UNIT 11:
PARALLELOGRAMS & TRIANGLES

Definition:

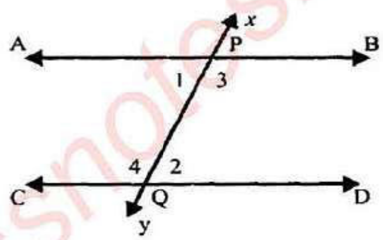
a) A quadrilateral is a closed figure having four sides and four angles. Line joining its opposite vertices is called its diagonal which divides the quadrilateral into two triangles.

b) Lines which are in the same plane and do not intersect each other, are called parallel lines.

c) If two lines lie in the same plane, they are said to be coplanar lines.

d) Lines drawn to cut two or more given lines is called transversal.

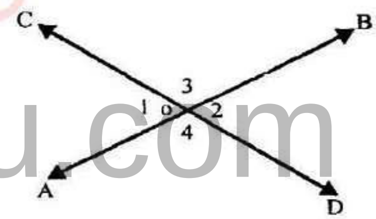
e) Line segment joining into midpoint of one side of a triangle to its opposite vertex is called the median of the triangle.



Alternate Angles:

When two coplanar lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are cut by a transversal $\overleftrightarrow{XY}$, two pairs of alternate angles are formed. If the coplanar lines are parallel, alternate angles are congruent.

From the figure, alternate angles $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$



Vertical Angles:

When two lines intersect each other, the angle having a common vertex and having no common arm are called vertical angles. In the figure the two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at O. $(\angle 1, \angle 2)$ and $(\angle 3, \angle 4)$ are two pairs of vertical angles. Also $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$.

THEOREM 11.1

Statement: In a parallelogram:

i) The opposite sides are congruent

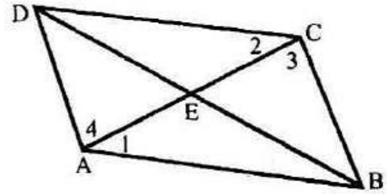
ii) The opposite angles are congruent

iii) The diagonals bisect each other

Given: ABCD is a parallelogram

i.e. $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$

$\overline{AC}$ and $\overline{BD}$ are the diagonals of parallelogram



To Prove: $\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$

$\angle A \cong \angle C$ and $\angle B \cong \angle D$

$\overline{AE} \cong \overline{CE}$ and $\overline{DE} \cong \overline{BE}$

Proof:

Statements	Reasons
$\because \overline{AB} \parallel \overline{DC}$ and $\overline{AC}$ intersects them	

$\therefore m\angle 1 \cong m\angle 2 \longrightarrow (1)$	Alternate angles
Similarly $\angle 3 \cong \angle 4 \longrightarrow (2)$	Alternate angles
In $\Delta ABC \longleftrightarrow \Delta CDA$	
$\overline{AC} \cong \overline{AC}$	Common
$m\angle 3 \cong m\angle 4$	Proved already
$m\angle 1 \cong m\angle 2$	Proved
$\therefore \Delta ABC \cong \Delta CDA$	(A.A.S)
Hence $\overline{AB} \cong \overline{CD}$	Corresponding sides of congruent triangles
And $\overline{BC} \cong DA$	Corresponding sides of congruent triangles
Also $\angle B \cong \angle D$	Corresponding angles of congruent triangles
$m\angle 1 + m\angle 4 \cong m\angle 2 + m\angle 3$	Adding (1) and (2)
Or $m\angle A \cong m\angle C$	$\angle 1 + \angle 4 = m\angle A$ and $\angle 2 + \angle 3 = m\angle C$
In $\Delta ABE \longleftrightarrow \Delta CDE$	
$\overline{AB} \cong \overline{CD}$	Proved
$m\angle 1 \cong m\angle 2$	Proved
$\angle AEB \cong \angle CED$	Vertical angles
$\therefore \Delta ABE \cong \Delta CDE$	(A.A.S $\cong$ A.A.S)
Hence $\overline{AE} \cong \overline{CE}$	
And $\overline{BE} \cong \overline{DE}$	Corresponding sides of congruent triangles

EXAMPLE 1

Quadrilateral *WXYZ* is a parallelogram. Find the value of x and y.

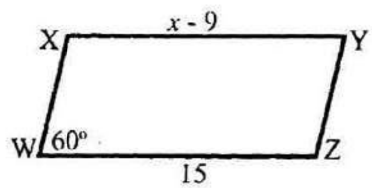
Solution:

$XY = WZ$ ($\because$ opposite sides of parallelogram are equal)

$x - 9 = 15$

$x = 24$

$y = 60$ ($\because$ opposite angles of a parallelogram are equal)



THEOREM 11.2

Statement: If two opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Given: ABCD is a quadrilateral in which

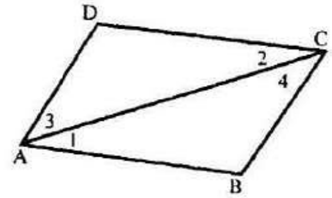
$\overline{AB} \cong \overline{DC}$ and $\overline{AB} \cong \overline{DC}$

To Prove:

ABCD is a parallelogram.

Construction:

Join A to C.



Proof:

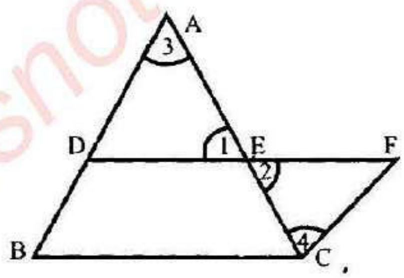
Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta CDA$	

$\overline{AB} \cong \overline{DC}$	Given
$\overline{AC} \cong \overline{AC}$	Common
$\angle 1 \cong \angle 2$	Alternate angles
$\therefore \Delta ABC \cong \Delta CDA$	(S.A.S ≅ S.A.S)
Hence $\angle 3 \cong \angle 4$	Corresponding angles of congruent triangles due to alternate angles
$\therefore \overline{AD} \parallel \overline{BC}$	
Hence ABCD is a parallelogram.	Opposite sides are parallel

THEOREM 11.3

Statement: The line segment, joining the midpoints of two sides of a triangle, is parallel to the third side and is equal to one half of its length.

Given: In $\Delta ABC, D$ and E are the midpoints of $\overline{AB}$ and $\overline{AC}$. $\overline{DE}$ joins them.



To Prove:

$\overline{DE} \parallel \overline{BC}$ and $m\overline{DE} = \frac{1}{2} m\overline{BC}$

Construction:

Takes F on $\overrightarrow{DE}$, such that $\overline{DE} \cong \overline{EF}$. Join F to C.

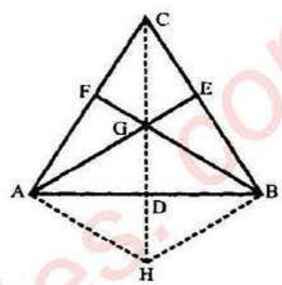
Proof:

Statements	Reasons
In $\Delta ADE \longleftrightarrow \Delta CFE$	
$\overline{DE} \cong \overline{EF}$	Construction
$\overline{AE} \cong \overline{CE}$	Given
$\angle 1 \cong \angle 2$	Vertical angles
$\therefore \Delta ADE \cong \Delta CFE$	(S.A.S ≅ (S.A.S)
$\therefore \overline{AD} \cong \overline{CF}$	Corresponding sides of congruent triangles
But $\overline{AD} \cong \overline{BD}$	Given
So $\overline{BD} \cong \overline{CF}$	Transitive property
Also $\overline{AB} \parallel \overline{CF}$	Alternate angles are congruent
Or $\overline{BD} \parallel \overline{CF}$	
i.e. BCFD is a parallelogram	Opposite sides are parallelogram
$\Rightarrow \overline{DE} \parallel \overline{BC}$	
And $m\overline{DF} = m\overline{BC}$	Opposite sides of parallelogram
But $m\overline{DE} = \frac{1}{2} m\overline{DF}$	Construction
$\therefore m\overline{DE} = \frac{1}{2} m\overline{BC}$	$(\because m\overline{DF} = m\overline{BC})$

Definition: A median of a triangle is a line segment from a vertex to the midpoint of the opposite side.

THEOREM 11.4

Statement: The medians of a triangle are concurrent and their point of concurrency is the point of trisection of each median.



Given: In ΔABC, E and F are the midpoints of $\overline{BC}$ and $\overline{AC}$ respectively. $\overline{AE}$ and $\overline{BF}$ intersect each other at G. $\overrightarrow{CG}$ is drawn to meet $\overline{AB}$ at D.

To Prove: i) Medians are concurrent

ii) G is the point of trisection of each median

Construction: On $\overrightarrow{CG}$ take a point H such that $\overline{CG} \cong \overline{GH}$. Join H to A and B.

Proof:

Statements	Reasons
In ΔACH $\overline{FG} \parallel \overline{AH}$ i.e. $\overline{FB} \parallel \overline{AH}$	$\overline{FG}$ joins midpoints of $\overline{CA}$ and $\overline{CH}$
Similarly in ΔBCH $\overline{GE} \parallel \overline{HB}$ Or $\overline{AE} \parallel \overline{HB}$	G and E are midpoints of $\overline{CH}$ and $\overline{CB}$
$\therefore$ $AHBG$ is a parallelogram	Opposite sides are parallel
i.e. $\overline{AD} \cong \overline{BD}$ Or D is the midpoint of $\overline{AB}$ Or $\overline{CGD}$ is a median Hence all the three medians are concurrent	Diagonals of a parallelogram bisect each other
Also $\overline{GD} \cong \overline{DH}$	Diagonals of a parallelogram bisect each other
Or $m\overline{GD} = \frac{1}{2} m\overline{GH} = \frac{1}{2} m\overline{GC}$	$\therefore \overline{CG} \cong \overline{GH}$ (construction)
Similarly it can be proved that $m\overline{GE} = \frac{1}{2} m\overline{AG}$ And $m\overline{FG} = \frac{1}{2} m\overline{GB}$ Hence G is the point of trisection of each median.	

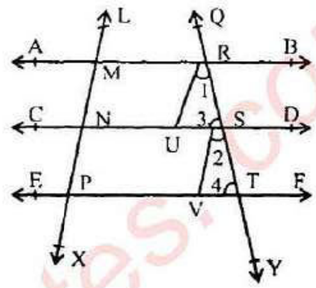
Definition: The two lines which do not intersect each other and which are coplanar are called *parallel lines*. The two lines which intersect each other at right angle are called *perpendicular lines*.

THEOREM 11.5

Statement: If three or more parallel lines make congruent intercepts on a transversal, they also intercept congruent segments on any other line that cuts them.

Given: Lines $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$.

The transversal $\overleftrightarrow{LX}$ intersects $\overleftrightarrow{AB}, \overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$ at the points M, N and P such that $\overline{MN} \cong \overline{NP}$. Another transversal $\overleftrightarrow{QY}$ intersects them at points R, S, T respectively.



To Prove: $\overline{RS} \cong \overline{ST}$

Construction: Draw $\overline{RU} \parallel \overleftrightarrow{LX}$ which meets $\overleftrightarrow{CD}$ at U and from S draw $\overline{RV} \parallel \overleftrightarrow{LX}$ which meets $\overleftrightarrow{EF}$ at V. Label the angles as $\angle 1, \angle 2, \angle 3$ and $\angle 4$.

Proof:

Statements	Reasons
$\overline{RU} \parallel \overline{LX}$	Construction
$\overline{AB} \parallel \overline{CD}$	Given
$\therefore$ MNUR is a parallelogram	By definition
Thus $\overline{MN} \cong \overline{RU} \longrightarrow (1)$	Opposite sides of parallelogram
Similarly $\overline{NP} \cong \overline{SV} \longrightarrow (2)$	
But $\overline{MN} \cong \overline{NP} \longrightarrow (3)$	Given
$\therefore \overline{RU} \cong \overline{SV}$	From (1), (2) and (3)
So $RU \parallel \overline{SV}$	Both are congruent to $\overline{LX}$
Now in $\Delta RUS \longleftrightarrow \Delta SVT$	
$\overline{RU} \cong \overline{SV}$	Proved
$\angle 1 \cong \angle 2$	Corresponding angles
$\angle 3 \cong \angle 4$	Corresponding angles
$\therefore \Delta RUS \cong \Delta SVT$	(A.S.A $\cong$ A.S.A)
Hence $\overline{RS} \cong \overline{ST}$	Corresponding sides of the congruent triangle

EXERCISE 11.1

Q1: Measure of one of the angles of parallelograms is 70°. Find the measure of the remaining angles.

Given: ABCD is a parallelogram in which $m\angle B = 70^{\circ}$.

To Prove: $m\angle C = 70^{\circ}$

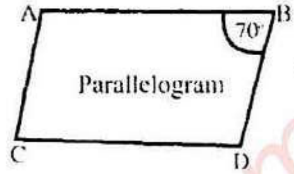
(Opposite angles of parallelogram are congruent)

$\angle A + \angle B + \angle C + \angle D = 360^\circ$ (Sum of all angles are 360°)

$\angle A + 70^\circ + 70^\circ + \angle D$

$\angle A + \angle D = 360^\circ - 70^\circ - 70^\circ$

$\angle A + \angle D = 220^\circ \Rightarrow \angle A = \angle D = \frac{220^\circ}{2} = 110^\circ$



Hence $m\angle C = 70^\circ$, $m\angle A = 110^\circ$ m and $m\angle D = 110^\circ$

Q2: Measure of one of the exterior angles of a parallelogram is 125°. Find the measure of all of its interior angles.

Solution: Given parallelogram $ABCD$ in which one exterior angle is $\angle BDX = 125^\circ$

i.e. $\angle BDX + \angle BDC = 180^\circ$ **(Supplementary angles)**

$125^\circ + \angle BDC = 180^\circ$

$\angle BDC = 180^\circ - 125^\circ = 55^\circ$ or $\angle D = 55^\circ$

So $\angle A = 55^\circ$ (Opposite angle of parallelogram)

$\angle A + \angle B + \angle C + \angle D = 360^\circ$ (Sum of all angles is 360°)

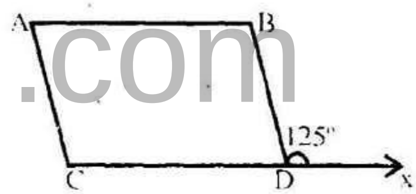
Putting the values of $\angle A = \angle D = 55^\circ$

$55^\circ + \angle B + \angle C + 55^\circ = 360^\circ$

$\angle B + \angle C + 110^\circ = 360^\circ$

$\Rightarrow \angle B + \angle C = 360^\circ - 110^\circ$

$\angle B + \angle C = 250^\circ$ or $\angle B = \frac{250^\circ}{2} = 125^\circ$



So $\angle C = 125^\circ$

Hence $\angle A = 55^\circ$, $\angle D = 55^\circ$, $\angle B = 125^\circ$ and $\angle C = 125^\circ$ Ans.

Q3: RSTU is a parallelogram. Find ST.

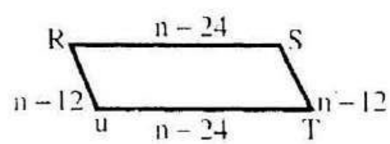
Solution: We know sum of all sides of parallelogram

$\Rightarrow n - 12 + n - 12 + n - 24 + n - 24 = 0$

$\Rightarrow 4n - 72 = 0$

$\Rightarrow 4n = 72$

$\Rightarrow n = \frac{72}{4} = 18$



Now $ST = n - 12$

$\Rightarrow ST = 18 - 12 = 6$

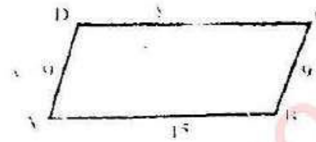
Hence $\boxed{ST = 6}$ Ans.

Q4: Find the value of each variable in the parallelogram using the properties:

i)

Solution:

Since ABCD is a parallelogram.



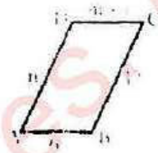
In parallelogram opposite sides are equal. Since $BC = AD$ and $AB = DC$

So $x = 9$, $y = 15$

ii)

Solution: Since $AB = DC$

$\Rightarrow m + 1 = 6 \quad \Rightarrow m = 6 - 1 = 5$

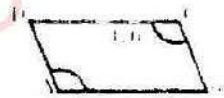


And $AD = BC$

$\Rightarrow \boxed{n = 12}$ and $\boxed{m = 5}$

iii)

Solution: Since opposite angles of parallelogram are equal.

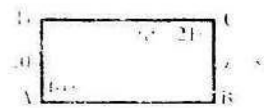


So $2P^\circ = 120$

$\Rightarrow P^\circ = \dfrac{120^\circ}{2} = 60^\circ$

$\boxed{P^\circ = 60^\circ}$ Ans.

iv)



Solution: Since $\overline{AD} = \overline{BC}$, so $Z - 8 = 20$

$\Rightarrow Z = 20 + 8 = 28$

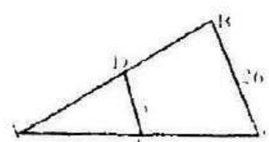
And $\angle A = \angle C \Rightarrow (d - 20)^\circ = 105^\circ$

$\Rightarrow d = 105^\circ + 20^\circ = 125$

Q5: $\overline{DE}$ is a mid segment of $\triangle ABC$. Find the value of x.

i)

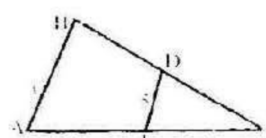
Solution: We know that.



$\overline{DE} = \dfrac{1}{2}\overline{BC}$ or $x = \dfrac{1}{2}(26)$

$\Rightarrow \boxed{x = 13}$ Ans.

ii)

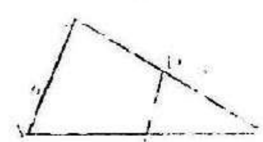


Solution: We know that.

$\overline{AB} = 2\overline{DE}$ or $x = 2(5)$

$\Rightarrow \boxed{x = 10}$ Ans.

iii)



Solution: Since $DE = \dfrac{1}{2}\overline{AB}$

$\Rightarrow x = \dfrac{1}{2}(6) \Rightarrow \boxed{x = 3}$ Ans.

Q6: Prove that diagonals of a rhombus bisect its angles.

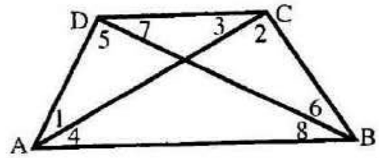
Solution: Let $\overline{AC}$ and $\overline{BD}$ be the two diagonals of the rhombus.

Since $\overline{AB} \parallel \overline{CD}$, so $\angle 3 \cong \angle 4$

And $\angle 1 \cong \angle 2$ because opposite angles of parallel sides are congruent.

Similarly $\angle 7 \cong \angle 8$ and $\angle 5 \cong \angle 6$

Hence the diagonals of a rhombus bisects the angles.



Q7: In rhombus *MNOP*, $m\angle N = 60^\circ$. What is $m\angle O$?

Solution: Since $m\angle N = 60^\circ$ and $\angle N \cong \angle P$

$\Rightarrow \angle P = 60^\circ$

But $\angle M + \angle N + \angle O + \angle P = 360^\circ$

(Sum of all angles are 360°)

$\angle M + 60^\circ + \angle O + 60^\circ = 360^\circ$

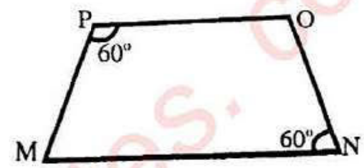
$\angle M + \angle O + 120^\circ = 360^\circ$

$\Rightarrow \angle M + \angle O = 360^\circ - 120^\circ$

$\angle M + \angle O = 240^\circ$

$\Rightarrow \angle M = \angle O = \dfrac{240^\circ}{2} = \boxed{120^\circ}$

Hence $\boxed{m\angle O = 120^\circ}$ Ans.



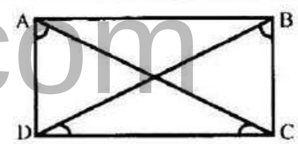
Q8: Prove that diagonals of a rectangle are congruent.

Given: ABCD is a rectangle in which $\overline{AB} \cong \overline{DC}$

And $\overline{AD} \cong \overline{BC}$.

To prove: $\overline{AC} \cong \overline{BD}$

Proof:



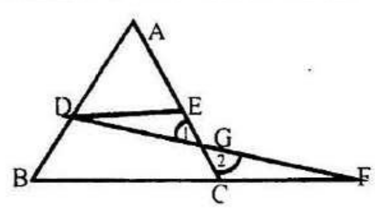
Statement	Reasons
In $\Delta ADC \longleftrightarrow \Delta BCD$	
$\overline{CD} \cong \overline{CD}$	Common
$m\angle D \cong m\angle C$	Each is 90°
$\overline{AD} \cong \overline{BC}$	Given
$\therefore \Delta ADC \cong \Delta BCD$	(A.A.S)
Hence $\overline{AC} \cong \overline{BD}$	Corresponding sides

Q9: In a $\Delta ABC, D$ and E are two points on $\overline{AB}$ and $\overline{AC}$, such that $m\overline{AD} = \frac{1}{4}$ $m\overline{AB}$ and $m\overline{AE} = \frac{1}{4}m\overline{AC}$. Prove that $m\overline{DE} = \frac{1}{4}m\overline{BC}$.

Given: ΔABC in which $\overline{AD} = \frac{1}{4}\overline{AB}$, $\overline{AE} = \frac{1}{4}\overline{AC}$

To Prove: $\overline{DE} = \frac{1}{4}\overline{BC}$

Construction: Join F with G and D such that



$m\overline{DG} \cong m\overline{GF}$ and $\overline{GC} \cong m\overline{GE}$. Produce $\overline{CF}$ such that $\overline{CF} = \frac{1}{4}\overline{BC}$.

Proof:

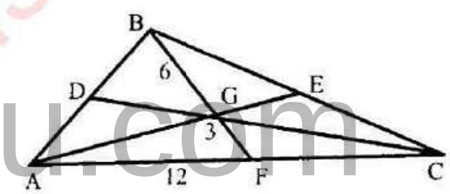
Statement	Reasons
In $\Delta DEG \longleftrightarrow \Delta CGF$	
$\overline{DG} \cong \overline{GF}$	Construction
$\overline{GE} \cong \overline{GC}$	Construction
$m\angle 1 \cong m\angle 2$	Vertical angles
$\therefore \Delta DEG \cong \Delta CGF$	$S.A.S \cong S.A.S$
So $\overline{DE} \cong \overline{CF}$	Corresponding sides
But $CF = \frac{1}{4}\overline{BC}$	Construction
$\therefore \overline{DE} \cong \frac{1}{4}\overline{BC}$	Transitive property

Q10: G is the centroid of $\Delta ABC, BG = 6, AF = 12$ and $AE = 15$. Find the length of the segment.

i) $\overline{FC}$ **ii)** $\overline{BF}$

iii) $\overline{AG}$ **iv)** $\overline{GE}$

Solution:



i) $\overline{FC}$

Since $\overline{BF}$ is the median of $\overline{AC}$ so $\overline{AF} = \overline{FC}$

Hence $\overline{FC} = 12$

ii) $\overline{BF}$

Since $BG : GF = 2:1$ or $BG : GF = 6:3$

Hence $\overline{BF} = \overline{BG} + GF = 6 + 3 = 9$

iii) $\overline{AG}$

Since $\overline{AE} = \overline{AG} + GE = 10 + 2 = 12$

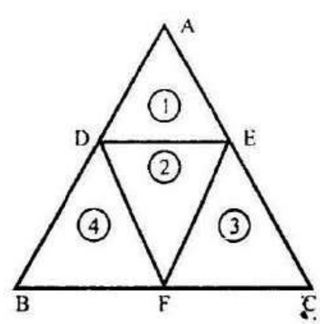
Hence $\overline{AG} = 10$

iv) $\overline{GE}$

Since $\overline{AG} : \overline{GE} = 10:2 \Rightarrow \overline{GE} = 2$ Ans.

Q11: Prove that the four triangles formed by joining the midpoints of the three sides of a triangle are congruent to one another.

Given: In ΔABC points D, E, F are midpoints of $\overline{AB}, \overline{AC}, \overline{BC}$. Four triangles are formed by joining these points.



Proof: $\overline{DE} \parallel \overline{BC}$ or $\overline{DE} \parallel \overline{BF}$

Or $m\overline{DE} = \frac{1}{2} m\overline{BL}$ or $\overline{DE} \cong m\overline{BF}$

So BFED is a parallelogram.

So $\Delta BFD \cong \Delta DEF$ $\qquad\qquad$ $\because \overline{DE} \cong \overline{BF}$ and $\overline{BD} \cong \overline{EF}$

Similarly $\Delta ADE \cong \Delta DEF$ and $\Delta CEF \cong \Delta DEF$

Hence all four triangles are congruent.

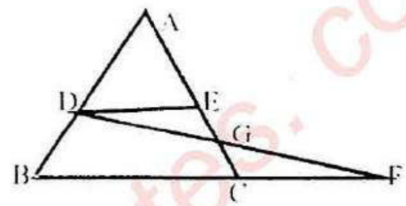
Q12: In the given $\Delta ABC, D$ and E are the midpoints of $\overline{AB}$ and $\overline{AC}$, and $m\overline{CF} = \frac{1}{2} m\overline{BC}$.

Solution: Given: In ΔABC

$\overline{AD} = \overline{BD}$

And $\overline{AE} \cong \overline{EC}$

To Prove: $m\overline{CG} = \frac{1}{3} m\overline{AG}$



Proof:

Statement	Reasons
In ΔABC	
$\overline{DE} \parallel \overline{BC}$	$\overline{DE}$ joins the midpoint of ΔABC
$\angle EDG \cong \angle CFG$	Alternate angles of parallel sides
$\therefore$ $m\overline{DE} = \frac{1}{2} m\overline{BC} \longrightarrow (1)$	
$\overline{DE} \cong \overline{BC}$	Given
$\therefore$ $m\overline{CF} = \frac{1}{2} m\overline{BC} \longrightarrow (2)$	
$\overline{DE} \parallel \overline{BE}$	From equation (1) and equation (2)
$\therefore$ $\overline{DE} \cong \overline{CF}$	
In $\Delta GED \longleftrightarrow CFG$	
$\angle EDG \cong \angle CFG$	Proved above
$\angle DGE \cong \angle FGE$	Vertical angles
$\overline{DE} \cong \overline{CF}$	Proved
$\therefore$ $\Delta GED \cong \Delta CFG$	
Hence $\overline{EG} \cong \overline{GC}$	Corresponding sides
$\overline{AE} \cong \overline{CE}$	Corresponding sides
$\overline{CG} + \overline{CG} = \overline{CE}$	
$2\overline{CG} = \overline{CE} \longrightarrow (1)$	
Now $\overline{AG} \cong \overline{CE} + \overline{EG}$	From $\overline{CE} \cong 2\overline{CG} + \overline{EG} = \overline{GC}$
$\overline{AG} \cong \overline{CG} + \overline{EG}$	
$\overline{AG} \cong 2\overline{CG} + \overline{GC}$	
$\overline{AG} \cong 3\overline{CG}$	
$\overline{CG} = \frac{1}{3}\overline{AG}$	

REVIEW EXERCISE 11

Q1: Select the correct answers:

i) Which quadrilateral must have diagonals that are congruent and perpendicular?
(a) Rhombus ✓ **(b) Square**
(c) Trapezoid (d) Parallelogram

ii) How many equilateral tri angles can be made by joining the midpoints of the sides of an equilateral triangle?
(a) 2 (b) 3 ✓ **(c) 4** (d) Cannot be determined

iii) Which statement must be true?
✓ **(a) If a parallelogram is not a rectangle, then it is not a square**
(b) If a parallelogram is not a square, then it is not a rectangle
(c) All rectangles are squares
(d) If a parallelogram is a rectangle, then it is a square

iv) Which quadrilateral's diagonals are perpendicular to each other and bisect the figure's opposite angles?
(a) Trapezoid (b) Rectangle
✓ **(c) Rhombus** (d) Parallelogram

v) If opposite angles of a quadrilateral are equal in measure and none of them is a right angle, then the quadrilateral is a:
(a) Square ✓ **(b) Parallelogram**
(c) Trapezoid (d) Rectangle

vi) Medians of a triangle are divided by the point of concurrency in the ratio:
✓ **(a) 2 : 1** (b) 2 : 3 (c) 1 : 3 (d) None of these

vii) Centroid is the point of concurrency is:
✓ **(a) Medians of a triangle** (b) Angle bisectors of a triangle
(c) Altitudes of a triangle (d) Perpendicular bisectors of a triangle

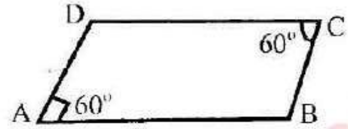
viii) If sum of the measures of $\angle A$ and $\angle C$ of a parallelogram ABCD is 120°, then $m\angle B = \ldots\ldots.$
(a) 25° (b) 50° (c) 65° ✓ **(d) None of these**

ix) Diagonals of a square are….to each other.
✓ **(a) Perpendicular** (b) Not congruent
(c) Congruent (d) Parallel

x) Sum of the measures of interior angles of a quadrilateral is:
(a) 2 right angles ✓ **(b) 4 right angles**
(c) 3 right angles (d) None of these

Q2: Measure of one of the angles of parallelogram is 60^o. Find the measure of the remaining angles.

Solution:



Since $\angle A = 60^o \quad \Rightarrow \angle A \cong \angle C = 60^o$

$\Rightarrow \angle A + \angle B + \angle C + \angle D = 360^o$

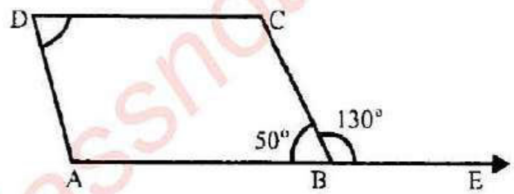
$\Rightarrow 60^o + \angle B + 60^o + \angle D = 360^o \qquad \therefore \angle B = \angle D = 120^o$

$\Rightarrow \angle B + \angle D + 120^o = 360^o$

$\Rightarrow \angle B = \dfrac{240^o}{2} = 120^o$

Q3: Measure of one of the exterior angles of a parallelogram is 130^o. Find the measure of all of its interior angles.

Solution:



Since $\angle CBE = 130^o$

So $\quad \angle ABC + \angle CBE = 180^o$

$\Rightarrow \angle ABC + 130^o = 180^o$

$\Rightarrow \angle ABC = 180^o - 130^o = 50^o$

$\therefore \angle D \cong \angle B = 50^o$

Now $\angle A + \angle B + \angle C + \angle D = 360^o$

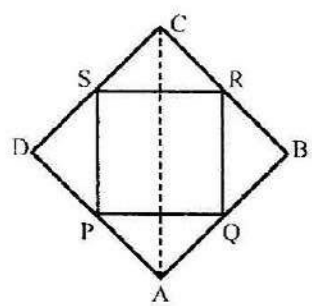
$\Rightarrow \angle A + 50^o + \angle C + 50^o = 360^o$

$\Rightarrow \angle A + \angle C = 360^o - 100^o = 260^o$

$\Rightarrow \angle A = \dfrac{260^o}{2} = 130^o$

$\Rightarrow \angle C = 130^o$

Q4: Prove that the line segments joining the midpoints of the sides of a quadrilateral taken in order, form a parallelogram.



Given: ABCD is a quadrilateral points P, Q, R, S are the mid-points of $\overline{AD}, \overline{AB}, \overline{BC}, \overline{CD}$.

To Prove: PQRS is a parallelogram.

Construction: Draw diagonal $\overline{AC}$.

Proof: In ΔADC, P and S are midpoints of $\overline{AD}$ and $\overline{CD}$.

So $\quad \overline{PS} \parallel \overline{AC} \longrightarrow (1)$

$\Rightarrow m\overline{PS} = \dfrac{1}{2} m\overline{AC} \longrightarrow (2)$

In ΔABC, Q and R are midpoints of $\overline{AB}$ and $\overline{BC}$

So $\quad \overline{QR} \parallel \overline{AC} \longrightarrow (3)$

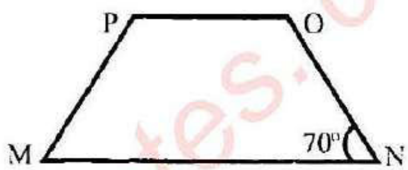
And $m\overline{QR} = \dfrac{1}{2} m\overline{AC} \longrightarrow (4)$

$\therefore \overline{PS} \parallel \overline{QR}$ from (1) and (3) and $\overline{PS} \parallel \overline{QR}$

From (2) and (4) similarly $\overline{PQ} \parallel \overline{RS}$

Hence PQRS is a parallelogram.

Q5: In rhombus *MNOP*, $m\angle N = 70^\circ$, what is $m\angle O$?



Solution: Given $m\angle N = 70^\circ$

As $m\angle N = m\angle P$, so $m\angle P = 70^\circ$

Now $\angle M + \angle N + \angle O + \angle P = 360^\circ$ (Sum of all angles are 360°)

$\angle M + 70^\circ + \angle O + 70^\circ = 360^\circ$

$\angle M + \angle O + 140^\circ = 360^\circ$

$\angle M + \angle O = 360^\circ - 140^\circ = 220^\circ$

OR $\angle O = \dfrac{220^\circ}{2} = 110^\circ$

$\therefore$ $\boxed{\angle M = \angle O = 110^\circ}$ Ans.

Q6: Measure of one of the angles of parallelogram is 50°. Find the measure of the remaining angles.

Solution:

Given ABCD is a parallelogram

Such that $m\angle A = 50^\circ$

Required:

$\angle B, \angle C, \angle D$

Proof:

Statement	Reasons
$\overline{AD} \parallel \overline{BC}$	Opposite sides
$m\angle A + m\angle B = 180^\circ$	Supplementary angles
$m\angle A = 50^\circ$	Given
$\Rightarrow$ $m\angle B = 180^\circ - 50^\circ = 130^\circ$	
$m\angle A = m\angle C = 50^\circ$	Opposite angles of parallelogram
$m\angle B = m\angle O = 130^\circ$	Opposite angles of parallelogram
Hence $m\angle B = 130^\circ$,	
$m\angle O = 130^\circ$,	
$m\angle C = 50^\circ$	

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Additional MCQs of Unit 11:
Parallelograms and Triangles

1. A polygone with four sides is called..........
   (a) Quadrilateral (b) Rectangle (c) Square (d) none
   ✓ **Ans. (a) Quadrilateral**
2. In geometry, a figure that lies in a plane is called..........figure.
   (a) Space (b) Cartesian (c) Plane (d) none
   ✓ **Ans. (c) Plane**
3. When all four sides are congruent then the figure is..........
   (a) Rectangle (b) Rhombus (c) Square (d) none
   ✓ **Ans. (c) Square**
4. A quadrilateral in which all sides and all angles are congruent is called..........
   (a) Rhombus (b) Square (c) Rectangle (d) none
   ✓ **Ans. (b) Square**
5. If two opposite sides of a quadrilateral are congruent then it is..........
   (a) Square (b) Rectangle (c) Parallelogram (d) none
   ✓ **Ans. (c) Parallelogram**
6. The diagonal divides which quadrilateral into two congruent triangles..........
   (a) Square (b) Rectangle (c) Parallelogram (d) All
   ✓ **Ans. (d) All**
7. The line segment joining the midpoints of two sides of a triangle is..........to third side.
   (a) Parallel (b) Perpendicular (c) Intersect (d) none
   ✓ **Ans. (a) Parallel**
8. A line segment from a vertex to the midpoint of the opposite side of a triangle is..........
   (a) Bisector (b) Altitude (c) Median (d) none
   ✓ **Ans. (c) Median**
9. The medians of a triangle are..........
   (a) Different (b) Concurrent (c) Proportional (d) none
   ✓ **Ans. (b) Concurrent**
10. The two lines which do not intersect and are Coplanar are ....lines.
    (a) Parallel (b) Perpendicular (c) Horizontal (d) none
    ✓ **Ans. (a) Parallel**