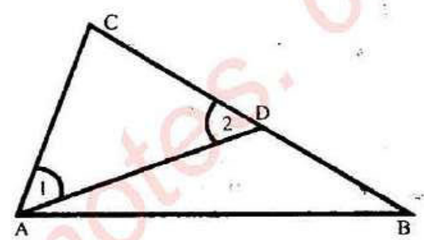
UNIT 13:
SIDES & ANGLES OF A TRIANGLE

THEOREM 13.1

If two sides of a triangle are unequal in length, the longer side has an angle of greater measure opposite to it.



Given:

ΔABC in which $m\overline{BC} > m\overline{AC}$

To Prove:

$m\angle CAB > m\angle B$

Construction:

Take a point D on a line segment $\overline{CB}$

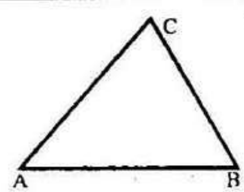
Such that $\overline{CD} \cong \overline{CA}$. Join D to A.

Proof:

Statements	Reasons
In ΔADC	
$\overline{CA} \cong \overline{CD}$	Construction
$\therefore$ $\angle 1 \cong \angle 2$	Angle opposite to congruent sides of ΔADC
But $m\angle 2 > m\angle B$	Exterior angle of triangle
$\therefore$ $m\angle 1 > m\angle B \longrightarrow (1)$	$\because \angle 1 \cong \angle 2$
Also $m\angle CAB > m\angle 1 \longrightarrow (2)$	$\because m\angle CAB = \angle 1 + \angle DAB$
$\therefore$ $m\angle CAB > m\angle B$	From (1) and (2)

THEOREM 13.2

If two angles of a triangle are unequal in measure, the side opposite the greater angle is longer than the side opposite to the smaller angle.



Given:

ΔABC in which $m\angle B > m\angle A$

To Prove:

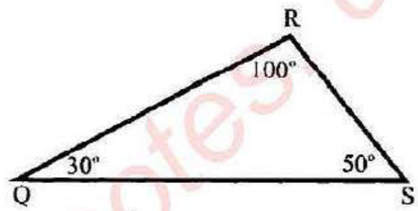
$m\overline{CA} > m\overline{CB}$

Proof:

Statements	Reasons
We have three options	
i) $m\overline{CA} < m\overline{CB}$	Trichotomy property
ii) $m\overline{CA} = m\overline{CB}$	
iii) $m\overline{CA} > m\overline{CB}$	
If $m\overline{CA} < m\overline{CB}$	

Then $m\angle B < m\angle A$	Angle opposite to smaller side
Which is contradiction to the given	
If $m\overline{CA} = m\overline{CB}$	
Then $m\angle B = m\angle A$	Base angles of an isosceles triangle
Which is again contradiction to the given.	
Hence the only possibility is	
$m\overline{CA} > m\overline{CB}$	

EXAMPLE 1



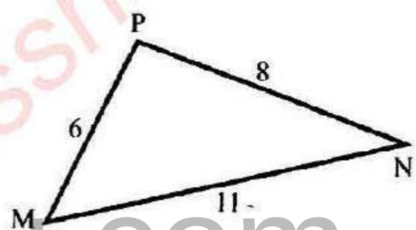
List the sides of the given triangle in increasing order.

Solution:

As $30 < 50 < 100$

Then $RS < QR < QS$

EXAMPLE 2



List the angles of the given triangle in increasing order.

Solution:

As $6 < 8 < 11$

Then $m\angle N < m\angle M < m\angle P$

THEOREM 13.3

Triangle inequality theorem:

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Given:

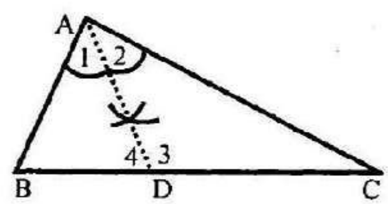
ΔABC is given.

To Prove:

$m\overline{AB} + m\overline{AC} > m\overline{BC}$

$m\overline{AB} + m\overline{BC} > m\overline{AC}$

$m\overline{AC} + m\overline{BC} > m\overline{AB}$



Construction:

Bisect $\angle A$. Let the bisector meets $\overline{BC}$ at D.

Proof:

Statements	Reasons
$m\angle 1 = m\angle 2 \longrightarrow (1)$	$\overline{AD}$ is the bisector of $\angle A$
But $m\angle 3 > m\angle 1 \longrightarrow (2)$	Exterior angle of ΔABD
$\therefore$ $m\angle 3 > m\angle 2$	From (1) and (2)

Statements		Reasons
$\Rightarrow$	$m\overline{AC} > m\overline{DC} \longrightarrow (3)$	Side opposite to greater angle
Similarly	$m\angle 1 = m\angle 2 \longrightarrow (4)$	
But	$m\angle 4 > m\angle 2 \longrightarrow (5)$	From (4) and (5)
$\therefore$	$m\angle 4 > m\angle 1$	
$\Rightarrow$	$m\overline{AB} > m\overline{BD} \longrightarrow (6)$	Side opposite to longer side
$\therefore$	$m\overline{AB} + m\overline{AC} > m\overline{BD} + m\overline{DC}$	Add (3) and (4)
i.e.	$m\overline{AB} + m\overline{AC} > m\overline{BC}$	$\because m\overline{BC} = \overline{BD} + \overline{DC}$
Similarly we can prove that		
	$m\overline{AB} + m\overline{BC} > m\overline{AC}$	
And	$m\overline{AC} + m\overline{BC} > m\overline{AB}$	

EXAMPLE 3

A triangle has one side of length 12 and another of length 8. Describe the possible length of the third side.

Solution:

Let x represent length of the third side.
Draw diagram to help visualize the small and large values of x.
Then use the triangle inequality theorem to write and solve inequalities.

Small values of x	Large values of x
$x + 8 > 12$	$8 + 12 > x$
$x > 4$	$20 > x$
	$\Rightarrow x < 20$

The length of the third side must be greater than 4 and less than 20.

THEOREM 13.4

From a point outside a line, the perpendicular is the shortest distance from the point to the line.

Given:

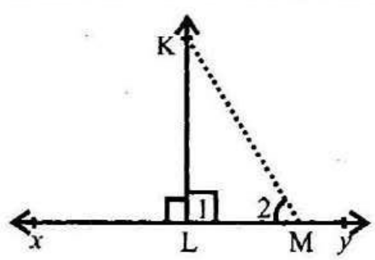
$\overline{KL} \perp \overleftrightarrow{XY}$ where K is a point outside $\overleftrightarrow{XY}$.

To Prove:

$m\overline{KL}$ is the shortest of all the distances from K to $\overleftrightarrow{XY}$

Construction:

Take a point M on $\overleftrightarrow{XY}$ other than L. Join M to K. ΔKLM is formed.



Proof:

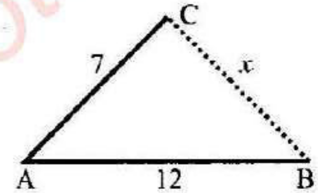
Statements		Reasons
In	ΔKLM	
	$m\angle 1 = 90^o$	Given
$\therefore$	$m\angle 1 > m\angle 2$	Only one angle is 90^o in one triangle
Or	$m\overline{KM} > m\overline{KL}$	Side opposite to greater angle

Or $m\overline{KL} < m\overline{KM}$ Similarly it can be proved that distance of K from any point taken on $\overleftrightarrow{XY}$ except L will be greater than $\overline{KL}$. Hence perpendicular $\overline{KL}$ is the shortest distance of point K from $\overleftrightarrow{XY}$.	Reflexive property

EXERCISE 13.1

Q1: In the given figure, what values of x will make a triangle possible?

Solution:



According to a theorem,

Sum of two sides of ΔABC is greater than the third side

$x + 7 > 12$

$\Rightarrow x > 12 - 7$ OR $x > 5$

Similarly $x < 7 + 12$ OR $x < 19$

So this triangle will be possible if,

$5 < x < 19$ Ans.

Q2: Could a triangle be formed by fastening three sticks that are 6 in, 10 in, 3 in long?

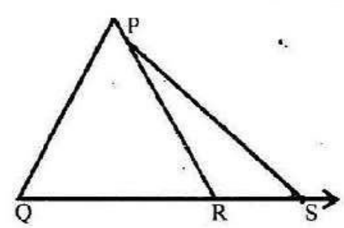
Solution:

No, such triangle is not possible because

$6 + 3 < 10$ OR $9 < 10$

As sum of two sides of a triangle must be greater than 3rd side.

Q3: Base $\overline{QR}$ of an isosceles triangle PQR is extended to S. Prove that $m\overline{PS} > m\overline{PQ}$.



Given:

ΔPQR is an isosceles in which $\overline{PQ} \cong \overline{PR}$.

$\overline{QR}$ is produced by S. Join S to P.

To Prove:

$m\overline{PS} > m\overline{PQ}$

Proof:

Statements	Reasons
In ΔPQS	
$m\angle Q \cong m\angle QRP \longrightarrow (1)$	
$m\angle QRS > m\angle PSR$	Exterior angle of triangle
$\therefore$ $m\angle Q > m\angle S$	From (1)
In ΔPQS, $m\overline{PS} > m\overline{PQ}$	Side opposite to greater angle

Q4: The length of two sides of a triangle is 11 and 23. If the third side is x, find the range of possible values for x.

Solution:

From the figure, we know that,

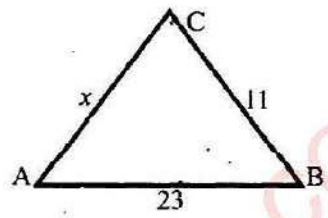
$$x+11>23$$
$$\Rightarrow x>23-11$$
$$\Rightarrow x>12$$

Similarly $x<11+23$

$$\Rightarrow x<34$$

Hence range of possible values of x is

$\boxed{12<x<34}$ Ans.



Q5: If S is any point on $\overline{PQ}$ of ΔPQR, S is joined to R. Prove that $\overline{PQ}+\overline{QR}>\overline{PS}+\overline{SR}$.

Solution:-

Since $\overline{PQ}:\overline{PS}=2:1$

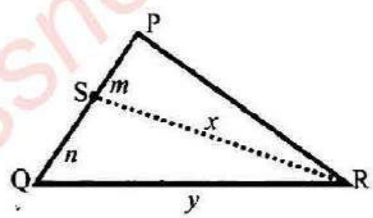
$$\Rightarrow \overline{PQ}>\overline{PS} \longrightarrow (1)$$

Similarly $\overline{QR}:\overline{SR}=2:1$

$$\Rightarrow \overline{QR}>\overline{SR} \longrightarrow (2)$$

Adding (1) and (2)

$$\overline{PQ}+\overline{QR}>\overline{PS}+\overline{SR}$$



Q6: If two angles of a triangle are 45° and 70° respectively,

i) Which is the greater of the two opposite sides?

ii) Which is the shortest sides of the triangle?

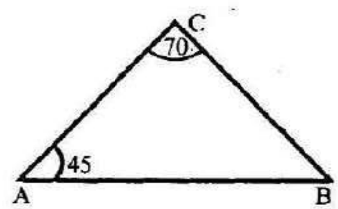
iii) Which is the longest?

Solution:

If $\theta=45^\circ$ and $\phi=70^\circ$ then the sum of the angles of triangle is 180°.

So $\theta+\phi+\psi=180^\circ$

$$45^\circ+70^\circ+\psi=180^\circ$$
$$\psi=180^\circ-115^\circ=65^\circ$$
$$\therefore \theta=45^\circ, \phi=70^\circ, \psi=65^\circ$$



The shortest side of triangle with $\theta=45^\circ$ is $\overline{BC}$.

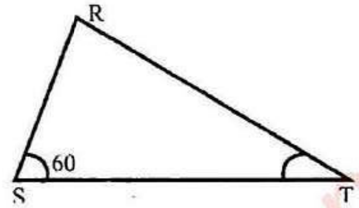
$\phi=70^\circ$ is the longest angle, so $\overline{AB}$ is the largest side of ΔABC.

Q7: If in Δ RST, RS > ST and $m\angle S=60^\circ$,

i) Which is the smallest side of the triangle?

ii) Which is the longest?

Solution:



Since $m\angle R$ is the greatest angle

So the opposite side $\overline{ST}$ is also greater

And $m\angle T$ is smaller

So $\overline{RS}$ is the smallest side of ΔRST.

Q8: Which of the following sets of lengths could be the length of the sides of triangle?

a) **2cm, 2cm, 2cm** **b) 3m, 4m, 5m**

c) **5cm, 8cm, 2cm** **d) 3m, 3m, 2m**

e) $\mathbf{1\frac{1}{2}m, 5m, 3\frac{1}{2}m}$ **f)** $\mathbf{2\frac{1}{2}cm, 3\frac{1}{2}cm, 4\frac{1}{2}cm}$

Solution:

a) 2cm, 2cm, 2cm

This could be the length of sides of triangle because sum of two sides is greater than third side i.e. $2+2=4>2$.

b) 3m, 4m, 5m

Since $3+4=7>5$

$4+5=9>3$ and $3+5=8>4$

Hence this could be the lengths of the sides of a triangle.

c) 5cm, 8cm, 2cm

Since $2+5=7<8$

$\therefore$ This could not be length of the sides of a triangle.

d) 3m, 3m, 2m

Since $3+3=6>2$

$3+2=5>3$ and $3+2=5>3$

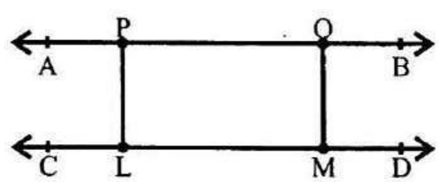
Hence this could be the length of the sides of a triangle.

e) $\mathbf{1\frac{1}{2}m, 5m, 3\frac{1}{2}m}$

This could be the length of the sides of a triangle.

Q9: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, if $\overline{PL}$ and $\overline{QM}$ are shortest distances between $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ then prove that $\overline{PL} \parallel \overline{QM}$.

Solution:



Given $\overline{PL}$ and $\overline{QM}$ are the shortest distance between two parallel lines $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$.

To Prove:

$\overline{PL} \parallel \overline{QM}$

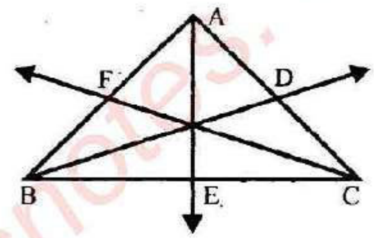
Proof:

Statement	Reasons
$\overline{PL} \perp \overleftrightarrow{CD}$	$\overline{PL}$ is the shortest distance between $\overleftrightarrow{AB}$ & $\overleftrightarrow{CD}$

And $\overline{QM} \perp \overline{CD}$	$\overline{QM}$ is the shortest distance between $\overline{AB}$ & $\overline{CD}$
$\therefore$ $m\angle PLM = m\angle PQM = 90^\circ$	
Also $m\angle LPQ = m\angle PQM = 90^\circ$	$\because PLMQ$ is a rectangle
Hence $\overline{PL} \parallel \overline{QM}$	Opposite sides of rectangle $PLMQ$

Q10: Prove that the perimeter of a triangle is greater than the sum of the measures of its altitudes.

Solution:



Given $\overline{BD} \perp \overline{AC}$

And $\overline{AE} \perp \overline{BC}$

$\overline{CF} \perp \overline{AB}$

Proof:

Statement	Reasons
$\overline{AC} + \overline{BC} > 2\overline{CF} \rightarrow (1)$	Sum of two sides of triangle is greater than double of altitudes.
$\overline{AB} + \overline{BC} > 2\overline{BD} \rightarrow (2)$	
$\overline{AC} + \overline{AB} > 2\overline{AE} \rightarrow (3)$	Adding equation (1), (2) and equation (3)
$\overline{AC} + \overline{BC} + \overline{BC} + \overline{AB} + \overline{AC}$ $+ \overline{AB} > 2\overline{CF} + 2\overline{BD} + 2\overline{AE}$	
Or $2\overline{AB} + 2\overline{BC} + 2\overline{AC} > 2(\overline{CF}$ $+ \overline{BD} + \overline{AE})$	
Or $\overline{AB} + \overline{BC} + \overline{AC} > \overline{CF} + \overline{BD} + \overline{AE}$	Dividing by 2

REVIEW EXERCISE 13

Q1: Select the correct answers:

i) An exterior angle of a triangle measures 120°. If measure of one of its remote interior angles is 40°, the measure of the second angle is:

(a) 40° ✓**(b) 80°** (c) 70° (d) 120°

ii) In $\Delta ABC, m\angle A = 90^\circ, m\angle B = 53^\circ$, and $m\angle C = 37^\circ$. Which expression correctly relates the lengths of the sides of this triangle?

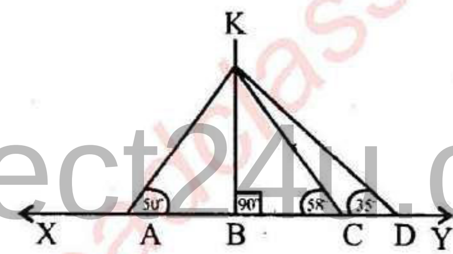
(a) $AB < BC < CA$ (b) $AC < BC < AB$

✓**(c) $AB < AC < BC$** (d) $BC < AC < AB$

iii) In the figure P lies outside $\overline{AB}$. $m\overline{PR}$ will be the shortest distance if $m\angle PRA$ is:

(a) 180° (b) 45° (c) 100° ✓**(d) 90°**

iv) In the given figure the point K lies outside $\overline{XY}$. Which of the following represents the shortest distance?



(a) $m\overline{KD}$ (b) $m\overline{KC}$ (c) $m\overline{KA}$ ✓**(d) $m\overline{KB}$**

v) Measures of two sides of a triangle are 10 and 14. Which of the following can be its third side?

(a) 2 (b) 4 ✓**(c) 22** (d) 24

vi) In ΔABC, $m\angle A = 50^\circ$ and $m\angle B = 30^\circ$, which of the following is correct?

(a) $m\overline{BC} > m\overline{AB}$ ✓**(b) $m\overline{AB} > m\overline{CA}$**

(c) $m\overline{BC} < m\overline{CA}$ (d) $m\overline{AB} < m\overline{CA}$

vii) Which of the following represents the sides of a triangle?

✓**(a) 3, 4 and 5** (b) 3, 4 and 7

(c) 3, 4 and 8 (d) 3, 4 and 1

viii. In $\Delta KLM, m\angle K = 45^\circ, m\angle L = 55^\circ$, $m\angle M = 80^\circ$. Which one of the following is the longest side?

✓**(a) KL** (b) LM (c) KM (d) None of these

Q2: If two sides of a triangle are 8 inches and 12 inches, what are the limiting values of the third side?

Solution:

Let $x = 8in,\ y = 12in$

Since sum of two sides of triangle is greater than third side, so

$x + y > z$ or $8 + 12 > z \Rightarrow 20 > z$

or $x - y < z \Rightarrow 8 - 12 < z \Rightarrow -4 < z$

hence the limiting value is $-4 < z < 20$ Ans.

Q3: Can a triangle have its sides equal to 7 inches, 5 inches and 12 inches?

Solution:

If $x = 7,\ y = 5$ and $z = 12$

Since sum of two sides of a triangle is greater than the third side.

$x + y > z,\ y + z > x$ and $x + z > y$

This is satisfied for the three given sides 7, 5, 12. Hence they form a triangle.

Q4: Prove that a line drawn from the vertex of an isosceles triangle to any point in the base is shorter than either of the equal sides.

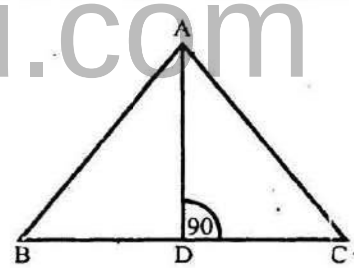
Solution:

In ΔABC, $m\angle D = 90°$ and $m\overline{AD} = m\overline{AD}$ (Common)

$\overline{AB} \cong \overline{AC}$ (As ΔABC is isosceles triangle)

Draw $\overline{AD} \perp \overline{BC}$ such that $\overline{BD} = \overline{DC}$

$\Rightarrow \overline{BD} = \frac{1}{2}\overline{BC}$.



This shows that base $\overline{BC}$ is shorter than either of the equal sides.

Q5: D is a point inside a ΔABC. Prove that

$$m\overline{DA} + m\overline{DB} + m\overline{DC} > \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{CA})$$

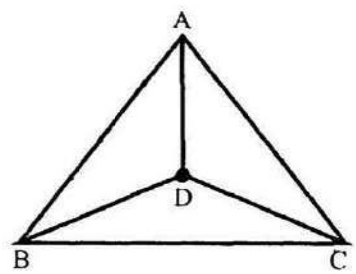
Solution:

Given ΔABC, D is at the centre.

To Prove:

$m\overline{AD} + m\overline{BD} + m\overline{CD}$

$> \frac{1}{2}\left(m\overline{AB} + m\overline{BC} + m\overline{CB}\right)$



Proof:

Statement	Reasons
In ΔACD	
$m\overline{AD} + m\overline{DC} > m\overline{AC} \rightarrow (1)$	Two sides of triangle is greater than third sides
In ΔABD	
$m\overline{AD} + m\overline{BD} > m\overline{AB} \rightarrow (2)$	
In ΔCBD	
$m\overline{BD} + m\overline{CD} > m\overline{BD} \rightarrow (3)$	
$m\overline{AD} + m\overline{DC} + m\overline{AD} + m\overline{BD}$	Adding (1), (2), (3)
$+m\overline{DC} + m\overline{DC} + m\overline{AD} + m\overline{BD}$	
$2(m\overline{AD} + m\overline{BD} + m\overline{CD})$	
$> m\overline{AB} + m\overline{BC} + m\overline{AC}$	
or $m\overline{AD} + m\overline{BD} + m\overline{CD}$	
$> \frac{1}{2}(m\overline{AB} + m\overline{BC} + m\overline{AC})$	

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Additional MCQs of Unit 13:
Sides and Angles of a Triangle

1. If two sides of a triangle are unequal then the larger side has an angle of.......... measure opposite to it.
(a) Greater (b) Smaller (c) Equal (d) none
✓ **Ans. (a) Greater**

2. The side opposite to the smaller angle of a triangle is..........in measure.
(a) Greater (b) Equal (c) Smaller (d) none
✓ **Ans. (b) Smaller**

3. The sum of any two sides of a triangle is..........of the third side.
(a) Equal (b) Greater (c) Smaller (d) none
✓ **Ans. (b) Greater**

4. From a point outside a line, the perpendicular is the..........distance from the point of the line.
(a) Perpendicular (b) Parallel (c) Shortest (d) Greatest
✓ **Ans. (c) Shortest**

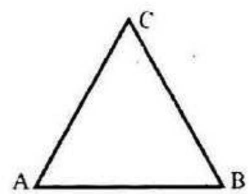
5. If a, b, c are the sides of ΔABC such that $a+b>c$ then it is..........theorem.
(a) Pythagoras (b) Factor (c) Remainder (d) Triangle inequality
✓ **Ans. (d) Triangle inequality**

6. What is the smallest number that can be exactly divisible by all the numbers 1 to 10.
(a) 2425 (b) 2025 (c) 2520 (d) 2735
✓ **Ans. (c) 2520**

7. If 2, 3, 5 are the measures then the triangle will be..........
(a) Right triangle (b) Isosceles (c) Impossible (d) Scalene
✓ **Ans. (c) Impossible**

8. Which are the sides of a triangle?
(a) 2,3 and 4 (b) 2,3 and 6 (c) 2,3 and 7 (d) 2,3 and 1
✓ **Ans. (a) 2,3 and 4**

9. In ΔABC if $m\angle A = 45^\circ$, $m\angle B = 55^\circ$, $m\angle C = 80^\circ$ then the longer side is..........
(a) AC (b) BC
(c) AB (d) none
✓ **Ans. (c) AB**



10. If $m\angle A = 40^\circ$, $m\angle B = 45^\circ$ and $m\angle C = 90^\circ$ then ΔABC will be..........
(a) Equilateral (b) Right angle (c) Isosceles (d) Impossible
✓ **Ans. (d) Impossible**