UNIT 8:
LINEAR GRAPHS & THEIR APPLICATIONS

Cartesian Plane & Linear Graphs:

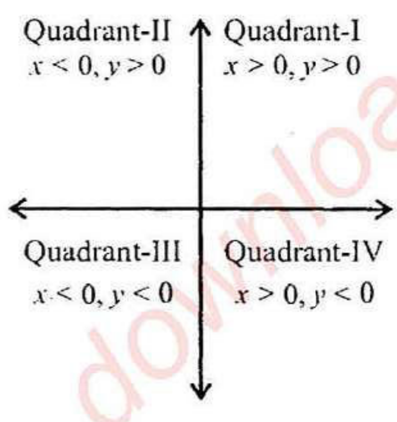
Let $A=\{1,2\}$ and $B=\{3,4\}$ are any two sets. Then the Cartesian product of A and B denoted by $A\times B$ i.e.

$$A\times B=\{(1,3),(1,4),(2,3),(2,4)\}$$

Cartesian product of a real number set is

$$R\times R=\{(a,b)\,|\,a,b\in R\}$$

A rectangular coordinate system is formed by two perpendicular number lines, one is horizontal and one is vertical which intersect at the zero point is called *Cartesian plane*. The point of intersection 0(0,0) is called the origin.



It is divided into 4 quadrants.

In first quadrant, $x>0, y>0$

In 2[nd] quadrant, $x<0, y>0$

In 3[rd] quadrant, $x<0, y<0$

In 4[th] quadrant, $x>0, y<0$

Each point which we draw in the xy-plane must be in ordered pair form (a,b). Where a is called abscissa and b is called ordinate. Both (a,b) are called coordinate.

EXAMPLE

Determine the x-coordinate and y-coordinate of the following points. Mention the quadrant in which each points lies. Also plot the points.

i) (3,1) ii) (-2, -4)

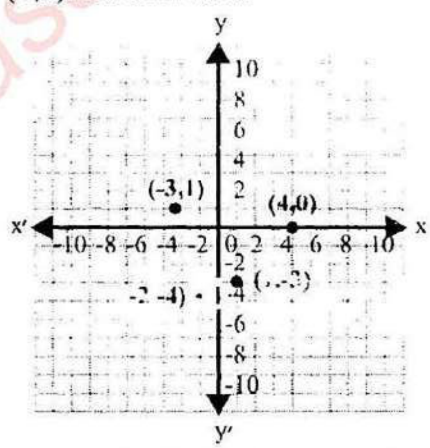
iii) (4,0) iv) (1, -3)

Solution:

i) In the point (-3,1), the x-coordinate is 3 and y-coordinate is 1. The point (-3,1) lies in quadrant II.

ii) In the point (-2,-4), the x-coordinate is -2 and y-coordinate is -4. The point (-2,-4), lies in quadrant III.

iii) In the point (4,0), the x-coordinate is 4 and y-coordinate is 0. The point (4,0) lies on x-axis.



iv) In the point (1,-3), the x-coordinate is 1 and y-coordinate is -3. The point (1,-3), lies in the quadrant IV.

Scale: 1 small square = 1 unit along x-axis

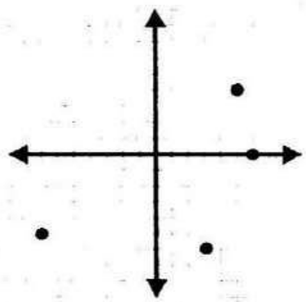
Scale: 1 small square = 1 unit along y-axis

EXAMPLE 5

The points A, B, C and D are shown in the graph. Write the coordinates of the points.

Solution:

The points are A(5,4), B(-6,-5), C(3,-6), D(6,0)

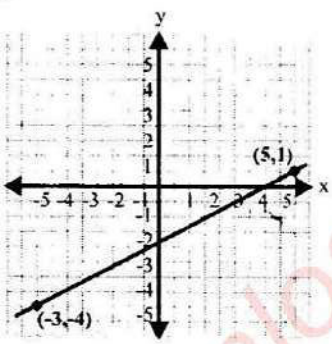


To draw line segment, triangle, rectangle, square and parallelogram by joining the set of given points:

EXAMPLE 7

Draw a triangle ABC by joining the points A(-2,3), B(4,2), C(1,-3).

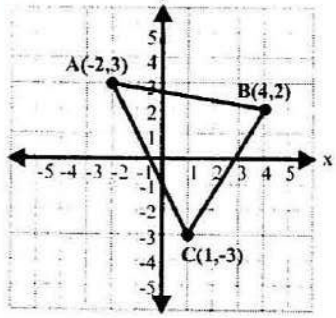
Solution:



Steps of Construction:

a) Plot the points A(-2,3), B(4,2) and C(1,-3) on the xy-plane.

b) Draw the straight line through the points A and B, B and C, A and C to get the sides AB, BC and AC of the triangle ABC. Thus ABC is the triangle.



EXAMPLE

Draw a parallelogram by joining the points O(0,0), A(1,4), B(4,2) and C(3,-2).

Solution:

a) Plot the points O(0,0), A(1,4), B(4,2) and C(3,-2) on the xy-plane.

b) Draw the straight line through O and A, A and B, B and C and O and C to get the sides OA, AB, BC, and OC respectively of the parallelogram or joined all the points by drawing straight line. OABC is a parallelogram.

EXERCISE 8.1

Q1: Determine the x and y coordinates of the following points:

i) $A(-7,5)$ **ii)** $B(0,7)$
iii) $C(-3,8)$ **iv)** $D(-3,-3)$
v) $E(10,12)$

Solution:

i) Given $A(-7,5)$ here
$x=-7,\ y=5$

ii) Given $B(0,7)$ here
$x=0,\ y=7$

iii) Given $C(-3,8)$ here
$x=-3,\ y=8$

iv) Given $D(-3,-3)$ here
$x=-3,\ y=-3$

v) Given $E(10,12)$ here
$x=10,\ y=12$

Q2: Mention the quadrant in which each of the following point lies.

i) $A(-1,\sqrt{2})$ **ii)** $B(-3,-2)$
iii) $C(5,5)$ **iv)** $D(3,-5)$
v) $E(-\sqrt{5},\sqrt{7})$

Solution:

i) Given point $A(-1,\sqrt{2})$, it lies in 2^{nd} quadrant.

ii) Given point $B(-3,-2)$, it lies in 3^{rd}

quadrant

iii) Given point $C(5,5)$, it lies in 1^{st} quadrant

iv) Given point $D(3,-5)$, it lies in 4^{th} quadrant

v) Given point $E(-\sqrt{5},\sqrt{7})$, it lies in 2^{nd} quadrant

Q3: Plot the points A, B, C and D on the xy-plane.

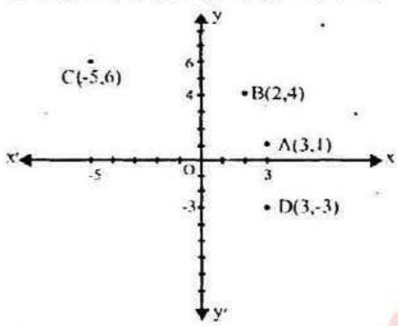
i) $A(3,1), B(2,4), C(-5,6), D(3,-3)$

ii) $A(-1,0), B(0,-1), C(2,-2), D(3,3)$

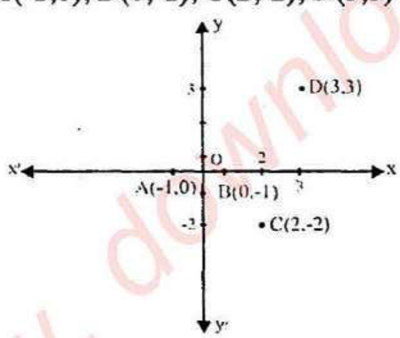
iii) $A(4,4), B(0,0), C(8,-6), D(-7,5)$

Solution:

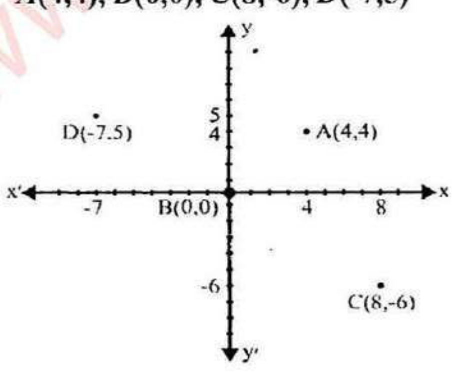
i) $A(3,1), B(2,4), C(-5,6), D(3,-3)$



ii) $A(-1,0), B(0,-1), C(2,-2), D(3,3)$



iii) $A(4,4), B(0,0), C(8,-6), D(-7,5)$

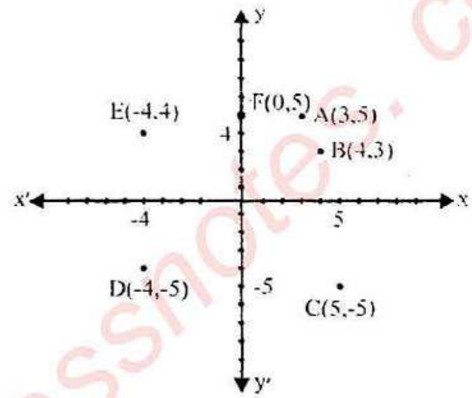


Q4: Plot the points associated with the ordered pairs $A(3,5)$, $B(4,3)$, $C(5,-5)$, $D(-4,-5)$, $E(-4,4)$ and $F(0,5)$.

Solution:

Given points are A(3,5), B(4,3), C(5,-5), D(-4,-5), E(-4,4) and F(0,5)

Their plotting is shown

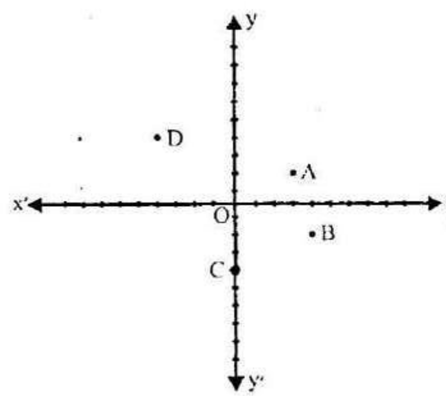


Q5: Write the coordinates of the points A, B, C and D in the given graph.

Solution:

From the graph, the coordinates of points A, B, C and D are written below:

$$A(3,2), B(4,-2), C(0,-3), D(-4,4)$$

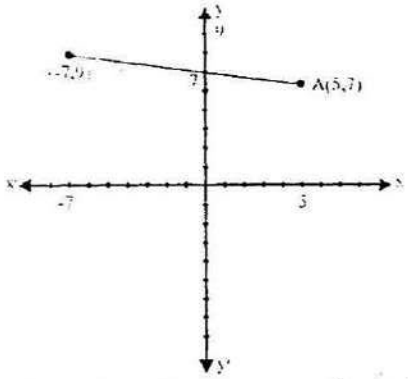


Q6: Draw a line segment by joining the points (5,7) and (-7, 9).

Solution:

Given points are

A(5, 7) and B(-7, 9) as shown in the figure.

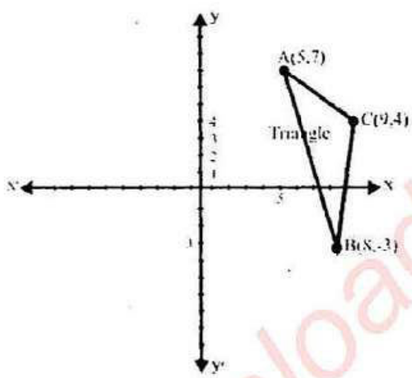


Q7: Draw a triangle *ABC* by joining the points A(5,7), B(8,-3), C(9,4).

Solution:

Given points are

A(5, 7), B(8, -3) and C(9, 4)

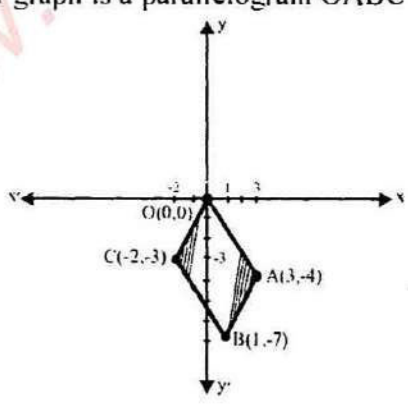


Q8: Draw a parallelogram *OABC* by joining the point O(0,0), A(3,-4), B(1,-7) and C(-2,-3).

Solution:

Given points are O(0,0), A(3,-4), B(1,-7) and C(-2,-3)

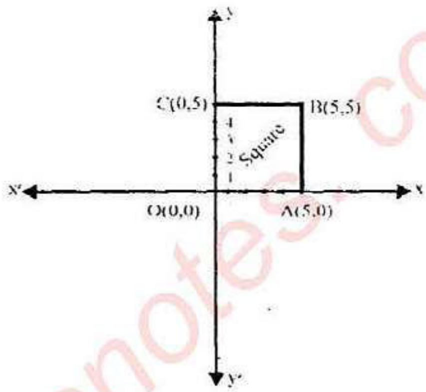
Their graph is a parallelogram OABC



Q9: Join the points O(0,0), A(5,0), B(5,5), C(0,5) to draw a square.

Solution: .

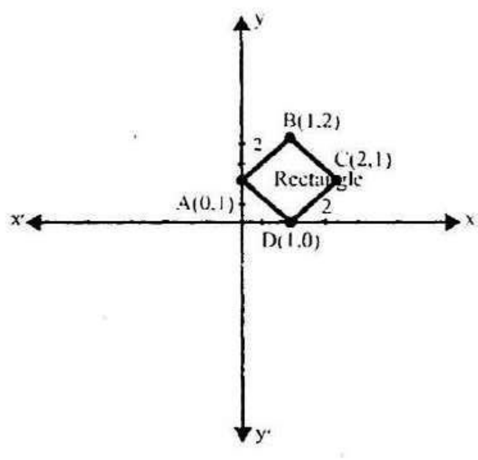
Given points are O(0,0), A(5,0), B(5,5), C(0,5). There graph is a square OABC.



Q10: By drawing the graph, that the points A(0,1), B(1,2), C(2,1) and D(1,0) are the vertices of a rectangle.

Solution:

Given points are A(0,1), B(1,2), C(2,1) and D(1,0). Their graph is a rectangle ABCD.



A linear Equation in two Variables:

Definition: An equation of the form $ax + by = c$ (where a, b and c are constants) is called a *linear equation in two variables*. The solution of a linear equation in two variables is a set of ordered pairs satisfying the given linear equation.

EXAMPLE 10

Graph: $3x - 5y = 10$

Solution:

We first solve for y:

$$3x - 5y = -10$$
$$\Rightarrow -y = -3x - 10$$
$$\Rightarrow y = \frac{3}{5}x + 2$$

Put $x = -5$, we get,

$$y = \frac{3}{5}x + 2 \Rightarrow \frac{3}{5}(-5) + 2$$
$$= -3 + 2 = -1$$

Similarly,

Put $x = 0$, we get $y = 2$

Put $x = 5$, we get $y = 5$

So the ordered pairs (-5, -1), (0,2), (5,5) satisfy the given linear equation.

Note that there are infinite number of ordered pairs that are solutions to the linear equation e.g. (10,8), (-10, -4) etc.

EXAMPLE 11

Draw the graph of $y = -2$.

Solution:

To draw the graph of $y = -2$, we first construct a table showing a few values of x and y.

x	-4	-2	0	2	3
y	-2	-2	-2	-2	-2

Plot the points (-4,2), (-2,-2), (0,-2), (2,-2), (3,-2) in the xy-plane. By joining the points we get the graph of $y = -2$.

In the graph of $y = -2$ the distance of the line is 2 units below the x-axis.

Drawing the graph of the equation of the for $x = a$.

The graph of the equation $x = a$ is a vertical line which is parallel to y-axis.

EXAMPLE 12

Graph the equation $x = 4$.

Solution:

We construct a table for some values of x and y.

x	4	4	4	4	4
y	-3	0	1	3	5

We plot the points (4,-3), (4,0) and (4,3) on the xy-plane. The line passing through the points (4,-3), (4,0) and (4,-3) is the graph of $x = 4$.

The distance of line from y-axis is 4 units.

Drawing the graph of the equation of the form $y = mx$.

EXAMPLE 13

Graph the equation $y = x$.

Solution:

Here $m = 1$, let $x = -3$ then $y = -3$

$x = 0$ then $y = 0$

$x = 1$ then $y = 1$

$x = 2$ then $y = 2$

$x = 4$ then $y = 4$

x	-3	0	1	2	4
y	-3	0	1	2	4

Now we plot the points (-3,-3), (1,1), (4,4) as shown in the graph. Thus the line joining the given points is the graph of $y = x$.

EXERCISE 8.2

Q1: Determine whether or not each of the following ordered pairs are solutions to the linear equations given:

a) **(6,1), $x - 5y = 1$**

b) **(5, -10), $2x + y = 6$**

c) **(0, 4), $x - y = 2$**

d) **(-3, 4), $x + 3y = 2$**

Solution:

a) (6,1), $x - 5y = 1$

Given $x - 5y = 1 \longrightarrow (1)$

Put $x = 6$, $y = 1$ in equation (1)

$$6 - 5(1) = 1$$
$$\Rightarrow 6 - 5 = 1 \Rightarrow 1 = 1$$

$L.H.S = R.H.S$ (True)

$\therefore (6,1)$ is the solution of given equation (1)

b) (5, -10), $2x + y = 6$

Given $2x + y = 6 \longrightarrow (1)$

Put $x = 5,\ y = -10$ in equation (1)

$$2(5) + (-10) = 6$$

$$\Rightarrow\ 10 - 10 = 6 \Rightarrow 0 \neq 6 \quad \text{(False)}$$

$$L.H.S \neq R.H.S$$

Which is not possible.

$\therefore$ (5,-10) is not the given solution of equation (1)

c) (0, 4), $x - y = 2$

Given $x - y = 2 \longrightarrow (1)$

Put $x = 0,\ y = 4$ in equation (1)

$$0 - 4 = 2$$

$$\Rightarrow\ -4 \neq 2 \qquad \text{(False)}$$

$$L.H.S \neq R.H.S$$

$\therefore$ (0,4) is not the solution of given equation (1)

d) (-3, 4), $x + 3y = 2$

Given $x + 3y = 2 \longrightarrow (1)$

Put $x = -3,\ y = 4$ in equation (1)

$$-3 + 3(4) = 2$$

$$\Rightarrow\ -3 + 12 = 2$$

$$\Rightarrow\ 9 \neq 2 \quad \text{(False)}$$

$$L.H.S \neq R.H.S$$

$\therefore$ (-3,4) is not the solution of equation (1)

Q2: Which of the following points are on the line $3x + 2y - 6 = 0$.

(1,1),(4,-3),(3,0),(2,0),(0,2),(0,3),(-2,6)

Solution:

Given equation of line

$$3x + 2y - 6 = 0 \rightarrow (1)$$

Put (1,1) in equation (1)

$$3(1) + 2(1) - 6 = 0$$

$$\Rightarrow\ 5 - 6 = 0$$

$$\Rightarrow\ -1 \neq 0 \qquad \text{(False)}$$

$$L.H.S \neq R.H.S$$

$\therefore (1,1)$ is not on the line.

Put $(4,-3)$ in equation (1)

$$3(4) + 2(-3) - 6 = 0$$

$$\Rightarrow\ 12 - 6 - 6 = 0$$

$$\Rightarrow 12 - 12 = 0$$

$$\Rightarrow\ 0 = 0 \qquad \text{(True)}$$

$$L.H.S = R.H.S$$

$\therefore (4,-3)$ is on the line.

Put $(3,0)$ in equation (1)

$$3(3) + 2(0) - 6 = 0$$

$$\Rightarrow\ 9 + 0 - 6 = 0$$

$$\Rightarrow\ 3 \neq 0 \qquad \text{(False)}$$

$$L.H.S \neq R.H.S$$

$\therefore (3,0)$ is not on the line.

Put (2,0) in equation (1)

$$3(2) + 2(0) - 6 = 0$$

$$\Rightarrow\ 6 - 6 = 0 \Rightarrow 0 = 0 \ \text{(True)}$$

$$L.H.S = R.H.S$$

$\therefore (2,0)$ is on the line.

Check for (0,2), (0,3), (-2,6)

Put (0,2) in equation (1)

$$\Rightarrow\ 3(0) + 2(2) - 6 = 0$$

$$\Rightarrow\ 0 + 4 - 6 = 0$$

$$\Rightarrow\ -2 \neq 0$$

$\therefore$ Point (0,2) is not on the line.

Put (0,3) in equation (1)

$$3(0) + 2(3) - 6 = 0$$

$$0 + 6 - 6 = 0$$

$$6 - 6 = 0$$

$$0 = 0 \qquad \text{(True)}$$

$$\text{L.H.S} = \text{R.H.S}$$

$\therefore (0,3)$ is on the line.

Put $(-2,6)$, put $x = -2, y = 6$ in eq. (1)

$$3(-2) + 2(6) - 6 = 0$$

$$\Rightarrow\ -6 + 12 - 6 = 0$$

$$\Rightarrow\ 0 = 0$$

$$-12 + 12 = 0$$

As $L.H.S = R.H.S$

The point (-2,6) is on the line.

Q3: Construct a table for four pair of values satisfying the equation $x - y = 4$.

Solution:

Given equation is $x - 4 = 4$

$$\Rightarrow x - 4 = y \Rightarrow y = x - 4 \rightarrow (1)$$

Put $x = -1$ in equation (1)

$y = -1 - 4 = -5$

Put $x = 0$ in equation (1)

$y = 0 - 4 = -4$

Put $x = 1$ in equation (1)

$y = 1 - 4 = -3$

Put $x = 2$ in equation (1)

$y = 2 - 4 = -2$

$\therefore$ The four pair of values satisfying the given equation are (-1,-5), (0,-4), (1,-3) and (2,-2).

Thus the table is

x	-1	0	1	2
$y = x - 4$	-5	-4	-3	-2

Q4: Draw the graphs of the equations:

a) $y - 2x = 6$ **b)** $y = 1 - x$

c) $y = 2$ **d)** $y = x$

Solution:

a) $y - 2x = 6$

Given $y - 2x = 6$

Or $y = 6 + 2x \longrightarrow (1)$

Put $x = -4$ in equation (1)

$y = 6 + 2(-4)$

$y = 6 - 8 = -2$

Put $x = -3$ in equation (1)

$y = 6 + 2(-3)$

$y = 6 - 6 = 0$

Put $x = -2$ in equation (1)

$y = 6 + 2(-2)$

$y = 6 - 4 = 2$

Put $x = -1$ in equation (1)

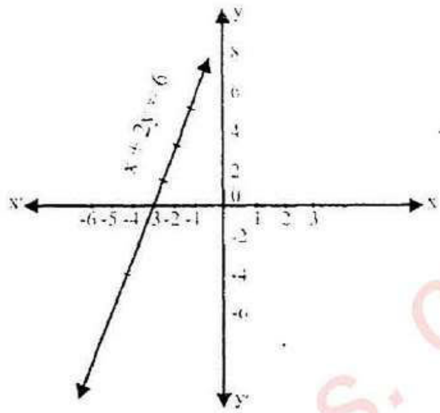
$y = 6 + 2(-1)$

$y = 6 - 2 = 4$

The table is under

x	-4	-3	-2	-1
y	-2	0	2	4

Now the graph of $x - 2y = 6$ is:



b) $y = 1 - x$

Given $y = 1 - x \longrightarrow (1)$

Put $x = -2$ in equation (1)

$y = 1 - (-2)$

$y = 1 + 2 = 3$

Put $x = -1$ in equation (1)

$y = 1 - (-1)$

$y = 1 + 1 = 2$

Put $x = 0$ in equation (1)

$y = 1 - 0 = 1$

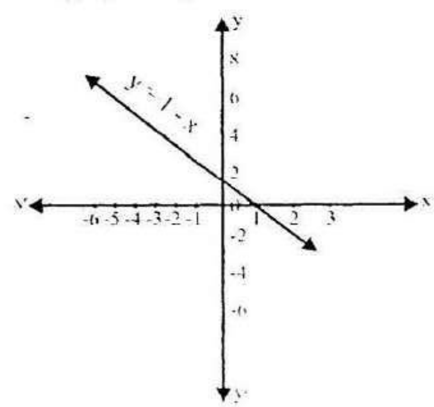
Put $x = 1$

$y = 1 - 1 = 0$

The table is under

x	-2	-1	0	1
y	3	2	1	0

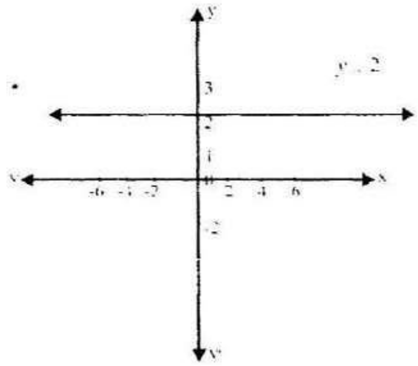
Now the graph of $y = 1 - x$ is:



c) $y = 2$

Given $y = 2$

Since y is constant and the graph of y intercept (y-axis) is 2 and parallel to x-axis.



d) $y = x$

Given $y = x \longrightarrow (1)$

Put $x = -1$ in equation $(1) \Rightarrow y = -1$

Put $x = 0$ in equation $(1) \Rightarrow y = 0$

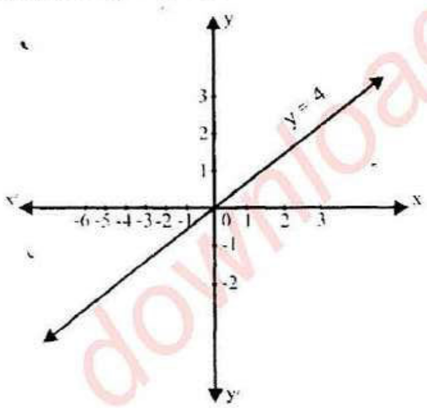
Put $x = 1$ in equation $(1) \Rightarrow y = 1$

Put $x = 2$ in equation $(1) \Rightarrow y = 2$

The table is under

x	0	1	-1	2
y	0	1	-1	2

The graph of $y = x$ is:



Q5: Complete each ordered pair so that it satisfies the given equation.

i) $3x - 7y = 21$; (,15),(14,)(-2,)

ii) $5y + 6x = 30$; (-5,),(,- 6), (,4)

iii) $2y + 9x = 36$; (6,),(0,), (,0)

iv) $4x + 7y = 56$; (,2), (,0), (0,)

Solution:

i) $3x - 7y = 21$; (,15),(14,)(-2,)

Given $3x - 7y = 21 \longrightarrow (1)$

And also given (,15), (14,), (−2,)

Put $y = 15$ in equation (1)

$$3x - 7(15) = 21$$
$$\Rightarrow 3x - 105 = 21$$
$$\Rightarrow 3x = 21 + 105$$
$$\Rightarrow 3x = 126$$
$$\Rightarrow x = \frac{\cancel{126}^{42}}{\cancel{3}} = 42$$

Put $x = 14$ in equation (1)

$$3(14) - 7y = 21$$
$$\Rightarrow 42 - 7y = 21$$
$$\Rightarrow 42 - 21 = 7y \text{ or } 7y = 21$$
$$\Rightarrow y = \frac{\cancel{21}^{3}}{\cancel{7}} = 3$$

Put $x = -2$ in equation (1)

$$3(-2) - 7y = 21$$
$$\Rightarrow -6 - 7y = 21$$
$$\Rightarrow -6 - 21 = 7y \text{ or } 7y = -27$$
$$\Rightarrow y = -\frac{27}{7}$$

Hence the complete ordered pairs are:

$$(42,15), (14,3), \left(-2, -\frac{27}{7}\right) \quad \text{Ans.}$$

ii) $5y + 6x = 30$; (-5,),(,- 6), (,4)

Given $5y + 6x = 30 \longrightarrow (1)$

And also given (−5,), (,−6), (,4)

Put $x = -5$ in equation (1)

$$5y + 6(-5) = 30$$
$$\Rightarrow 5y - 30 = 30$$
$$\Rightarrow 5y = 30 + 30$$
$$\Rightarrow 5y = 60$$
$$\Rightarrow y = \frac{\cancel{60}^{12}}{\cancel{5}} = 12$$

Put $y = -6$ in equation (1)

$$5(-6) + 6x = 30$$
$$\Rightarrow -30 + 6x = 30$$
$$\Rightarrow 6x = 30 + 30$$
$$\Rightarrow 6x = 60$$
$$\Rightarrow x = \frac{\cancel{60}^{10}}{\cancel{6}} = 10$$

Put $y=4$ in equation (1)

$5(4)+6x=30$

$\Rightarrow 20+6x=30$

$\Rightarrow 6x=30-20$

$\Rightarrow 6x=10$

$\Rightarrow x=\frac{\cancel{10}^{5}}{\cancel{6}_{3}}=\frac{5}{3}$

Hence the complete ordered pairs are:

$(-5,12),(10,-6),\left(\frac{5}{3},4\right)$ Ans.

iii) $2y+9x=36$; (6,),(0,), (,0)

Given $2y+9x=36\longrightarrow(1)$

And also given (6,), (0,),(,0)

Put $x=6$ in equation (1)

$2y+9(6)=36$

$\Rightarrow 2y+54=36$

$\Rightarrow 2y=36-54$

$\Rightarrow 2y=-18$

$\Rightarrow y=-\frac{\cancel{18}^{9}}{\cancel{2}}=-9$

Put $x=0$ in equation (1)

$2y+9(0)=36$

$\Rightarrow 2y+0=36 \Rightarrow 2y=36$

$\Rightarrow y=\frac{\cancel{36}^{18}}{\cancel{2}}=18$

Put $y=0$ in equation (1)

$2(0)+9x=36$

$\Rightarrow 0+9x=36 \Rightarrow 9x=36$

$\Rightarrow x=\frac{\cancel{36}^{4}}{\cancel{9}}=4$

Hence the required ordered pairs are:

$(6,-9),(0,18),(4,0)$ Ans.

iv) $4x+7y=56$; (,2), (,0), (0,)

Given $4x+7y=56\longrightarrow(1)$

And (,2), (,0), (0,)

Put $y=2$ in equation (1)

$\Rightarrow 4x+7(2)=56$

$\Rightarrow 4x+14=56$

$\Rightarrow 4x=56-14 \Rightarrow 4x=42$

$\Rightarrow x=\frac{\cancel{42}^{21}}{\cancel{2}}=\frac{21}{2}$

Put $y=0$ in equation (1)

$\Rightarrow 4x+7(0)=56$

$\Rightarrow 4x+0=56 \Rightarrow 4x=56$

$\Rightarrow x=\frac{\cancel{56}^{14}}{\cancel{4}}=14$

Put $x=0$ in equation (1)

$\Rightarrow 4(0)+7y=56$

$\Rightarrow 0+7y=56 \Rightarrow 7y=56$

$\Rightarrow y=\frac{\cancel{56}^{8}}{\cancel{7}}=8$

Hence the complete ordered pairs are:

$\left(\frac{21}{2},2\right),(14,0),(0,8)$ Ans.

Q6: The weight in kilogram and age in years of a person is expressed by the equation $y=2x$, where y (in kg) and x (in years). Draw the age-weight graph from the values of the following table:

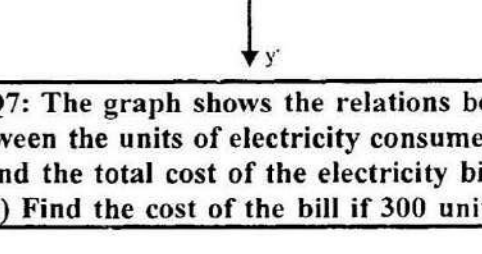
x	5	10	15	20	25	30
y	10	20	30	40	50	60

Scale:

Using the above table and draw its graph in xy-plane as follow:

Let 1 square = 5 on x-axis

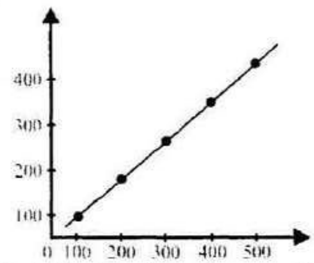
1 square = 10 on y-axis



Q7: The graph shows the relations between the units of electricity consumed and the total cost of the electricity bill (i) Find the cost of the bill if 300 units

(n) are consumed. (ii) The number of units used when the bill is Rs. 1500.

Solution:



Cost of the bill is using the graph we shall answer the following:

i) We can see from the graph that if 300 units are consumed, then

Cost $(c) = Rs.2500$

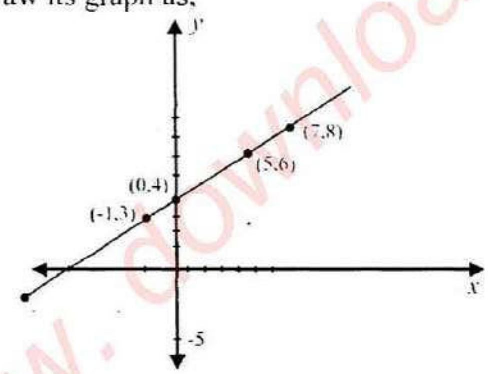
ii) When the bill is Rs. 1500, then the number of units used = 150 units.

Q8: Draw the graph from the following table by using a suitable scale.

x	0	-1	5	7	-4
y	4	3	6	8	-5

Solution:

Using values of the given table we can draw its graph as;

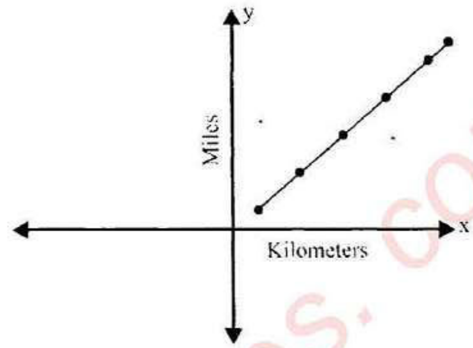


Conversion of Graphs:

If two quantities in a relation either both are increasing or decreasing then the graph of the relation will be straight line.

Conversion Graph of Miles into Kilometers:

Let the variable 'M' denotes the distance in miles indicated along y-axis and 'K' denotes along x-axis the distance in kilometers. The relation is represented by the equation $M = f(K)$.



$\because 1M = 1.60\,km$

M	1	2	3	4	5
Km	1.60	3.20	4.80	6.40	8.00

Scale:

1 mile = 1 unit along y-axis

$1\,Km = 1$ unit along x-axis

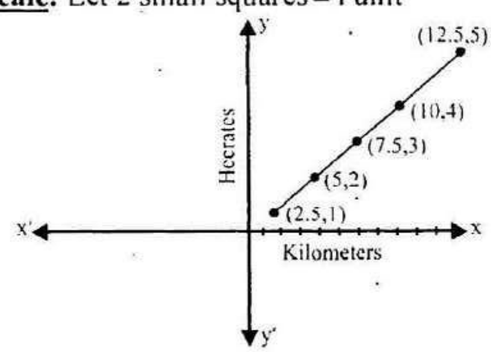
Conversion of Hectares into Acres:

Note: $1H = 2.5A$ where H = hectares, A = acres.

First we write a table in which hectare are converted into acre.

H	1	2	3	4	5	6
A	2.5	5	7.5	10	12.5	15

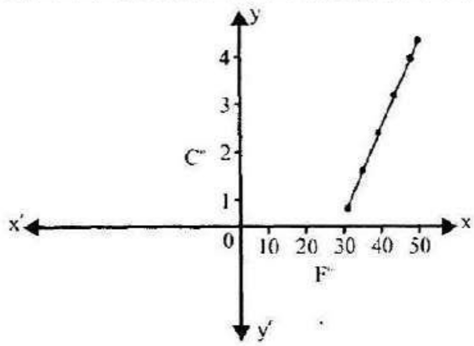
Scale: Let 2 small squares = 1 unit



Conversion of Degree Celsius into Degree Fahrenheit:

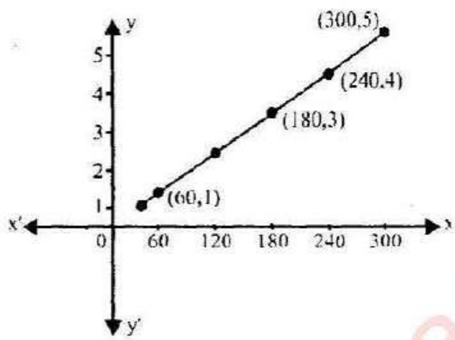
Note: $F^o = \frac{9}{5}C^o + 32$

C°	1	2	3	4	5
F°	33.8 = 34	35.6 = 36	37.4 = 37	39.2 = 39	41



<u>Conversion of Pakistani Currency into Foreign Currency:</u>

US #	1	2	3	4
PKR	160	320	480	640



EXERCISE 8.3

Q1: Using the conversion formula 1 mile $=1.6\,km$. Draw the conversion graph of Miles-Kilometers if distance in miles are given as:
1M, 3M, 4M, 5M (M is used for miles)

<u>Solution</u>:

Since $1M = 1.60\,Km$

The table values are;

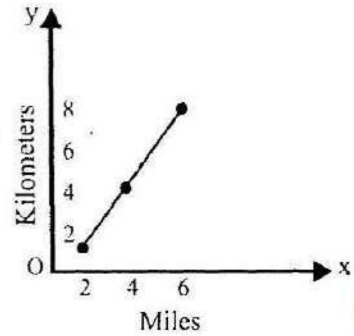
x (M)	1	3	4	5
y (Km)	1.60	4.80	6.40	8.00

<u>Scale</u>:

Take $1M = 1$ unit along x-axis

$1\,Km = 1$ unit along y-axis

The graph is shown below:



Q2: Draw the miles-kilometers graph. If distance in kilometers are given as 1km, 2km, 3km, 4km and 5km.

<u>Solution</u>:

As $1.60\,Km = 1\,M$

$$\Rightarrow 1\,Km = \frac{1}{1.60} = 0.625$$

Then

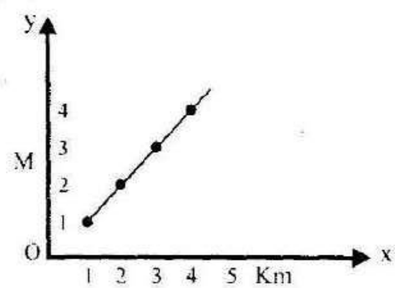
$x(Km)$	1	2	3	4	5
$y(M)$	0.625	1.25	1.875	2.5	3.125

<u>Scale</u>:

Take $1\,Km = 1$ unit along x-axis

$1M = 1$ unit along y-axis

The graph is shown below:



Q3: Given that 1 hectare = 2.5 acres (approximately), draw a conversion graph of hectare. Acre from the given values of hectare $2H$, $4H$, $8H$.

<u>Solution</u>:

Given $1H = 2.5A$, then

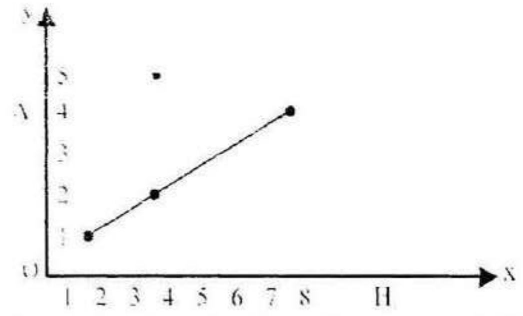
H	2	4	8
A	5	10	20

<u>Scale</u>:

$1H = 1$ unit along x-axis

$4A = 1$ unit along y-axis

The graph is shown below;



Q4: Convert the following temperature given in Celsius degree into Fahrenheit and then draw its graph. Degree Celsius=0°C, 2°C, 3°C. Conversion formula is: $F^{o} = \frac{9}{5}C^{o} + 32$

Solution: Degree Celsius $= 0^{o}, 2^{o}, 3^{o}$

Given $F^{o} = \frac{9}{5}C^{o} + 32$

Put $C^{o} = 0$

$\Rightarrow F^{o} = \frac{9}{5}(0) + 32 = 32^{o}$

Put $C^{o} = 2$

$\Rightarrow F^{o} = \frac{9}{5}(2) + 32 = \frac{18+160}{5}$

$\Rightarrow F^{o} = \frac{178}{5} = 35.6$

Put $C^{o} = 3$

$\Rightarrow F^{o} = \frac{9}{5}(3) + 32 = \frac{27+160}{5}$

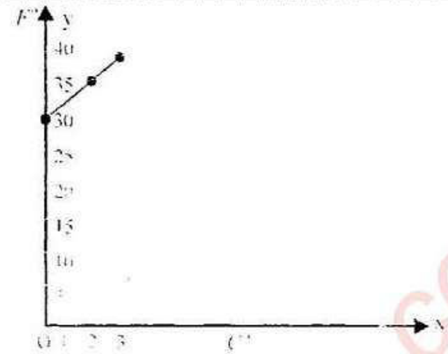
$\Rightarrow F^{o} = \frac{187}{5} = 37.4$

The table values are:

C^{o}	0	2	3
F^{o}	32	$35.6 = 36$	$37.4 = 37$

Now the graph let $1C^{o} = 1$ unit along x-axis

$5F^{o} = 1$ unit along y-axis



Q5: Draw the following conversion graph:

(i) PKR (Pakistan rupees) – US$ (1$, 3$, 5$)

(ii) PKR (Pakistani rupees) – GB£ (1£, 2£, 3£)

"$" is used for US dollar and "£" is used for pound sterling, where 1$ = Rs. (60) approximately.

1GB£ = Rs. 100 (approx)

Solution:

i) Given 1$ = Rs. 60 then.

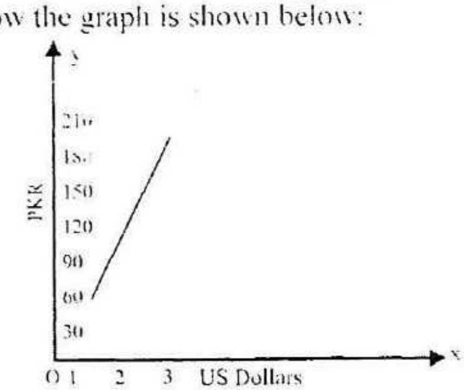
x(US$)	1	2	3
y(PKR)	60	120	180

Scale:

Let $1\$ = 1$ unit along x-axis

30 PKR = 1 unit along y-axis

Now the graph is shown below:



ii) Given 1GBB = Rs. 100 then,

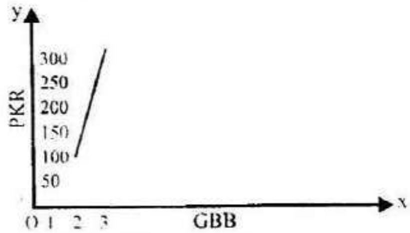
x(US$)	1	2	3
y(PKR)	100	200	300

Scale:

i) Let 1GBB = 1 unit along x-axis

50PKR = 1 unit along y-axis

Now the graph is shown below:



Q6: Solve the following simultaneous equations by using graphical method.

i) $2x + y = 3,\ x - y = 0$

ii) $y = 2x + 2,\ y = x - 1$

iii) $x + 4y = 5,\ 2x + 3y = 0$

iv) $3x + 5y = 2,\ 3x + 5y = 8$

v) $3x - 2y = 13,\ 2x + 3y = 13$

Solution:

i) $2x + y = 3,\ x - y = 0$

Given $2x + y = 3 \longrightarrow (1)$

$x - y = 0 \longrightarrow (2)$

The tables are constructed showing the values of x and y satisfying both equations.

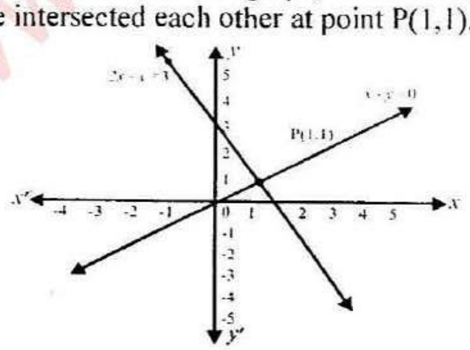
Table for $2x + y = 3$:

x	-1	0	1	2	3	4
y	5	3	1	-1	-3	-5

Table for $x - y = 0$

x	-1	0	1	2	3	4
y	-1	0	1	2	3	4

Take the points from both tables on the graph and then draw straight line joining all the points. In the graph, the two lines are intersected each other at point P(1,1).



Hence the solution of (1) and (2) is the point P(1,1).

ii) $y = 2x + 2,\ y = x - 1$

Given $y = 2x + 2 \longrightarrow (1)$

$y = x - 1 \longrightarrow (2)$

The following tables are constructed showing the values of x and y satisfying both equations.

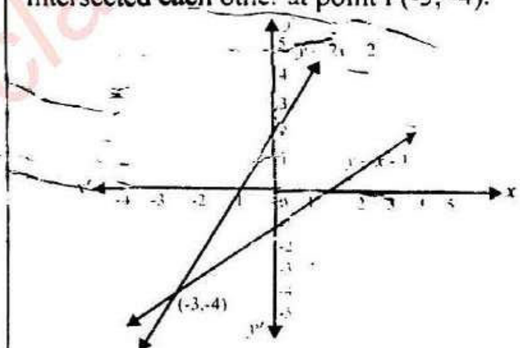
Table for $y = 2x + 2$

x	-3	-2	-1	0	1	2
y	-4	-2	0	2	4	6

Table for $y = x - 1$

x	-3	-2	-1	0	1	2
y	-4	-3	-2	-1	0	1

Take the points from both tables on the graph and then draw straight line joining all the points. In the graph, the two lines are intersected each other at point P(-3, -4).



Hence solution is the point (-3,-4).

iii) $x + 4y = 5,\ 2x + 3y = 0$

Given $x + 4y = 5 \longrightarrow (1)$

$2x + 3y = 0 \longrightarrow (2)$

The following tables are constructed showing the values of x and y satisfying both equations.

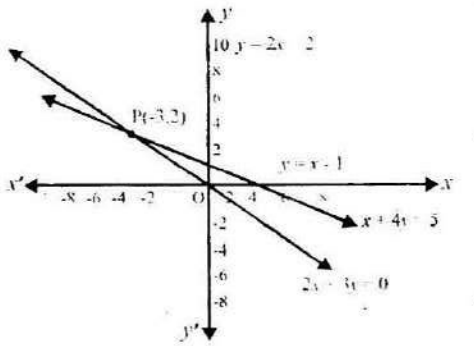
Table for $x + 4y = 5$

x	-7	-3	1	5	9	13
y	3	2	1	0	-1	-2

Table for $2x + 3y = 0$

x	-3	0	3	6	9	12
y	2	0	-2	-4	-6	-8

Take the points from both tables on the graph and then draw straight line joining all the points. In the graph, the two lines are intersected each other at point (-3,2).



Hence the point (-3, 2) is the required solution.

iv) $\mathbf{3x+5y=2,\ \ 3x+5y=8}$

Given $3x+5y=2 \longrightarrow (1)$

$3x+5y=8 \longrightarrow (2)$

The following tables are formed showing the values of x and y satisfying both equations.

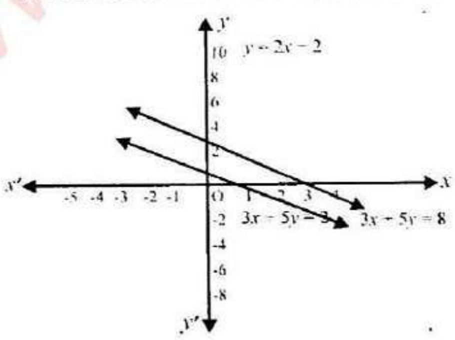
Table for $3x+5y=2$

x	-1	0	1	2	3	4
y	1	$\frac{2}{5}$	$\frac{-1}{5}$	$\frac{-4}{5}$	$-\frac{7}{5}$	-2

Table for $3x+5y=8$

x	-1	0	1	2	3
y	$\frac{11}{5}=2.5$	$\frac{8}{5}=1.6$	1	$\frac{2}{5}=.4$	$-\frac{1}{5}$

Take the points of both tables on the graph and then draw straight line joining all the points. Keep in mind that in the graph the two lines does not intersect each other.



Because they are parallel.

As two lines does not intersect each other, hence the solution set is $=\{\ \}$.

v) $\mathbf{3x-2y=13,\ \ 2x+3y=13}$

Given $3x-2y=13 \longrightarrow (1)$

$2x+3y=13 \longrightarrow (2)$

The following tables are constructed showing the values of x and y satisfying both equations.

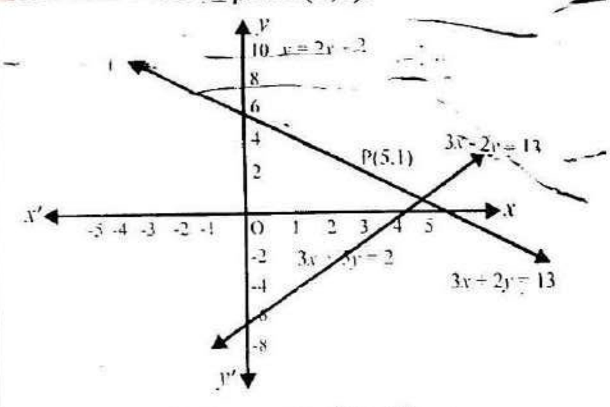
Table for $3x-2y=13$

x	-1	1	3	5	7
y	-8	-5	-2	1	4

Table for $2x+3y=13$

x	-4	-1	2	5	8
y	7	5	3	1	-1

Plot the points from both tables on the graph and then draw straight line joining all the points corresponding to the given equations. In the graph, the two lines intersect each other at point (5,1).



Hence the solution set is $\{(5,1)\}$. Ans.

Review Exercise 8

Q1: Select the correct answer.

i) The point (5, -2) is located in:
(a) Quadrant I
(b) Quadrant II
(c) Quadrant II
✓ **(d) Quadrant IV**

ii) The two coordinate axes intersect at an angle of:
(a) 30° (b) 60°
✓ **(c) 90°** (d) 45°

iii) The point (-3,8) is located in:
(a) Quadrant I
✓ **(b) Quadrant II**
(c) Quadrant III
(d) Quadrant IV

iv) The lines represented by the equations $x+y=1$ and $x+y=4$ are:
✓ **(a) Parallel**
(b) Inclined
(c) Intersecting
(d) Perpendicular

v. The point (-6,-6) is located in:
(a) Quadrant I
(b) Quadrant II
✓ **(c) Quadrant III**
(d) Quadrant IV

vi. The line $x=a$ where a is a real number is parallel to:
✓ **(a) y-axis**
(b) x-axis
(c) Both x-axis and y-axis
(d) Neither x-axis nor y-axis

vii. The point (2,11) is located in:
✓ **(a) Quadrant I**
(b) Quadrant II
(c) Quadrant III
(d) Quadrant IV

viii. The solution set of the lines $y=2$ and $y=3$ is:
(a) {5,3} (b) {4,0}
(c) {0,0} ✓ **(d) { }**

ix. The line $y=5$ is parallel to:
(a) y-axis
✓ **(b) x-axis**
(c) Both x-axis and y-axis
(d) Neither y-axis nor x-axis

x. Solution set of simultaneous equations $y=x+1$, $y=2x-2$
(a) $x=2,\ y=4$
✓ **(b) $x=3,\ y=4$**
(c) $x=2,\ y=4$
(d) $x=2,\ y=4$

Q2: Determine the x-coordinate and y-coordinate of the following points. Also mention the quadrant in which each point lies.

i) (2,3) ii) (-4, 5)
iii) (4,0)

Solution:

i) (2,3)
Given point (2,3)
x-coordinate is 2
y-coordinate is 3
Since in (2,3) the x-coordinate is positive and the y-coordinate is also positive. So the point (2,3) lies in the 1^{st} quadrant.
Hence x-coordinate is 2 and y-coordinate is 3 and the point (2,3) lies in the 1^{st} quadrant.

ii) (-4, 5)
Given point (-4, 5)
x-coordinate is -4
y-coordinate is 5
Since in (-4, 5) the x-coordinate is negative and the y-coordinate is positive. So the point (-4, 5) lies in the 2^{nd} quadrant.
Hence x-coordinate is -4 and y-coordinate is 5 and the point (-4,5) lies in the 2^{nd} qua-

drant.

iii) (4, 0)

Given point is (4,0)

x-coordinate is 4

y-coordinate is 0

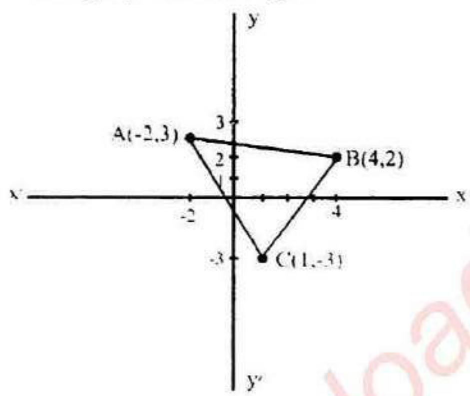
Since in (4,0) the x-coordinate is positive and the y-coordinate is zero. So the point (4,0) lies on x-axis.

Hence x-coordinate is 4 and y-coordinate is 0 and the point (4,0) lies on x-axis.

Q3: Draw a triangle ABC by joining the points A(-2,3), B(4,2), C(1,-3).

Solution:

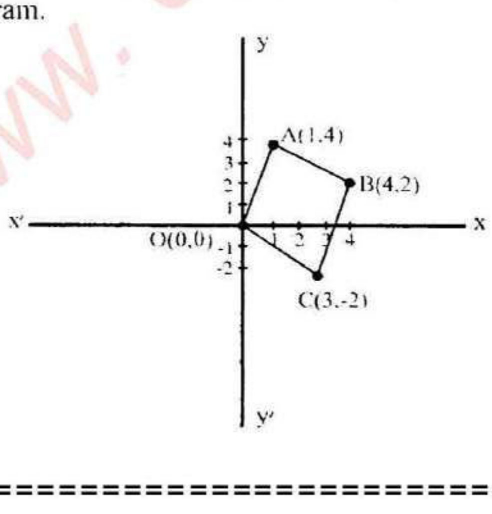
Given points are A(-2,3), B(4,2) and C(1,-3). Their graph is a triangle.



Q4: Draw a parallelogram by joining the points O(0,0), A(1,4), B(4,2) and C(3,-2).

Solution:

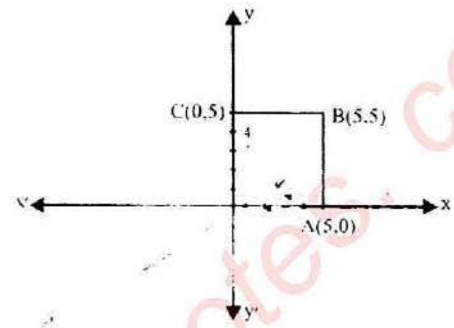
Given points are O(0,0), A(1,4), B(4,2) and C(3,-2). Their graph is a parallelogram.



Q5: By joining the points O(0,0), A(5,0), B(5,5) and C(0,5), draw a square.

Solution:

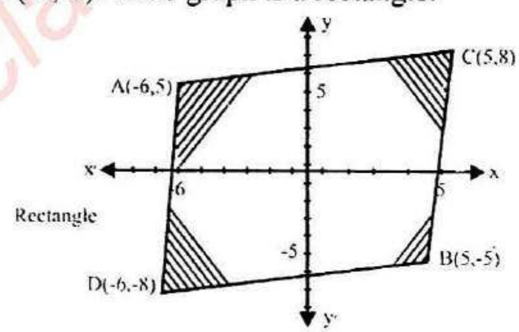
Given points are O(0,0), A(5,0), B(5,5), C(0,5). Their graph is a square.



Q6: Draw a rectangle by joining the points A(-6,5), B(5,-5), C(5,8) & D(-6, -8)

Solution:

Given points are A(-6,5), B(5,-5), C(5,8), D(-6,-8). Their graph is a rectangle.



Q7: Draw the graph of the equation $x + y = 4$.

Solution:

Given that $x + y = 4$

$\Rightarrow y = 4 - x \rightarrow (1)$

Put $x = -1$

$\Rightarrow y = 4 - (-1)$

$= 4 + 1 = 5$

Put $x = 0$

$\Rightarrow y = 4 - 0 = 4$

Put $x = 1$

$\Rightarrow y = 4 - 1 = 3$

Put $x = 2$

$\Rightarrow y = 4 - 2 = 2$

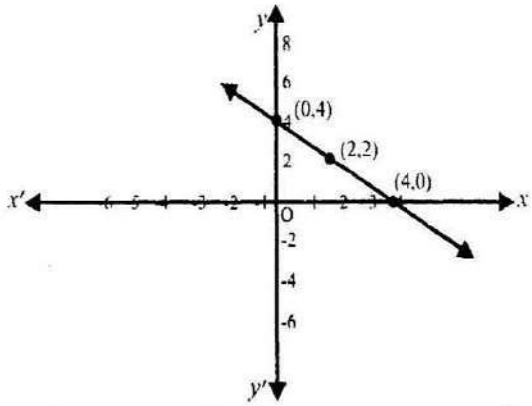
Put $x=3$ then $y=4-3=1$

Put $x=4$ then $y=4-4=0$

The table is under:

x	-1	0	1	2	3	4
y	5	4	3	2	1	0

The graph of $x+y=4$ is:

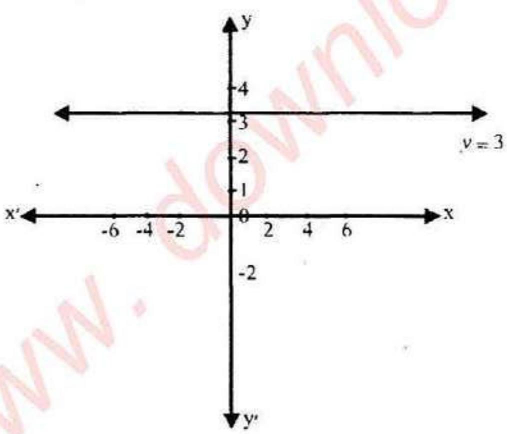


Q8: Draw the graph of $y=3$.

Solution:

Given $y=3$

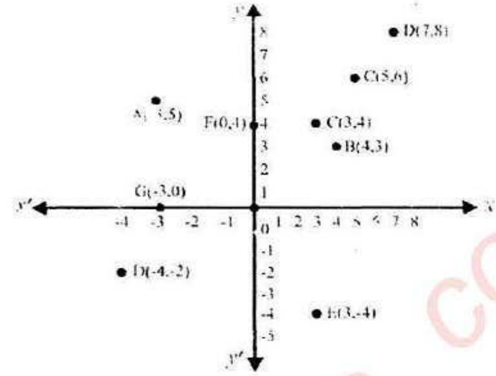
Since y is constant function and its graph will be parallel to x-axis, this type of line is called horizontal line which intersects y-axis at $y=3$.



Q9: Draw a graph from the following table by using a suitable scale.

x	-3	4	3	-4	3	0	-3	0
y	5	3	4	-2	-4	4	0	0

Solution: The graph of the given table can be shown in dots.



Think:

Q10: Solve the following system of equations graphically, $x+y=2$, $x-y=4$

Solution:

Given $x+y=2 \rightarrow (1)$

$x-y=4 \rightarrow (2)$

The tables are constructed showing the values of x and y satisfying both equations.

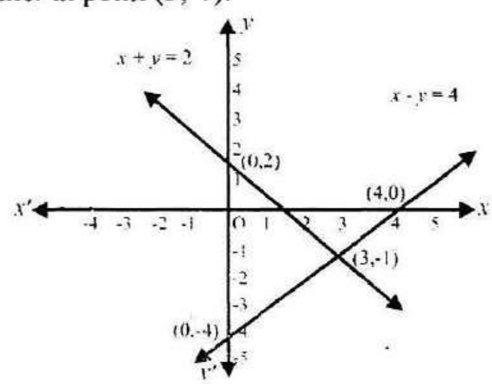
Table for $x+y=2$ or $y=2-x$

x	-1	0	1	2	3	4
y	3	2	1	0	-1	-2

Table for $x-y=4$ or $y=x-4$

x	-1	0	1	2	3	4
y	-5	-4	-3	-2	-1	0

Plot the points from both tables on the graph and then draw straight line joining all the points. The two lines intersect each other at point (3,-1).



Hence the solution of equation (1) and equation (2) is the point (3,-1).

***

Additional MCQs of Unit 8:
Linear Graphs and their Applications

1. $A \times B = \{(a,b) : a \in A \wedge b \in B\}$ is called..........
(a) Cartesian product (b) Relation (c) Function (d) Ordered pair
✓ **Ans. (a) Cartesian product**

2. The ordered pair $(a,b) =$ ....
(a) (b,a) (b) $(-a,-b)$ (c) $(-b,-a)$ (d) none
✓ **Ans. (d) none**

3. The two dimensional coordinate system is called..........plane.
(a) Real (b) Space (c) Cartesian (d) none
✓ **Ans. (c) Cartesian**

4. If $A = m$ and $b = n$ elements then $A \times B =$ ....elements.
(a) $m+n$ (b) 2^{mn} (c) $m \times n$ (d) none
✓ **Ans. (c) $m \times n$**

5. The graph of $ax + by + c = 0$ is..........
(a) Circle (b) Straight line (c) Rectangle (d) none
✓ **Ans. (b) Straight line**

6. The point $P(2,-3)$ lies in..........quadrant.
(a) 1^{st} (b) 2^{nd} (c) 3^{rd} (d) 4^{th}
✓ **Ans. (d) 4^{th}**

7. Which of the following points lies in 3^{rd} quadrant?
(a) $(2,-3)$ (b) $(-2,-3)$ (c) $(-2,3)$ (d) none
✓ **Ans. (b) (-2, -3)**

8. $ax + by = c$ if $a \neq 0$ and $b \neq 0$ is called linear equation in..........variables.
(a) One (b) Three (c) Two (d) none
✓ **Ans. (c) Two**

9. The graph of $y = a$ is..........line.
(a) Vertical (b) Horizontal (c) Straight (d) none
✓ **Ans. (b) Horizontal**

10. $x = a$ represents..........line.
(a) Vertical (b) Horizontal (c) Parallel (d) none
✓ **Ans. (a) Vertical**