Perpendicular (c) Parallel (d) none
✓ **Ans. (c) Parallel**

3. If two triangles are similar then the measure of their..........sides are proportional.
(a) Opposite (b) Corresponding (c) Similar (d) none
✓ **Ans. (b) Corresponding**

4. If area of ΔABC is $18cm^2$ and ΔDEF is $24cm^2$ then the ratio between their areas is..........
(a) 2:3 (b) 4:5 (c) 3:4 (d) 5:6
✓ **Ans. (c) 3:4**

5. If the ratio between a, b and c, d is same then the quantities are said to be..........
(a) Equal (b) Proportion (c) Parallel (d) Unequal
✓ **Ans. (b) Proportion**

6. $a:b::c:d$ means that..........
(a) $\frac{a}{c}=\frac{b}{d}$ (b) $ab=cd$ (c) $\frac{a}{b}=\frac{c}{d}$ (d) none
✓ **Ans. (c) $\frac{a}{b}=\frac{c}{d}$**

7. If the corresponding angles of two triangles are congruent then the triangles are said to have..........
(a) Concurrency (b) Similarity (c) Proportional (d) none
✓ **Ans. (b) Similarity**

8. To prove the similarity of two triangles which is impossible in the following:
(a) SS (b) SSS (c) SSA (d) SAS
✓ **Ans. (a) SS**

9. In a school 40 students are admitted the ratio of boys to girls is 6 to 2 then the boys are..........
(a) 30 (b) 20 (c) 32 (d) 28
✓ **Ans. (a) 30**