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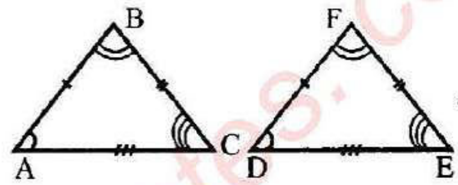
UNIT 10:
CONGRUENT TRIANGLES

Two plane figures are congruent if they have same size and same shape. A triangle has six elements. In which three are sides and three are angles.

Definition:

Two triangles ΔABC and ΔDEF are said to be congruent if the following six conditions are hold:



i) $\angle A \cong \angle D$

ii) $\angle B \cong \angle E$

iii) $\angle C \cong F$

iv) $\overline{AB} \cong \overline{DF}$

v) $\overline{BC} \cong \overline{EF}$

vi) $\overline{AC} \cong \overline{DE}$

Then we write $\Delta ABC \cong \Delta DEF$

Properties of Congruent Triangles:

1. $\Delta ABC \cong \Delta ABC$, this is called identity congruence which means every triangle is congruent to itself.
2. If $\Delta ABC \cong \Delta DEF$ then $\Delta DEF \cong \Delta ABC$, this is called symmetric property of congruence.
3. If $\Delta ABC \cong \Delta DEF$ and $\Delta DEF \cong \Delta XYZ$ then $\Delta ABC \cong \Delta XYZ$, this is called transitive property of congruence.

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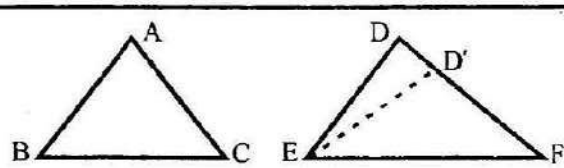
Remember: $\boxed{S.A.S}$

If two sides and their included angle of one triangle are congruent to two sides and their included angle of another triangle then the two triangles are congruent.

THEOREM 10.1

Statement: If two angles and a non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the triangles are congruent.



Given: In $\Delta ABC \longleftrightarrow \Delta DEF$

$\overline{BC} \cong \overline{EF}$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$

To Prove: $\Delta ABC \cong \Delta DEF$

Construction: Suppose $\overline{AC} \not\cong \overline{DF}$ and $\overline{AC} \cong \overline{D'F}$, where D' is a point on $\overline{DF}$. Join D' to E.

Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta D'EF$	

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$\overline{BC} \cong \overline{EF}$	Given
$\angle C \cong \angle F$	Given
$\overline{AC} \cong \overline{D'F}$	Supposition
$\therefore \Delta ABC \cong \Delta D'EF$	(S.A.S ≅ S.A.S)
Hence $\angle B \cong \angle D'EF$	Corresponding angles of congruent triangles
But $\angle DEF \cong \angle B$	Given
$\therefore \angle D'EF \cong DEF$	Transitive property
$\Rightarrow \overline{ED} \cong \overline{ED'}$	
Thus $\overline{AC} \cong \overline{DF}$	
In $\Delta ABC \longleftrightarrow \Delta DEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\overline{AC} \cong \overline{DF}$	Already proved
$\angle C \cong \angle F$	Given
$\therefore \Delta ABC \cong \Delta DEF$	(S.A.S) ≅ (S.A.S)

THEOREM 10.2

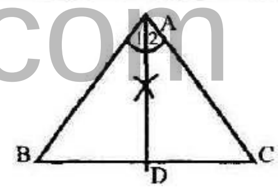
Statement: If two angles of a triangle are congruent then the side opposite to these angles must be congruent.

Given: In $\Delta ABC, \angle B \cong \angle C$

To Prove: $\overline{AB} \cong \overline{AC}$

Construction:

Draw bisector of $\angle A$ cutting $\overline{BC}$ at point D.



Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$m\angle 1 \cong m\angle 2$	Construction
$m\angle B \cong m\angle C$	Given
$\therefore \Delta ABD \cong \Delta ACD$	(A.A.S ≅ A.A.S)
Hence $\overline{AB} \cong \overline{AC}$	Corresponding sides of congruent triangles

THEOREM 10.3

Statement: In a correspondence of two triangle, if three sides of one triangle are congruent to the corresponding three sides of the other, the two triangles are congruent.

Given: In $\Delta ABC \longleftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$

To Prove: $\Delta ABC \cong \Delta DEF$

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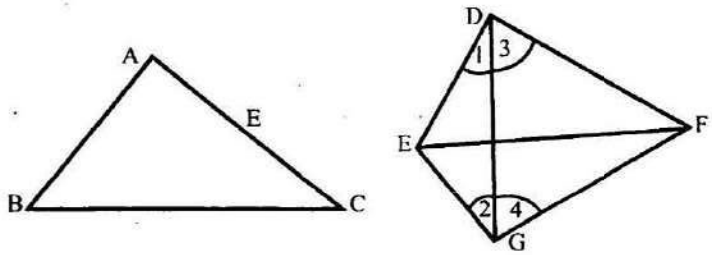
Construction: Suppose $\overline{BC}$ is not shorter than any of the other two sides of ΔABC. Construct ΔGEF such that $\overline{GE} \cong \overline{AB}$. Join G with D.



Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta GEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\therefore m\angle B \cong \angle GEF$	Construction
$\overline{AB} \cong \overline{GE}$	Construction
$\therefore \Delta ABC \cong \Delta GEF$	(S.A.S ≅ S.A.S)
So $\overline{GF} \cong \overline{AC}$	Corresponding sides of congruent triangles
But $\overline{DF} \cong \overline{AC}$	Given
$\therefore \overline{GF} \cong \overline{DF} \longrightarrow (1)$	Transitive property
In ΔFDG,	
$\angle 3 \cong \angle 4 \longrightarrow (2)$	ΔDFG is isosceles
In ΔEGD,	
$\overline{DE} \cong \overline{EG}$	Each is congruent to $\overline{AB}$
$\therefore \angle 1 \cong \angle 2 \longrightarrow (3)$	By (2) and (3)
$\therefore \angle 3 + \angle 1 \cong \angle 4 + \angle 2$	
$\therefore m\angle D \cong m\angle G$	Addition of angles postulate
In the correspondence	
In $\Delta DEF \longleftrightarrow \Delta GEF$	
$\overline{DE} \cong \overline{GE}$	Proved
$\angle D \cong \angle G$	Proved
$\overline{DE} \cong \overline{FG}$	From equation (1)
$\therefore \Delta DEF \cong \Delta GEF$	(S.A.S)
But $\Delta ABC \cong \Delta GEF$	Proved
Hence $\Delta ABC \cong \Delta GEF$	From transitive property

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THEOREM 10.4

Statement: If in the correspondence of two triangles, the hypotenuse and one side of one are congruent to the hypotenuse and one side of other, then triangles are congruent.

Given: In $\Delta ABC, \Delta DEF$

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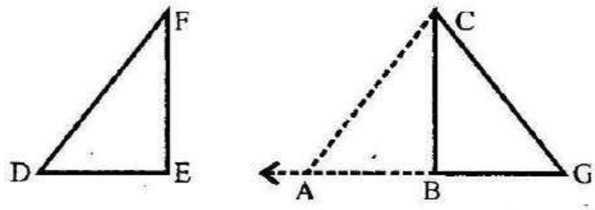
$\angle ABC \cong \angle E$ (Each is right angle)

$\overline{AC} \cong \overline{DF}$ and $\overline{BC} \cong \overline{EF}$



To Prove: $\Delta ABC \cong \Delta DEF$

Construction:

Produce $\overline{AB}$ to the point G such that $\overline{BG} \cong \overline{DE}$. Join C with G.

Proof:

Statements	Reasons
In ΔGBC	
$m\angle CBG = 90^\circ$	Given
In $\Delta GBC \longleftrightarrow \Delta DEF$	
$\overline{GB} \cong \overline{DE}$	Construction
$\angle GBC \cong \angle DEF$	Each is right angle or 90°
$\therefore \Delta GBC \cong \Delta DEF$	(S.A.S $\cong$ S.A.S)
$\overline{GC} \cong \overline{DF}$	Correspondence sides of congruent triangles
But $\overline{DF} \cong \overline{AC}$	Given
$\therefore \overline{GC} \cong \overline{AC}$	
$\Rightarrow \angle A \cong \angle G$	Angles opposite to congruent sides of ΔCAG
In $\Delta ABC \longleftrightarrow \Delta GBC$	
$\overline{AC} \cong \overline{GC}$	Proved
$\angle A \cong \angle G$	Proved
$\angle ABC \cong \angle GBC$	Each is of 90°
$\therefore \Delta ABC \cong \Delta GBC$	(A.A.S)
But $\Delta GBC \cong \Delta DEF$	Proved
$\therefore \Delta ABC \cong \Delta DEF$	From transitive property

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EXERCISE 10.1

Q1: Prove that the perpendiculars drawn from the endpoints of the base of an isosceles triangle to their opposite sides are congruent.

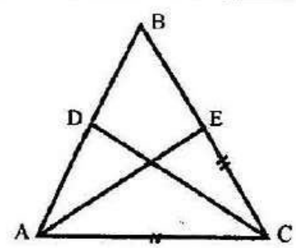
Given: ΔABC in which $\overline{AC} \cong \overline{BC}$

And $\overline{AE} \perp \overline{BC}$

And $\overline{CD} \perp \overline{AB}$

To Prove: $m\overline{AE} \cong m\overline{CD}$

Proof: In $\Delta CAD \longleftrightarrow \Delta ACE$



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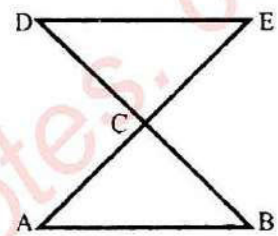
Statements	Reasons
$m\angle CAD \cong m\angle ACE$	Angles opposite to congruent sides of triangle
$m\overline{AC} \cong m\overline{AC}$	Common
$m\angle CDA \cong m\angle AEC$	Both are of 90°
$\therefore \Delta CAD \cong \Delta ACE$	(A.A.S)
Thus $m\overline{AE} \cong m\overline{CD}$	Corresponding sides of congruent triangles

Q2: In the given figure $\overline{AC} \cong \overline{CE}$ and $\angle B \cong \angle D$ prove that $\overline{BC} \cong \overline{CD}$.

Given: $\overline{AC} \cong \overline{CE}$

And $\angle B \cong \angle D$



To Prove:

$\overline{BC} \cong \overline{CD}$

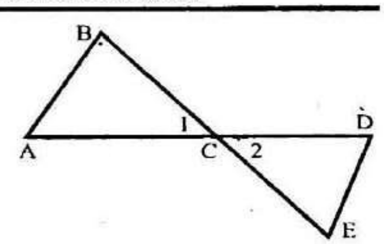
Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta DEC$	
$\overline{AC} \cong \overline{CE}$	Given
$\angle B \cong \angle D$	Given
$\angle ACB \cong \angle DCE$	Vertical angles
$\therefore \Delta ABC \cong \Delta DEC$	(A.A.S)
So $\overline{BC} \cong \overline{CD}$	Corresponding sides of congruent triangles

Q3: Given: C is the midpoint of $\overline{BE}$. $\angle B \cong \angle C$, Prove: $\Delta ABC \cong \Delta DEC$.

Given: C is midpoint

And $\angle B \cong \angle E$



To Prove:

$\Delta ABC \cong \Delta DEC$

Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta DEC$	
$m\overline{BC} \cong \overline{CE}$	Given $\quad \therefore$ is mid point
$m\angle B \cong m\angle E$	Given
$m\angle 1 \cong m\angle 2$	Vertical angles
$\therefore \Delta ABC \cong \Delta DEC$	(A.A.S)

Q4: ABC is a triangle in which $m\angle A = 35^\circ$ and $m\angle B = 100^\circ$, $\overline{BD} \perp \overline{AC}$. Prove that ΔBDC is an isosceles triangle.

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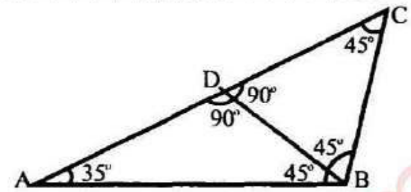
Given: ΔABC in which $m\angle A = 45^o$, $m\angle B = 100^o$

And $\overline{BD} \perp \overline{AC}$



To Prove:

$\overline{BD} \cong \overline{CD}$ or ΔBDC is isosceles triangle.

Proof:

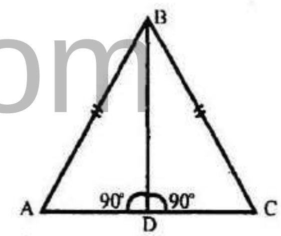
Statements	Reasons
$\angle A = 35^o$, $\angle ABC = 100^o$	Given
$\overline{BD} \perp \overline{AC}$	Given
$m\angle ADB = \angle BDC = 90^o$	Both are supplementary angles
In ΔADB	
$m\angle ABD = 90^o - 35^o = 55^o$	
In ΔBDC	
$m\angle DBC = 100^o - 55^o = 45^o$	
So $m\angle BCD = 90^o - 45^o = 45^o$	
So in ΔBDC	Sides opposite to congruent angles
$\overline{BD} \cong \overline{CD}$	

Q5: Prove that the bisector of the vertex angle of an isosceles triangle is the perpendicular bisector of the base.

Given:

ΔABC in which $\overline{AC}$ is the base of ΔABC

And $\overline{BD} \perp \overline{AC}$



To Prove:

$\overline{AB} \cong \overline{BC}$

Proof:

Statements	Reasons
In $\Delta ABD \longleftrightarrow \Delta CBD$	
$\overline{BD} \cong \overline{BD}$	Common
$m\angle ADB \cong \angle CDB$	Each is right angle
$m\angle ABD \cong \angle CBD$	Given
$\therefore \Delta ABD \cong \Delta CBD$	(A.A.S)
Thus $\overline{AB} \cong \overline{BC}$	Corresponding sides of congruent ΔS

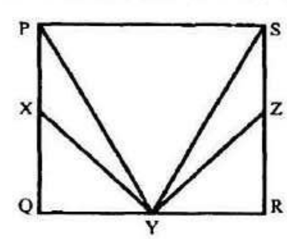
Q6: PQRS is a square. X, Y and Z are the midpoints of $\overline{PQ}, \overline{QR}$ and $\overline{RS}$ respectively. Prove that $\Delta PXY = \Delta SZY$.

Given:

PQRS is a square in which X,

And Z are midpoints of $\overline{PQ}, \overline{QR}$

And $\overline{RS}$ respectively.



To Prove: $\Delta PXY \cong \Delta SZY$

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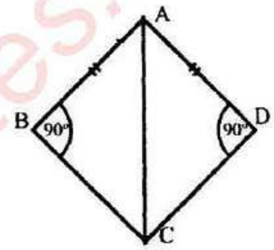
Proof:

Statements	Reasons
In $\Delta PXY \longleftrightarrow \Delta SZY$	
$\overline{PX} \cong \overline{SZ}$	Given X and Z are midpoints
$\overline{PY} \cong \overline{SY}$	Corresponding sides of congruent triangles isosceles Δ
$\overline{XY} \cong \overline{ZY}$	Corresponding sides of congruent triangles
$\therefore \Delta PXY \cong \Delta SZY$	S.S.S

Q7: Given: $\overline{AB} \perp \overline{BC}$, $\overline{AD} \perp \overline{DC}$, $\overline{AB} \cong \overline{AD}$. Prove that: $\Delta ABC \cong \Delta ADC$.

Given: $\overline{AB} \perp \overline{BC}$

$\overline{AD} \perp \overline{CD}$, $\overline{AB} \cong \overline{AD}$



To Prove:

$\Delta ABC \cong \Delta ADC$

Proof:

Statements	Reasons
In $\Delta ABC \longleftrightarrow \Delta ADC$	
$\overline{AB} \cong \overline{AD}$	Given
$m\angle ABC \cong m\angle ADC$	(Each is of 90^o) Given
$\overline{AC} \cong \overline{AC}$	Common
$\therefore \Delta ABC \cong \Delta ADC$	(S.A.S)

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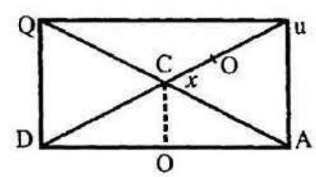
Q8: QUAD is a rectangle. Find x. $OC = x$, $DC = 3x - 8$

Solution:

If O is the midpoint of $\overline{AD}$, join O to C.

Then in ΔOCD, $(CD)^2 - (OC)^2 + (OD)^2$

Pythagoras theorem



$$(3x-8)^2 = (x)^2 + \left(\frac{x}{2}\right)^2$$

$$(3x-8)^2 = x^2$$

$$9x^2 + 64 - 48x = x^2$$

OR $9x^2 - x^2 - 48x + 64 = 0$

$8x^2 - 48x + 64 = 0$ (Divide by 8)

$x^2 - 6x + 8 = 0 \Rightarrow x^2 - 2x - 4x + 8 = 0$

$x(x-2) - 4(x-2) = 0 \Rightarrow (x-2)(x-4) = 0$

$x - 2 = 0$ OR $x - 4 = 0$

$\Rightarrow$ $x = 2$ OR $x = 4$

But $x \neq 2$

Hence $\boxed{x = 4}$ Ans.

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Review Exercise 10

Q1: Select the correct answer.

i) Which of the following is not a sufficient condition for the congruency of two triangles?

(a) $A.S.A \cong A.S.A$ (b) $H.S \cong H.S$
(c) $S.A.A \cong S.A.A$ ✓**(d) $A.A.A \cong A.A.A$**

ii) The diagonal of…does not divide it into two congruent triangles.

(a) Rectangle (b) Square
(c) Parallelogram ✓**(d) Trapezium**

iii) In a given correspondence of two triangles, if $\Delta ABC \cong \Delta DEF$, then which of the following is not correct?

(a) $m\angle B = m\angle E$ (b) $m\overline{CA} \cong m\overline{FD}$
(c) $m\angle CBA \cong m\angle FED$ ✓**(d) $\angle ABC \cong \angle EFD$**

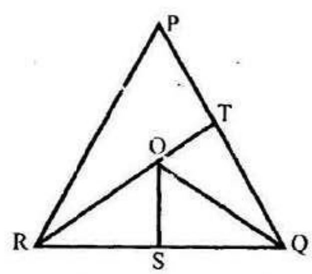
iv) In ΔABC, if $m\angle A \cong m\angle B$ then bisector of……divides the ΔABC into two congruent triangles.

(a) $\angle A$ (b) $\angle B$ ✓(c) $\angle C$ (d) Any one of its angles

v) Which of the following is a legitimate reason for declaring two triangles to be congruent?

(a) $SSA \cong SSA$ ✓**(b) $SAS \cong SAS$**
(c) $AAA \cong AAA$ (d) All of these

Q2: In a ΔPQR, the bisectors of $\angle Q$ and $\angle R$ meet at O. Prove that O is equidistant from the sides of the triangle.



Solution: In ΔPQR, $\overline{OR}$ and $\overline{OQ}$ are the bisectors of $\angle R$ and $\angle Q$. Both bisectors meet at point O.

To Prove: $m\overline{OS} \cong m\overline{OT}$

Construction:

Draw $\overline{OS} \perp \overline{QR}$ and $\overline{OT} \cong \overline{PQ}$.

Proof:

Statements	Reasons
In $\Delta QTO \longleftrightarrow \Delta QSO$	
$m\angle QTO \cong \angle QSO$	Each is of 90°
$m\overline{OQ} \cong m\overline{OQ}$	Common
$m\angle OQS \cong m\angle OQT$	Given
$\therefore \Delta QTO \cong \Delta QSO$	(A.A.S)

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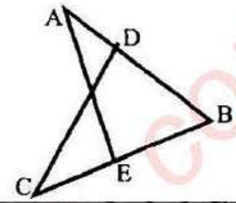
So $m\overline{OS} \cong m\overline{OT}$	Corresponding sides of congruent triangles

Q3: In the given figure $\overline{AB} \cong \overline{CB}$ and $\angle A \cong \angle C$, prove that $\overline{AE} \cong \overline{CD}$.

Solution:

$\overline{AB} \cong \overline{BC}$ and $\angle A \cong \angle C$



To Prove:

$\overline{AE} \cong \overline{CD}$

Proof:

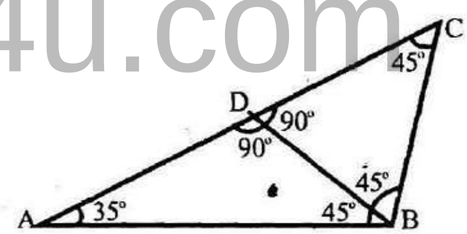
Statements	Reasons
In $\Delta ABE \longleftrightarrow \Delta CBD$	
$m\angle A \cong m\angle C$	Given
$m\angle B \cong m\angle B$	Common
$m\overline{AB} \cong m\overline{BC}$	Given
$\therefore \Delta ABE \cong \Delta CBD$	(A.A.S)
So $\overline{AE} \cong \overline{CD}$	Corresponding sides of congruent triangles

Q4: ABC is a triangle in which $m\angle A = 35^\circ$, $m\angle B = 100^\circ$, $\overline{BD} \perp \overline{AC}$. Prove that ΔBDC is an isosceles triangle.

Given: ΔABC, in which $m\angle A = 35^o$,

$m\angle B = 100^o$

And $\overline{BD} \perp \overline{AC}$



To Prove: $\overline{BD} \cong \overline{CD}$

Or ΔBDC is an isosceles triangle

Proof:

Statements	Reasons
$\angle A = 35^o, \angle ABC = 100^o$	Given
$m\overline{BD} \perp m\overline{AC}$	Given
$m\angle ADB = \angle BDC = 90^o$	Each is right angles
In ΔADB	
$m\angle ABD = 90^o - 35^o = 55^o$	
$m\angle DBC = 100^o - 55^o = 45^o$	
So in ΔBDC	
$m\overline{BD} = m\overline{CD}$	Sides opposite to congruent triangles

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Additional MCQs of Unit 10:
Congruent Triangles

1. When the three sides and three angles are equal then the triangles are...........
   (a) Parallel (b) Different (c) Congruent (d) none
   ✓ **Ans. (c) Congruent**

2. SAS postulate means two sides and...........
   (a) One angle (b) Interior angle (c) Included angle (d) none
   ✓ **Ans. (c) Included angle**

3. $\Delta ABC \longleftrightarrow \Delta DEF$ shows...........between two triangles.
   (a) Relation (b) Correspondence (c) Equality (d) none
   ✓ **Ans. (b) Correspondence**

4. If two angles and their included side are congruent then we write...........
   (a) AAS (b) SAA (c) ASA (d) none
   ✓ **Ans. (c) ASA**

5. If two sides of a triangle are congruent then it is...........
   (a) Isosceles (b) Scalene (c) Equilateral (d) none
   ✓ **Ans. (a) Isosceles**

6. A scalene triangle having three sides...........
   (a) Unequal (b) Parallel (c) Perpendicular (d) Equal
   ✓ **Ans. (a) Unequal**

7. If hypotenuse and one side are congruent between two triangles then we write...........
   (a) S.A.S (b) A.S.A (c) S.S.S (d) $H.L \cong H.L$
   ✓ **Ans. (d) $H.L \cong H.L$**

8. The diagonals of trapezium does not divide into two equal...........
   (a) Squares (b) Triangles (c) Rectangles (d) none
   ✓ **Ans. (b) Triangles**

9. If two opposite sides are equal and parallel then it is called...........
   (a) Quadrilateral (b) Rectangle (c) Trapezium (d) none
   ✓ **Ans. (c) Trapezium**

10. If $m\angle ABC = 90^{o}$ then such angle is called...........
    (a) Acute (b) Abtuse (c) Right (d) none
    ✓ **Ans. (c) Right**

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