



# Chapter # 1

## Matrices

### Exercise# 1.1

#### Matrix

A matrix is a rectangular array (arrangements) of real numbers enclosed in square brackets. Each number in a matrix is called an element or entry of the matrix. Matrices are mostly denoted by capital letters.

#### Examples

$$A = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

#### Rows and Columns of a Matrix

The rows of a matrix run horizontally, and the columns of a matrix run vertically.

#### Order or Dimension of a Matrix

The number of rows and columns that a matrix has is called order of a matrix.

#### Order of a matrix is represented by:

Order of matrix =  $m \times n$

OR

Order of matrix = m-by-n

Here "m" represents number of Rows

And "n" represents number of columns

#### Note

Order of a matrix is also called dimension or size of a matrix.

#### Examples

$$D = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

#### Examples

$$D = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

In this example

2, 5, 1, 3 all are the elements of a matrix D.

2, 5 and 1, 3 are the rows of a matrix D.

2, 1 and 5, 3 are the columns of matrix D.

As No. of Rows= 2

And No. of Columns= 2

So order is 2-by - 2 (OR)  $2 \times 2$

#### Equal Matrix

When two matrices of the same order and the corresponding elements are same.

### Exercise # 1.1

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Q1: Which of the following are square and which are rectangular matrices?

(i)  $A = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

As No. of Rows = No. of Columns

So it is Square matrix.

(ii)  $B = \begin{bmatrix} 6 & 3 & -1 \\ 1 & 5 & 2 \end{bmatrix}$

As No. of Rows  $\neq$  No. of Columns

So it is Rectangular matrix.

(iii)  $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

As No. of Rows = No. of Columns

So it is Square matrix.



(iv)  $D = \begin{bmatrix} -5 \end{bmatrix}$

As No. of Rows = No. of Columns  
So it is Square matrix.

(v)  $E = \begin{bmatrix} -3 & 4 \end{bmatrix}$

As No. of Rows  $\neq$  No. of Columns  
So it is Rectangular matrix.

(vi)  $F = \begin{bmatrix} -1 \\ 7 \end{bmatrix}$

As No. of Rows  $\neq$  No. of Columns  
So it is Rectangular matrix.

**Q2: List the order of the following matrices.**

(i)  $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 2 \end{bmatrix}$

As No. of Rows= 2  
And No. of Columns= 3  
So order is 2 – by – 3 (OR)  $2 \times 3$

(ii)  $B = \begin{bmatrix} -4 \end{bmatrix}$

As No. of Rows= 1  
And No. of Columns= 1  
So order is 1–by–1 (OR)  $1 \times 1$

(iii)  $C = \begin{bmatrix} 2 & 3 & -1 \\ 1 & 2 & 5 \end{bmatrix}$

As No. of Rows= 2  
And No. of Columns= 3  
So order is 2 – by – 3 (OR)  $2 \times 3$

(iv)  $F = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 4 & -1 \end{bmatrix}$

As No. of Rows= 3  
And No. of Columns= 2  
So order is 3–by–2(OR)  $3 \times 2$

(v)  $E = \begin{bmatrix} 3 & 2 \end{bmatrix}$

As No. of Rows= 1  
And No. of Columns= 2  
So order is 1–by–2 (OR)  $1 \times 2$

(vi)  $D = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 5 & 9 \\ 0 & 0 & 0 \end{bmatrix}$

As No. of Rows= 3  
And No. of Columns= 3  
So order is 3–by–3 (OR)  $3 \times 3$

**Q3: If  $A = \begin{bmatrix} 3 & 2 & -4 \\ -2 & 5 & 0 \\ 2 & 1 & 5 \\ -3 & 4 & 6 \end{bmatrix}$ , give the following elements.**

**Solution**

$$A = \begin{bmatrix} 3 & 2 & -4 \\ -2 & 5 & 0 \\ 2 & 1 & 5 \\ -3 & 4 & 6 \end{bmatrix}$$

$$\text{As } \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

**Answers:**

- (i)  $a_{12} = 2$
- (ii)  $a_{23} = 0$
- (iii)  $a_{32} = 1$
- (iv)  $a_{43} = 6$
- (v)  $a_{13} = -4$
- (vi)  $a_{43} = 6$



**Q4: Which of the following matrices are equal?**

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix},$$

$$B = \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix},$$

$$C = \begin{bmatrix} 1+1 & 3+2 \\ 4 & 2+1 \end{bmatrix}, \quad D = \begin{bmatrix} 2 & 4+1 \\ 1 & 3 \end{bmatrix}$$

**Solution:**

As  $C = \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix}$  and  $D = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

So A and D are equal i.e.  $A = D$

And B and C are equal i.e.  $B = C$

**Q5: Let  $A = \begin{bmatrix} 2 & -3 \\ u & 0 \end{bmatrix}$  and  $B = \begin{bmatrix} v & -3 \\ 5 & w \end{bmatrix}$ , for what values of  $u, v$ , and  $w$  are when A and B equal.**

**Solution**

$$A = \begin{bmatrix} 2 & -3 \\ u & 0 \end{bmatrix}, \quad B = \begin{bmatrix} v & -3 \\ 5 & w \end{bmatrix}$$

As A and B are equal. So

$$\begin{bmatrix} 2 & -3 \\ u & 0 \end{bmatrix} = \begin{bmatrix} v & -3 \\ 5 & w \end{bmatrix}$$

Now compare the corresponding elements

$$2 = v$$

$$\text{Or } v = 2$$

$$u = 5$$

$$0 = w$$

$$\text{Or } w = 0$$

**Q6: If**

$$\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$$

**find the values of a, b, c, x, y and z.**

**Solution:**

**As**

$$\begin{bmatrix} x+3 & z+4 & 2y-7 \\ -6 & a-1 & 0 \\ b-3 & -21 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ -6 & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$$

Now compare the corresponding elements

$$x + 3 = 0$$

$$x = -3$$

**Now**

$$z + 4 = 6$$

$$z = 6 - 4$$

$$z = 2$$

**Now**

$$2y - 7 = 3y - 2$$

$$-7 + 2 = 3y - 2y$$

$$-5 = y$$

$$y = -5$$

**Now**

$$a - 1 = -3$$

$$a = -3 + 1$$

$$a = -2$$

**Now**

$$0 = 2c + 2$$

$$0 - 2 = 2c$$

$$-2 = 2c$$

$$\frac{-2}{2} = c$$

$$-1 = c$$

$$c = -1$$

**Now**

$$b - 3 = 2b + 4$$

$$-3 - 4 = 2b - b$$

$$-7 = b$$

$$b = -7$$

### Answers:

$$a = -2$$

$$b = -7$$

$$c = -1$$

$$x = -3$$

$$y = -5$$

$$z = 2$$



**Q7:** Solve the following equation for  $a, b, c, d$ .

$$\begin{bmatrix} a+b & b+2c \\ 2c+d & 2a-d \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 8 & 0 \end{bmatrix}$$

**Solution**

$$\begin{bmatrix} a+b & b+2c \\ 2c+d & 2a-d \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 8 & 0 \end{bmatrix}$$

Now compare the corresponding elements

$$a + b = -1 \quad \text{equ(i)}$$

$$b + 2c = 4 \quad \text{equ(ii)}$$

$$2c + d = 8 \quad \text{equ(iii)}$$

$$2a - d = 0 \quad \text{equ(iv)}$$

Subtract equ(ii) from equ(i)

$$(a + b) - (b + 2c) = -1 - 4$$

$$a + b - b - 2c = -5$$

$$a - 2c = -5 \quad \text{equ(v)}$$

Now Add equ(iii) and equ(v)

$$2c + d + (a - 2c) = 8 + (-5)$$

$$2c + d + a - 2c = 8 - 5$$

$$d + a = 3 \quad \text{equ(vi)}$$

Now add equ(iv) and equ(vi)

$$2a - d + d + a = 0 + 3$$

$$2a + a = 3$$

$$3a = 3$$

$$a = \frac{3}{3}$$

$$a = 1$$

Put  $a = 1$  in equ(i)

$$1 + b = -1$$

$$b = -1 - 1$$

$$b = -2$$

Put  $b = -2$  in equ(ii)

$$-2 + 2c = 4$$

$$2c = 4 + 2$$

$$2c = 6$$

$$c = \frac{6}{2}$$

$$c = 3$$

Put  $c = 3$  in equ(iii)

$$2(3) + d = 8$$

$$6 + d = 8$$

$$d = 8 - 6$$

$$d = 2$$

**Answers:**

$$a = 1$$

$$b = -2$$

$$c = 3$$

$$d = 2$$

**Ex 1.1 End**

## Exercise # 1.2

### Types of matrices

#### Row matrix

A matrix having just one row is called row matrix.

$$A = \begin{bmatrix} 1 & 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 5 \end{bmatrix}$$

#### Column matrix

A matrix having just one column is called column matrix.

$$A = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \quad B = \begin{bmatrix} 5 \end{bmatrix}$$

#### Square matrix

A matrix in which number of rows and columns are equal is called square matrix.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 5 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 3 & 2 \\ -2 & 0 & 4 \\ -1 & 3 & 6 \end{bmatrix}$$

#### Rectangular matrix

A matrix in which number of rows and columns are not equal is called rectangular matrix.

$$A = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \quad D = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 9 & 8 \end{bmatrix}$$



### Ex # 1.2

#### Zero matrix or Null matrix

A matrix in which all the elements are zero is called Zero or Null matrix. A null matrix is generally denoted by  $O$ .

$$O = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad O = [0 \ 0 \ 0]$$

#### Diagonal matrix

A square matrix on which all elements are zero except diagonal elements is known as diagonal matrix.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

#### Scalar matrix

A matrix in which diagonal elements are same is called scalar matrix.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}, \quad C = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$$

#### Note

Every scalar matrix is a diagonal matrix but every diagonal matrix is not necessarily a scalar matrix.

#### Identity or Unit matrix

A matrix in which the diagonal elements are equal to "1" is called identity matrix. It is generally denoted by "I".

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

### Ex # 1.2

#### Transpose of a matrix

A matrix obtained by interchanging all rows and columns with each other is called transpose of a matrix. The transpose of a matrix  $B$  is written as  $B^t$ .

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^t = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

#### Symmetric matrix

In a square matrix, when  $A^t = A$ , then  $A$  is said to be symmetric matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

$$A^t = A$$

#### Skew-Symmetric matrix

In a square matrix, when  $A^t = -A$ , then  $A$  is said to be skew-symmetric matrix.

$$A = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$A^t = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$$

$$A^t = -\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$A^t = -A$$



## Exercise # 1.2

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Q1: Write the transpose of the following matrices.

(i)  $P = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

Solution:

$$P = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$$

Taking transpose on both sides

$$P^t = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}^t$$

$$P^t = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}$$

(ii)  $Q = \begin{bmatrix} l & m \\ n & p \end{bmatrix}$

Solution:

$$Q = \begin{bmatrix} l & m \\ n & p \end{bmatrix}$$

Taking transpose on both sides

$$Q^t = \begin{bmatrix} l & m \\ n & p \end{bmatrix}^t$$

$$Q^t = \begin{bmatrix} l & n \\ m & p \end{bmatrix}$$

(iii)  $R = [6]$

Solution

$$R = [6]$$

Taking transpose on both sides

$$R^t = [6]^t$$

$$R^t = [6]$$

Ex # 1.2

(iv)  $S = \begin{bmatrix} -5 & 1 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}$

Solution

$$S = \begin{bmatrix} -5 & 1 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}$$

Taking transpose on both sides

$$S^t = \begin{bmatrix} -5 & 1 \\ -2 & 1 \\ 4 & 4 \end{bmatrix}^t$$

$$S^t = \begin{bmatrix} -5 & -2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

(v)  $T = \begin{bmatrix} 6 & 7 & 8 \\ 13 & 1 & 3 \\ 2 & 4 & 5 \end{bmatrix}$

Solution

$$T = \begin{bmatrix} 6 & 7 & 8 \\ 13 & 1 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$

Taking transpose on both sides

$$T^t = \begin{bmatrix} 6 & 7 & 8 \\ 13 & 1 & 3 \\ 2 & 4 & 5 \end{bmatrix}^t$$

$$T^t = \begin{bmatrix} 6 & 13 & 2 \\ 7 & 1 & 4 \\ 8 & 3 & 5 \end{bmatrix}$$



## Ex # 1.2

Q2: Which of the following matrices are transpose of the each other?

$$A = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}, B = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

$$C = \begin{bmatrix} -3 & 1 & -1 \\ 4 & 2 & 7 \end{bmatrix}, D = \begin{bmatrix} -3 & 4 \\ 1 & 2 \\ -1 & 7 \end{bmatrix}$$

**Solution:**

**As**  $A^t = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} = B$

**And**  $B^t = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} = A$

Thus A and B are the transpose of each other.

**As**  $C^t = \begin{bmatrix} -3 & 4 \\ 1 & 2 \\ -1 & 7 \end{bmatrix} = D$

**And**  $D^t = \begin{bmatrix} -3 & 1 & -1 \\ 4 & 2 & 7 \end{bmatrix} = C$

Thus C and D are the transpose of each other.

Q3: Which of the following are symmetric?

(i)  $A = \begin{bmatrix} 5 & -7 \\ -1 & 5 \end{bmatrix}$

**Solution:**

$$A = \begin{bmatrix} 5 & -7 \\ -1 & 5 \end{bmatrix}$$

By taking transpose, we get

$$A^t = \begin{bmatrix} 5 & -1 \\ -7 & 5 \end{bmatrix}$$

$$A^t \neq A$$

Thus A is not symmetric matrix

## Ex # 1.2

(ii)  $B = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$

**Solution:**

$$B = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$$

By taking transpose, we get

$$B^t = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$$

$$B^t = B$$

Thus B is symmetric matrix

(iii)  $C = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$

**Solution**

$$C = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$$

By taking transpose, we get

$$C^t = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$C^t \neq C$$

Thus C is not symmetric matrix

(iv)  $D = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

**Solution**

$$D = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

By taking transpose, we get

$$D^t = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$D^t \neq D$$

Thus D is not symmetric matrix



## Ex # 1.2

Q4: Which of the following matrices are skew-symmetric?

$$(i) \quad A = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

**Solution**

$$A = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

By taking transpose, we get

$$A^t = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix}$$

$$A^t = -\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$A^t = -A$$

Thus A is a skew-symmetric matrix

$$(ii) \quad B = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

**Solution**

$$B = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

By taking transpose, we get

$$B^t = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$$

$$B^t = -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$$

$$B^t = -B$$

Thus B is a skew-symmetric matrix

$$(iii) \quad C = \begin{bmatrix} 0 & 7 \\ 7 & 0 \end{bmatrix}$$

**Solution**

$$C = \begin{bmatrix} 0 & 7 \\ 7 & 0 \end{bmatrix}$$

By taking transpose, we get

$$C^t = \begin{bmatrix} 0 & 7 \\ 7 & 0 \end{bmatrix}$$

## Ex # 1.2

$$C^t = -\begin{bmatrix} 0 & -7 \\ -7 & 0 \end{bmatrix}$$

$$C^t \neq -C$$

Thus C is not a skew-symmetric matrix

$$(iv) \quad D = \begin{bmatrix} 0 & 3 & 2 \\ -3 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$$

**Solution**

$$D = \begin{bmatrix} 0 & 3 & 2 \\ -3 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$$

By taking transpose, we get

$$D^t = \begin{bmatrix} 0 & -3 & -2 \\ 3 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$$

$$D^t = -\begin{bmatrix} 0 & 3 & 2 \\ -3 & 0 & 1 \\ -2 & -1 & 0 \end{bmatrix}$$

$$D^t = -D$$

Thus D is a skew-symmetric matrix



## Exercise # 1.3

### Conformability for Addition or Subtraction

When two matrices have the same order, then they are conformability for Addition and Subtraction.

### Adding and Subtracting of matrices

- (a) Addition can be obtained by adding the corresponding elements of the matrices.
- (b) Subtraction can be obtained by subtracting the corresponding elements of the matrices.

### Example

$$A = \begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3+4 & 8+0 \\ 4+1 & 6-9 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 3-4 & 8-0 \\ 4-1 & 6+9 \end{bmatrix}$$

$$A - B = \begin{bmatrix} -1 & 8 \\ 3 & 15 \end{bmatrix}$$

### Multiplication of a matrix by a real number

The real number is multiplying to each element of the matrix. The real number is called the scalar multiplication of that matrix i.e. 3 is scalar multiplication in the following matrix.

$$\text{Multiply } A = \begin{bmatrix} 6 & 2 \\ -3 & 1 \end{bmatrix} \text{ by } 3$$

$$3A = 3 \begin{bmatrix} 6 & 2 \\ -3 & 1 \end{bmatrix}$$

$$3A = \begin{bmatrix} 18 & 6 \\ -9 & 3 \end{bmatrix}$$

### Commutative Property w.r.t Addition

If two matrices of same order then  $A + B = B + A$  is called the Commutative law under addition.

### Associative Property w.r.t Addition

If three matrices of same order, then  $A + (B + C) = (A + B) + C$  is called the Associative law under addition.

### Additive Identity of matrices

Normally zero (0) is called additive identity. Thus Zero or Null matrix is additive identity matrix.

### Additive Inverse of a matrix

When the sum of two matrices is zero (0), then these matrices are called inverse of each other.  $A = -B$  or  $B = -A$

## Exercise # 1.3

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- Q1:** Let A & B be 2 – by – 3 matrices and let C & D be 2 – square matrices. Which of the following matrix operation are defined? For those which are defined, give the dimension of the resulting matrix.

- (i)  **$A + B$**

As the order of A is 2 – by – 3

And the order of B is 2 – by – 3

**So  $A + B$  are conformable**

- (ii)  **$B + D$**

As the order of B is 2 – by – 3

And the order of D is 2 – by – 2

**So  $B + D$  are not conformable**

- (iii)  **$3A - 2C$**

As the order of A is 2 – by – 3

And the order of C is 2 – by – 2

**So  $3A - 2C$  are not conformable**

- (iv)  **$7C - 2D$**

As the order of C is 2 – by – 2

And the order of D is 2 – by – 2

**So  $7C - 2D$  are conformable**



## Ex # 1.3

**Q2:** Multiply the following matrices by real numbers as indicated.

(i) Multiply  $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$  by 2

**Solution:**

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

**Multiply B.S by 2**

$$2A = 2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$2A = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

(ii) Multiply  $B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$  by  $p \in \mathbb{R}$

**Solution:**

$$B = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

**Multiply B.S by p**

$$pB = p \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$pB = \begin{bmatrix} pa & pb & pc \\ pd & pe & pf \end{bmatrix}$$

**Q3:** Find a matrix X such that  $4X = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 2 & 3 \\ -1 & 9 & 7 \end{bmatrix}$

**Solution:**

$$4X = \begin{bmatrix} 1 & 2 & 1 \\ 4 & 2 & 3 \\ -1 & 9 & 7 \end{bmatrix}$$

## Ex # 1.3

**Multiply B.S by  $\frac{1}{4}$**

$$\frac{1}{4} \times 4X = \frac{1}{4} \times \begin{bmatrix} 1 & 2 & 1 \\ 4 & 2 & 3 \\ -1 & 9 & 7 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \times \frac{1}{4} & 2 \times \frac{1}{4} & 1 \times \frac{1}{4} \\ 4 \times \frac{1}{4} & 2 \times \frac{1}{4} & 3 \times \frac{1}{4} \\ -1 \times \frac{1}{4} & 9 \times \frac{1}{4} & 7 \times \frac{1}{4} \end{bmatrix}$$

$$X = \begin{bmatrix} \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 1 & \frac{1}{2} & \frac{3}{4} \\ -\frac{1}{4} & \frac{9}{4} & \frac{7}{4} \end{bmatrix}$$

**Q4:** If  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$  and  $B = \begin{bmatrix} -3 & -2 \\ 1 & -5 \\ 4 & 3 \end{bmatrix}$ , then find  $3A - B$ .

**Solution:**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} -3 & -2 \\ 1 & -5 \\ 4 & 3 \end{bmatrix}$$

**Now**

$$3A - B = 3 \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} - \begin{bmatrix} -3 & -2 \\ 1 & -5 \\ 4 & 3 \end{bmatrix}$$

$$3A - B = \begin{bmatrix} 3 & 6 \\ 9 & 12 \\ 15 & 18 \end{bmatrix} - \begin{bmatrix} -3 & -2 \\ 1 & -5 \\ 4 & 3 \end{bmatrix}$$



$$3A - B = \begin{bmatrix} 3+3 & 6+2 \\ 9-1 & 12+5 \\ 15-4 & 18-3 \end{bmatrix}$$

$$3A - B = \begin{bmatrix} 6 & 8 \\ 8 & 17 \\ 11 & 15 \end{bmatrix}$$

Q5: Given  $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 3 & 0 \end{bmatrix}$ ,

find the matrix C such that  $A + 2B = C$

**Solution:**

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 3 & 0 \end{bmatrix}$$

As  $A + 2B = C$

Or  $C = A + 2B$

$$C = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + 2 \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 3 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 4 \\ 8 & 4 & 10 \\ 4 & 6 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1+6 & 2-2 & -3+4 \\ 5+8 & 0+4 & 2+10 \\ 1+4 & -1+6 & 1+0 \end{bmatrix}$$

$$C = \begin{bmatrix} 7 & 0 & 1 \\ 13 & 4 & 12 \\ 5 & 5 & 1 \end{bmatrix}$$

Q6: If  $A = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$  and  $B = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ , then find the matrix X such that  $2A + 3X = 5B$

### Ex # 1.3

**Solution:**

$$A = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}, B = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

As we have

$$2A + 3X = 5B$$

$$2 \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix} + 3X = 5 \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -4 \\ 8 & 4 \\ -10 & 2 \end{bmatrix} + 3X = \begin{bmatrix} 40 & 0 \\ 20 & -10 \\ 15 & 30 \end{bmatrix}$$

$$3X = \begin{bmatrix} 40 & 0 \\ 20 & -10 \\ 15 & 30 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 4 \\ -10 & 2 \end{bmatrix}$$

$$3X = \begin{bmatrix} 40-4 & 0+4 \\ 20-8 & -10-4 \\ 15+10 & 30-2 \end{bmatrix}$$

$$3X = \begin{bmatrix} 36 & 4 \\ 12 & -14 \\ 25 & 28 \end{bmatrix}$$

Multiply B.S by  $\frac{1}{3}$ , we get

$$\frac{1}{3} \times 3X = \begin{bmatrix} 36 \times \frac{1}{3} & 4 \times \frac{1}{3} \\ 12 \times \frac{1}{3} & -14 \times \frac{1}{3} \\ 25 \times \frac{1}{3} & 28 \times \frac{1}{3} \end{bmatrix}$$

$$X = \begin{bmatrix} 12 & \frac{4}{3} \\ 4 & \frac{-14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$



## Ex # 1.3

Q7: Find  $x, y, z$  and  $w$  if

$$3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ 3+w & 3 \end{bmatrix}$$

**Solution:**

$$3 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ 3+w & 3 \end{bmatrix}$$

$$\begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+3+w & 2w+3 \end{bmatrix}$$

**By comparing their corresponding elements**

$$3x = x + 4$$

$$3x - x = 4$$

$$2x = 4$$

$$x = \frac{4}{2}$$

$$x = 2$$

**Now**

$$3y = 6 + x + y$$

$$3y - y = 6 + 2 \quad \text{Putting } x = 2$$

$$2y = 8$$

$$y = \frac{8}{2}$$

$$y = 4$$

**Now**

$$3w = 2w + 3$$

$$3w - 2w = 3$$

$$w = 3$$

**Now**

$$3z = -1 + 3 + w$$

$$3z = 2 + 3 \quad \text{Putting } w = 3$$

$$3z = 5$$

$$z = \frac{5}{3}$$

**Answers:**

$$x = 2, y = 4, z = \frac{5}{3} \text{ and } w = 3$$

Q8: Find X and Y if  $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ ,

$$X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

**Solution:**

$$X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} \dots\dots\dots \text{equ (i)}$$

$$X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix} \dots\dots\dots \text{equ (ii)}$$

**Add equ (i) and equ (ii)**

$$X + Y + X - Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$X + X + Y - Y = \begin{bmatrix} 5+3 & 2+6 \\ 0+0 & 9-1 \end{bmatrix}$$

$$2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\frac{1}{2} \times 2X = \frac{1}{2} \times \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$X = \begin{bmatrix} 8 \times \frac{1}{2} & 8 \times \frac{1}{2} \\ 0 \times \frac{1}{2} & 8 \times \frac{1}{2} \end{bmatrix}$$

$$X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

**Put the values of X in equ (i)**

$$\begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix} + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$$

$$Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

$$Y = \begin{bmatrix} 5-4 & 2-4 \\ 0-0 & 9-4 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

**Thus**

$$X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix} \text{ and } Y = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

**Ex # 1.3**

**Q9:** Let  $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix}$  and  $C = \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}$ .

If  $c = 2$  and  $d = -4$  then verify that:

$$A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 3 & -1 \\ 0 & 4 \end{bmatrix}$$

$$c = 2, d = -4$$

$$(c + d)A = cA + dA$$

(i) **Solution:**

L.H.S:

$$(c + d)A$$

$$(c + d)A = (2 + (-4)) \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$$

$$(c + d)A = (2 - 4) \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$$

$$(c + d)A = -2 \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$$

$$(c + d)A = \begin{bmatrix} -4 & 6 \\ -8 & -10 \end{bmatrix}$$

RHS

$$cA + dA$$

$$cA + dA = 2 \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} + (-4) \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$$

$$cA + dA = \begin{bmatrix} 4 & -6 \\ 8 & 10 \end{bmatrix} + \begin{bmatrix} -8 & 12 \\ -16 & -20 \end{bmatrix}$$

$$cA + dA = \begin{bmatrix} 4 - 8 & -6 + 12 \\ 8 - 16 & 10 - 20 \end{bmatrix}$$

$$cA + dA = \begin{bmatrix} -4 & 6 \\ -8 & -10 \end{bmatrix}$$

Hence  $(c + d)A = cA + dA$

(ii)  $c(A + B) = cA + cB$

**Solution:**

LHS

$$c(A + B)$$

$$c(A + B) = 2 \left( \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix} \right)$$

**Ex # 1.3**

$$c(A + B) = 2 \begin{bmatrix} 2+2 & -3+5 \\ 4-1 & 5+3 \end{bmatrix}$$

$$c(A + B) = 2 \begin{bmatrix} 4 & 2 \\ 3 & 8 \end{bmatrix}$$

$$c(A + B) = \begin{bmatrix} 8 & 4 \\ 6 & 16 \end{bmatrix}$$

RHS

$$cA + cB = 2 \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} + 2 \begin{bmatrix} 2 & 5 \\ -1 & 3 \end{bmatrix}$$

$$cA + cB = \begin{bmatrix} 4 & -6 \\ 8 & 10 \end{bmatrix} + \begin{bmatrix} 4 & 10 \\ -2 & 6 \end{bmatrix}$$

$$cA + cB = \begin{bmatrix} 4+4 & -6+10 \\ 8-2 & 10+6 \end{bmatrix}$$

$$cA + cB = \begin{bmatrix} 8 & 4 \\ 6 & 16 \end{bmatrix}$$

Hence  $c(A + B) = cA + cB$

Proved

(iii)  $cd(A) = c(dA)$

**Solution:**

$$cd(A) = c(dA)$$

LHS

$$cd(A)$$

$$cd(A) = (2)(-4) \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$$

$$cd(A) = -8 \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$$

$$cd(A) = \begin{bmatrix} -16 & 24 \\ -32 & -40 \end{bmatrix}$$

LHS

$$c(dA)$$

$$c(dA) = 2 \left( -4 \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix} \right)$$

$$c(dA) = 2 \begin{bmatrix} -8 & 12 \\ -16 & -20 \end{bmatrix}$$

$$c(dA) = \begin{bmatrix} -16 & 24 \\ -32 & -40 \end{bmatrix}$$

Hence  $cd(A) = c(dA)$



**Q10** Let  $A = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 2 & 0 \\ -3 & 2 & 5 \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 & -1 & 2 \\ -5 & 3 & 4 \\ -3 & -4 & 0 \end{bmatrix}$  and  $C = \begin{bmatrix} 2 & -3 & 6 \\ 0 & 4 & -1 \\ -5 & 1 & 3 \end{bmatrix}$ . Compute the following if possible.

(i)  $A + 2B$

**Solution:**

$$A + 2B = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 2 & 0 \\ -3 & 2 & 5 \end{bmatrix} + 2 \begin{bmatrix} 3 & -1 & 2 \\ -5 & 3 & 4 \\ -3 & -4 & 0 \end{bmatrix}$$

$$A + 2B = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 2 & 0 \\ -3 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 4 \\ -10 & 6 & 8 \\ -6 & -8 & 0 \end{bmatrix}$$

$$A + 2B = \begin{bmatrix} -1+6 & 2-2 & 3+4 \\ 4-10 & 2+6 & 0+8 \\ -3-6 & 2-8 & 5+0 \end{bmatrix}$$

$$A + 2B = \begin{bmatrix} 5 & 0 & 7 \\ -6 & 8 & 8 \\ -9 & -6 & 5 \end{bmatrix}$$

(ii)  $3A - 4B$

**Solution:**

$$3A - 4B = 3 \begin{bmatrix} -1 & 2 & 3 \\ 4 & 2 & 0 \\ -3 & 2 & 5 \end{bmatrix} - 4 \begin{bmatrix} 3 & -1 & 2 \\ -5 & 3 & 4 \\ -3 & -4 & 0 \end{bmatrix}$$

$$3A - 4B = \begin{bmatrix} -3 & 6 & 9 \\ 12 & 6 & 0 \\ -9 & 6 & 15 \end{bmatrix} - \begin{bmatrix} 12 & -4 & 8 \\ -20 & 12 & 16 \\ -12 & -16 & 0 \end{bmatrix}$$

$$3A - 4B = \begin{bmatrix} -3-12 & 6+4 & 9-8 \\ 12+20 & 6-12 & 0-16 \\ -9+12 & 6+16 & 15-0 \end{bmatrix}$$

$$3A - 4B = \begin{bmatrix} -15 & 10 & 1 \\ 32 & -6 & -16 \\ 3 & 22 & 15 \end{bmatrix}$$

(iii)  $(A + B) - C$   
**Solution:**

$$(A+B)-C = \left( \begin{bmatrix} -1 & 2 & 3 \\ 4 & 2 & 0 \\ -3 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ -5 & 3 & 4 \\ -3 & -4 & 0 \end{bmatrix} \right) - \begin{bmatrix} 2 & -3 & 6 \\ 0 & 4 & -1 \\ -5 & 1 & 3 \end{bmatrix}$$

$$(A+B)-C = \begin{bmatrix} -1+3 & 2-1 & 3+2 \\ 4-5 & 2+3 & 0+4 \\ -3-3 & 2-4 & 5+0 \end{bmatrix} - \begin{bmatrix} 2 & -3 & 6 \\ 0 & 4 & -1 \\ -5 & 1 & 3 \end{bmatrix}$$

$$(A+B)-C = \begin{bmatrix} 2 & 1 & 5 \\ -1 & 5 & 4 \\ -6 & -2 & 5 \end{bmatrix} - \begin{bmatrix} 2 & -3 & 6 \\ 0 & 4 & -1 \\ -5 & 1 & 3 \end{bmatrix}$$

$$(A+B)-C = \begin{bmatrix} 2-2 & 1+3 & 5-6 \\ -1-0 & 5-4 & 4+1 \\ -6+5 & -2-1 & 5-3 \end{bmatrix}$$

$$(A+B)-C = \begin{bmatrix} 0 & 4 & -1 \\ -1 & 1 & 5 \\ -1 & -3 & 2 \end{bmatrix}$$

(iv)  $A + (B + C)$

**Solution:**

$$A + (B+C) = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 2 & 0 \\ -3 & 2 & 5 \end{bmatrix} + \left( \begin{bmatrix} 3 & -1 & 2 \\ -5 & 3 & 4 \\ -3 & -4 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -3 & 6 \\ 0 & 4 & -1 \\ -5 & 1 & 3 \end{bmatrix} \right)$$

$$A + (B+C) = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 2 & 0 \\ -3 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 3+2 & -1-3 & 2+6 \\ -5+0 & 3+4 & 4-1 \\ -3-5 & -4+1 & 0+3 \end{bmatrix}$$

$$A + (B+C) = \begin{bmatrix} -1 & 2 & 3 \\ 4 & 2 & 0 \\ -3 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 5 & -4 & 8 \\ -5 & 7 & 3 \\ -8 & -3 & 3 \end{bmatrix}$$

$$A + (B+C) = \begin{bmatrix} -1+5 & 2-4 & 3+8 \\ 4-5 & 2+7 & 0+3 \\ -3-8 & 2-3 & 5+3 \end{bmatrix}$$

$$A + (B+C) = \begin{bmatrix} 4 & -2 & 11 \\ -1 & 9 & 3 \\ -11 & -1 & 8 \end{bmatrix}$$

**Ex # 1.3**

**Q11 Prove that the following matrices commutative law of addition holds.**

$$A = \begin{bmatrix} 7 & 1 \\ 2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

**Solution:**

(i)  $A = \begin{bmatrix} 7 & 1 \\ 2 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

$$A + B = B + A$$

**LHS**

$$A + B = \begin{bmatrix} 7 & 1 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 7+1 & 1+1 \\ 2+2 & 4+2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 8 & 2 \\ 4 & 6 \end{bmatrix}$$

**RHS**

$$B + A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 7 & 1 \\ 2 & 4 \end{bmatrix}$$

$$B + A = \begin{bmatrix} 1+7 & 1+1 \\ 2+2 & 2+4 \end{bmatrix}$$

$$B + A = \begin{bmatrix} 8 & 2 \\ 4 & 6 \end{bmatrix} \text{ Hence } A + B = B + A \text{ Proved}$$

(ii)  $C = \begin{bmatrix} -3 & 4 & -5 \\ 2 & 3 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} -3 & -4 & 5 \\ 1 & 2 & 3 \end{bmatrix}$

**Solution:**

$$C = \begin{bmatrix} -3 & 4 & -5 \\ 2 & 3 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} -3 & -4 & 5 \\ 1 & 2 & 3 \end{bmatrix}$$

$$C + D = D + C$$

**LHS:**

$$C + D = \begin{bmatrix} -3 & 4 & -5 \\ 2 & 3 & 1 \end{bmatrix} + \begin{bmatrix} -3 & -4 & 5 \\ 1 & 2 & 3 \end{bmatrix}$$

$$C + D = \begin{bmatrix} -3-3 & 4-4 & -5+5 \\ 2+1 & 3+2 & 1+3 \end{bmatrix}$$

$$C + D = \begin{bmatrix} -6 & 0 & 0 \\ 3 & 5 & 4 \end{bmatrix}$$

**Ex # 1.3**

**RHS**

$$D + C = \begin{bmatrix} -3 & -4 & 5 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} -3 & 4 & -5 \\ 2 & 3 & 1 \end{bmatrix}$$

$$D + C = \begin{bmatrix} -3-3 & -4+4 & 5-5 \\ 1+2 & 2+3 & 3+1 \end{bmatrix}$$

$$D + C = \begin{bmatrix} -6 & 0 & 0 \\ 3 & 5 & 4 \end{bmatrix}$$

$$\text{Hence } C + D = D + C \quad \underline{\text{Proved}}$$

**Q12: Verify  $A + (B + C) = (A + B) + C$  for the following matrices.**

(i)  $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & -2 \\ 3 & 6 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 7 \\ -6 & -3 \end{bmatrix}$

**Solution:**

$$A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & -2 \\ 3 & 6 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 7 \\ -6 & -3 \end{bmatrix}$$

$$A + (B + C) = (A + B) + C$$

**LHS:**  $A + (B + C)$

$$B + C = \begin{bmatrix} 5 & -2 \\ 3 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 7 \\ -6 & -3 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 5+1 & -2+7 \\ 3-6 & 6-3 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 6 & 5 \\ -3 & 3 \end{bmatrix}$$

$$A + (B + C) = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 6 & 5 \\ -3 & 3 \end{bmatrix}$$

$$A + (B + C) = \begin{bmatrix} 2+6 & -3+5 \\ 4-3 & 1+3 \end{bmatrix}$$

$$A + (B + C) = \begin{bmatrix} 8 & 2 \\ 1 & 4 \end{bmatrix}$$

**RHS:**  $(A + B) + C$

$$A + B = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 5 & -2 \\ 3 & 6 \end{bmatrix}$$



$$A+B = \begin{bmatrix} 2+5 & -3-2 \\ 4+3 & 1+6 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 7 & -5 \\ 7 & 7 \end{bmatrix}$$

$$(A+B)+C = \begin{bmatrix} 7 & -5 \\ 7 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 7 \\ -6 & -3 \end{bmatrix}$$

$$(A+B)+C = \begin{bmatrix} 7+1 & -5+7 \\ 7-6 & 7-3 \end{bmatrix}$$

$$(A+B)+C = \begin{bmatrix} 8 & 2 \\ 1 & 4 \end{bmatrix}$$

**Hence  $A+(B+C) = (A+B)+C$  Proved**

$$(ii) \quad A = \begin{bmatrix} a & b & c \\ 3 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 4 \end{bmatrix}, \\ C = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \end{bmatrix}$$

**Solution**

$$A = \begin{bmatrix} a & b & c \\ 3 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 4 \end{bmatrix}, \\ C = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \end{bmatrix}$$

$$A + (B+C) = (A+B) + C$$

**LHS:**  $A+(B+C)$

$$B+C = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 4 \end{bmatrix} + \begin{bmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \end{bmatrix}$$

$$B+C = \begin{bmatrix} 1+2 & 2+1 & 3-1 \\ -2+3 & 1+1 & 4-2 \end{bmatrix}$$

$$B+C = \begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} a & b & c \\ 3 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 3 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} a+3 & b+3 & c+2 \\ 3+1 & 4+2 & 5+2 \end{bmatrix}$$

$$A+(B+C) = \begin{bmatrix} a+3 & b+3 & c+2 \\ 4 & 6 & 7 \end{bmatrix}$$

**RHS:**  $(A+B)+C$

$$A+B = \begin{bmatrix} a & b & c \\ 3 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & 4 \end{bmatrix}$$

$$A+B = \begin{bmatrix} a+1 & b+2 & c+3 \\ 3-2 & 4+1 & 5+4 \end{bmatrix}$$

$$A+B = \begin{bmatrix} a+1 & b+2 & c+3 \\ 1 & 5 & 9 \end{bmatrix}$$

$$(A+B)+C = \begin{bmatrix} a+1 & b+2 & c+3 \\ 1 & 5 & 9 \end{bmatrix} + \begin{bmatrix} 2 & 1 & -1 \\ 3 & 1 & -2 \end{bmatrix}$$

$$(A+B)+C = \begin{bmatrix} a+1+2 & b+2+1 & c+3-1 \\ 1+3 & 5+1 & 9-2 \end{bmatrix}$$

$$(A+B)+C = \begin{bmatrix} a+3 & b+3 & c+2 \\ 4 & 6 & 7 \end{bmatrix}$$

**Hence  $A + (B + C) = (A + B) + C$  Proved**

**Q13: Find the additive inverse of the following matrices.**

$$(i) \quad A = \begin{bmatrix} 3 & 4 \\ 6 & 2 \end{bmatrix}$$

**Additive Inverse:**

$$-A = \begin{bmatrix} -3 & -4 \\ -6 & -2 \end{bmatrix}$$

$$(ii) \quad B = \begin{bmatrix} a & -a & b \\ -c & a & -b \\ l & m & n \end{bmatrix}$$

**Additive Inverse:**

$$-B = \begin{bmatrix} -a & a & -b \\ c & -a & b \\ -l & -m & -n \end{bmatrix}$$

**Q14: Show that the following matrices are additive inverse of the each other.**

$$(i) \quad A = [1 \ -2 \ 3], B = [-1 \ 2 \ -3]$$

$$A+B = [1 \ -2 \ 3] + [-1 \ 2 \ -3]$$

$$A+B = [1-1 \ -2+2 \ 3-3]$$

$$A+B = [0 \ 0 \ 0]$$


**Ex # 1.3**

$$(ii) \quad C = \begin{bmatrix} a & -b \\ -c & d \end{bmatrix}, D = \begin{bmatrix} -a & b \\ c & -d \end{bmatrix}$$

$$C + D = \begin{bmatrix} a & -b \\ -c & d \end{bmatrix} + \begin{bmatrix} -a & b \\ c & -d \end{bmatrix}$$

$$C + D = \begin{bmatrix} a-a & -b+b \\ -c+c & d-d \end{bmatrix}$$

$$C + D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$(iii) \quad E = \begin{bmatrix} 1 & -2 & -4 \\ 2 & 1 & 3 \\ -3 & 4 & -2 \end{bmatrix}, F = \begin{bmatrix} -1 & 2 & 4 \\ -2 & -1 & -3 \\ 3 & -4 & 2 \end{bmatrix}$$

$$E + F = \begin{bmatrix} 1 & -2 & -4 \\ 2 & 1 & 3 \\ -3 & 4 & -2 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 4 \\ -2 & -1 & -3 \\ 3 & -4 & 2 \end{bmatrix}$$

$$E + F = \begin{bmatrix} 1-1 & -2+2 & -4+4 \\ 2-2 & 1-1 & 3-3 \\ -3+3 & 4-4 & -2+2 \end{bmatrix}$$

$$E + F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

## Exercise # 1.4

### Conformability for multiplication of matrices

Two matrices are conformable for multiplication, when number of columns of first matrix is equal to number of rows of second matrix.

### Multiplication of Matrices

For multiplication, multiply each element of a row of first matrix by the corresponding element of column of second matrix and then add these products.

### **OR**

Multiply first row of the matrix A with each corresponding elements of the first column of the matrix B and then add these products.

### Commutative Law of Multiplication

Commutative law of multiplication of matrices may or may not be holds.

$$(i) \quad AB \neq BA \text{ (Mostly)}$$

$$(ii) \quad AB = BA$$

### Associative Law under Multiplication

$A(BC) = (AB)C$  is called Associative law of matrices under multiplication

### Distributive Law of Multiplication over Addition

$$A(B + C) = AB + AC$$

$$(A + B)C = AC + BC$$

### Multiplicative Identity of a Matrix

Any matrix multiplied with Identity matrix will be the same matrix. e.g.  $A \cdot I = I \cdot A = A$

### Transpose of a Matrix

A matrix obtained by interchanging all rows and column with each other is called transpose of a matrix. The transpose of a matrix B is written as  $B^t$ .

### Note:

$$(AB)^t = B^t A^t$$



## Exercise # 1.4

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- Q1:** Show that which of the following matrices are conformable for multiplication.

$$A = \begin{bmatrix} a \\ b \end{bmatrix}, B = \begin{bmatrix} p & q \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}, D = \begin{bmatrix} p & r & s \end{bmatrix}$$

- (i) **AB**

As number of Columns in matrix  $A = 1$

And number of Rows in matrix  $B = 1$

**Thus AB is conformable for multiplication.**

- (ii) **AC**

As number of Columns in matrix  $A = 1$

And number of Rows in matrix  $C = 2$

**Thus AC is not conformable for multiplication.**

- (iii) **AD**

As number of Columns in matrix  $A = 1$

And number of Rows in matrix  $D = 1$

**Thus AD is conformable for multiplication.**

- (iv) **BA**

As number of Columns in matrix  $B = 2$

And number of Rows in matrix  $A = 2$

**Thus BA is conformable for multiplication.**

- (vi) **BC**

As number of Columns in matrix  $B = 2$

And number of Rows in matrix  $C = 2$

**Thus BC is conformable for multiplication.**

- (vii) **BD**

As number of Columns in matrix  $B = 2$

And number of Rows in matrix  $D = 1$

**Thus BD is not conformable for multiplication.**

- (viii) **CA**

As number of Columns in matrix  $C = 2$

And number of Rows in matrix  $A = 2$

**Thus CA is conformable for multiplication.**

- (ix) **CB**

As number of Columns in matrix  $C = 2$

And number of Rows in matrix  $B = 1$

**Thus CB is not conformable for multiplication.**

- (x) **CD**

As number of Columns in matrix  $C = 2$

And number of Rows in matrix  $D = 1$

**Thus CD is not conformable for multiplication.**

- (xi) **DA**

As number of Columns in matrix  $D = 3$

And number of Rows in matrix  $A = 2$

**Thus DA is not conformable for multiplication.**

- xii) **DB**

As number of Columns in matrix  $D = 3$

And number of Rows in matrix  $B = 1$

**Thus DB is not conformable for multiplication.**

- xiii) **DC**

As number of Columns in matrix  $D = 3$

And number of Rows in matrix  $C = 2$

**Thus DC is not conformable for multiplication.**

- Q2:

$$\text{If } A = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

- (i) Is it possible to find **AB**?

- (ii) Is it possible to find **BA**?

- (iii) Find the possible product/ products.

Solution:

$$A = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

- (i) **AB**

As number of Columns in matrix  $A = 2$

And number of Rows in matrix  $B = 2$

**Thus AB is possible for multiplication.**

- (ii) **BA**

As number of Columns in matrix  $B = 1$

And number of Rows in matrix  $A = 2$

**Thus BA is not possible for multiplication.**



## Ex # 1.4

(iii) Now

$$AB = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

$$AB = \begin{bmatrix} (-1)(3) + (0)(-2) \\ (2)(3) + (1)(-2) \end{bmatrix}$$

$$AB = \begin{bmatrix} -3+0 \\ 6+(-2) \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 \\ 6-2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

Q3:  $A = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}, C = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$  and

$$D = \begin{bmatrix} 1 & -2 \\ -\frac{1}{2} & \frac{2}{3} \end{bmatrix} \text{ Find (i) } AB \text{ and (ii) } CD$$

**Solution:**

(i)  $AB = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$

$$AB = \begin{bmatrix} (4)(1) + (1)(-3) & (4)(-1) + (1)(4) \\ (3)(1) + (1)(-3) & (3)(-1) + (1)(4) \end{bmatrix}$$

$$AB = \begin{bmatrix} 4+(-3) & -4+4 \\ 3+(-3) & -3+4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4-3 & 0 \\ 3-3 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(ii)  $CD = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -\frac{1}{2} & \frac{2}{3} \end{bmatrix}$

$$CD = \begin{bmatrix} (3)(1) + (4)\left(-\frac{1}{2}\right) & (3)(-2) + (4)\left(\frac{2}{3}\right) \\ (1)(1) + (2)\left(-\frac{1}{2}\right) & (1)(-2) + (2)\left(\frac{2}{3}\right) \end{bmatrix}$$

$$CD = \begin{bmatrix} 3+(2)(-1) & -6+\frac{8}{3} \\ 1+(1)(-1) & -2+\frac{4}{3} \end{bmatrix}$$

$$CD = \begin{bmatrix} 3-2 & \frac{-18+8}{3} \\ 1-1 & \frac{-6+4}{3} \end{bmatrix}$$

$$CD = \begin{bmatrix} 1 & \frac{-10}{3} \\ 0 & \frac{-2}{3} \end{bmatrix}$$

Q4: Given that  $A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$  (i) Find  $AB$

**(ii) Does  $BA$  exist?****Solution:**

(i)  $AB = \begin{bmatrix} 2 & 1 \\ 3 & 0 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} (2)(1) + (1)(2) & (2)(0) + (1)(1) \\ (3)(1) + (0)(2) & (3)(0) + (0)(1) \\ (-1)(1) + (4)(2) & (-1)(0) + (4)(1) \end{bmatrix}$$

$$AB = \begin{bmatrix} 2+2 & 0+1 \\ 3+0 & 0+0 \\ -1+8 & 0+4 \end{bmatrix} \rightarrow AB = \begin{bmatrix} 4 & 1 \\ 3 & 0 \\ 7 & 4 \end{bmatrix}$$



**(ii) Does  $BA$  exists?**

**BA**

As number of Columns in matrix  $B = 2$

And number of Rows in matrix  $A = 3$

Thus  $BA$  is not possible for multiplication.

**Q5:** If  $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ ,  $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$  then show that

$AB \neq BA$

**Solution:**

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$AB \neq BA$

**LHS:**

$$AB = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} (1)(0)+(1)(0) & (1)(1)+(1)(0) \\ (0)(0)+(0)(0) & (0)(1)+(0)(0) \end{bmatrix}$$

$$AB = \begin{bmatrix} 0+0 & 1+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

**RHS**

$$BA = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

**Hence**

$AB \neq BA$     Proved:

**Ex # 1.4**

**Q6:** If  $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ , then find  $A \times A$

**Solution:**

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A \times A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$$A \times A = \begin{bmatrix} (1)(1)+(1)(0) & (1)(1)+(1)(0) \\ (0)(1)+(0)(0) & (0)(1)+(0)(0) \end{bmatrix}$$

$$A \times A = \begin{bmatrix} 1+0 & 1+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$A \times A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

**Q7:** If  $A = \begin{bmatrix} -2 & 3 \\ 2 & -1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$ . Is  $AB = BA$

**Solution:**

$$AB = \begin{bmatrix} -2 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$$

$$AB = \begin{bmatrix} (-2)(1)+(3)(2) & (-2)(-1)+(3)(4) \\ (2)(1)+(-1)(2) & (2)(-1)+(-1)(4) \end{bmatrix}$$

$$AB = \begin{bmatrix} -2+6 & 2+12 \\ 2-2 & -2-4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 14 \\ 0 & -6 \end{bmatrix}$$

**Now**

$$BA = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 2 & -1 \end{bmatrix}$$

$$BA = \begin{bmatrix} (1)(-2)+(-1)(2) & (1)(3)+(-1)(-1) \\ (2)(-2)+(4)(2) & (2)(3)+(4)(-1) \end{bmatrix}$$

$$BA = \begin{bmatrix} -2-2 & 3+1 \\ -4+8 & 6-4 \end{bmatrix}$$

$$BA = \begin{bmatrix} -4 & 4 \\ 4 & 2 \end{bmatrix}$$

**Hence  $AB$  is not equal to  $BA$**



**Q8:** If  $A = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & -2 \end{bmatrix}$ ,  $C = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ , then

(i) find  $(AB)C$  and  $A(BC)$

**Solution:**

$$A = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -2 \end{bmatrix}, C = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$(AB)C$ :

$$AB = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \end{bmatrix}$$

$$AB = \begin{bmatrix} (-1)(1) & (-1)(-2) \\ (1)(1) & (1)(-2) \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}$$

Now

$$(AB)C = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} (-1)(3) + (2)(-1) & (-1)(1) + (2)(2) \\ (1)(3) + (-2)(-1) & (1)(1) + (-2)(2) \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -3 - 2 & -1 + 4 \\ 3 + 2 & 1 - 4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 & 3 \\ 5 & -3 \end{bmatrix}$$

$A(BC)$ :

$$BC = \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$BC = \begin{bmatrix} (1)(3) + (-2)(-1) & (1)(1) + (-2)(2) \end{bmatrix}$$

$$BC = \begin{bmatrix} 3 + 2 & 1 - 4 \end{bmatrix}$$

$$BC = \begin{bmatrix} 5 & -3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \begin{bmatrix} 5 & -3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} (-1)(5) & (-1)(-3) \\ (1)(5) & (1)(-3) \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 & 3 \\ 5 & -3 \end{bmatrix}$$

#### Ex # 1.4

(ii) Determine whether  $(AB)C = A(BC)$

Yes

$$(AB)C = \begin{bmatrix} -5 & 3 \\ 5 & -3 \end{bmatrix} = A(BC) = \begin{bmatrix} -5 & 3 \\ 5 & -3 \end{bmatrix}$$

(iii) Interpret which law of multiplication this result shows?

This shows Associative Property of Multiplication

**Q9:** Verify that  $(A(B + C)) = AB + AC$  for the following matrices.

(i)  $A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$

**Solution:**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, C = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$$

$$A(B+C) = AB + AC$$

LHS:  $A(B + C)$

Now

$$B + C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 1+3 & 0-1 \\ 0+0 & 2+2 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$$

Now

$$A(B+C) = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 4 & -1 \\ 0 & 4 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} (1)(4) + (2)(0) & (1)(-1) + (2)(4) \\ (3)(4) + (-1)(0) & (3)(-1) + (-1)(4) \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 4+0 & -1+8 \\ 12+0 & -3-4 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 4 & 7 \\ 12 & -7 \end{bmatrix}$$

**Ex # 1.4****RHS:  $AB + AC$** **Now**

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} (1)(1) + (2)(0) & (1)(0) + (2)(2) \\ (3)(1) + (-1)(0) & (3)(0) + (-1)(2) \end{bmatrix}$$

$$AB = \begin{bmatrix} 1+0 & 0+4 \\ 3+0 & 0-2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$$

**Now**

$$AC = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}$$

$$AC = \begin{bmatrix} (1)(3) + (2)(0) & (1)(-1) + (2)(2) \\ (3)(3) + (-1)(0) & (3)(-1) + (-1)(2) \end{bmatrix}$$

$$AC = \begin{bmatrix} 3+0 & -1+4 \\ 9+0 & -3-2 \end{bmatrix}$$

$$AC = \begin{bmatrix} 3 & 3 \\ 9 & -5 \end{bmatrix}$$

**Now**

$$AB + AC = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 9 & -5 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 1+3 & 4+3 \\ 3+9 & -2-5 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 4 & 7 \\ 12 & -7 \end{bmatrix}$$

**Hence  $A(B+C) = AB + AC$       Proved:**

(ii)  $A = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, C = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

**Solution**

$$A = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, C = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

**$A(B + C) = AB + AC$**

**LHS:  $A(B + C)$**

**Now**

$$B + C = \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 1-1 \\ 2+1 \end{bmatrix}$$

$$B + C = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

**Now**

$$A(B + C) = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} (3)(0) + (-1)(3) \\ (0)(0) + (2)(3) \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} 0-3 \\ 0+6 \end{bmatrix}$$

$$A(B + C) = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$$

**RHS:  $AB + AC$**

$$AB = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} (3)(1) + (-1)(2) \\ (0)(1) + (2)(2) \end{bmatrix}$$

$$AB = \begin{bmatrix} 3-2 \\ 0+4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$AC = \begin{bmatrix} 3 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$AC = \begin{bmatrix} (3)(-1) + (-1)(1) \\ (0)(-1) + (2)(1) \end{bmatrix}$$

$$AC = \begin{bmatrix} -3-1 \\ 0+2 \end{bmatrix}$$

$$AC = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

**Ex # 1.4****Now**

$$AB + AC = \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} 1-4 \\ 4+2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$$

**Hence**  $A(B+C) = AB + AC$     **Proved:**

**Q10:** Let  $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ,  $A = \begin{bmatrix} 5 & -3 \\ 4 & 6 \end{bmatrix}$ ,  $B = \begin{bmatrix} -7 & 3 \\ 2 & 8 \end{bmatrix}$

**(i)  $AI$** **Solution:**

$$AI = \begin{bmatrix} 5 & -3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AI = \begin{bmatrix} (5)(1)+(-3)(0) & (5)(0)+(-3)(1) \\ (4)(1)+(6)(0) & (4)(0)+(6)(1) \end{bmatrix}$$

$$AI = \begin{bmatrix} 5+0 & 0-3 \\ 4+0 & 0+6 \end{bmatrix}$$

$$AI = \begin{bmatrix} 5 & -3 \\ 4 & 6 \end{bmatrix} = A$$

**(ii)  $BI$** **Solution:**

$$BI = \begin{bmatrix} -7 & 3 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BI = \begin{bmatrix} (-7)(1)+(3)(0) & (-7)(0)+(3)(1) \\ (2)(1)+(8)(0) & (2)(0)+(8)(1) \end{bmatrix}$$

$$BI = \begin{bmatrix} -7+0 & 0+3 \\ 2+0 & 0+8 \end{bmatrix}$$

$$BI = \begin{bmatrix} -7 & 3 \\ 2 & 8 \end{bmatrix} = B$$

**Q11:** Let  $A = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} -3 & 4 & 2 \end{bmatrix}$ , then prove that

(i)  $(A + B)^t = A^t + B^t$

**LHS:**  $(A + B)^t$ 

$$A + B = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 4 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3-3 & 2+4 & 1+2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 0 & 6 & 3 \end{bmatrix}$$

**Now**

$$(A + B)^t = \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$$

**RHS:**  $A^t + B^t$ 

$$\text{As } A^t = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\text{And } B^t = \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix}$$

**Now**

$$A^t + B^t = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix}$$

$$A^t + B^t = \begin{bmatrix} 3-3 \\ 2+4 \\ 1+2 \end{bmatrix}$$

$$A^t + B^t = \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}$$

**Hence**  $(A + B)^t = A^t + B^t$     **Proved:**

$(A - B)^t = A^t - B^t$

**LHS:**  $(A - B)^t$ 

$$A - B = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -3 & 4 & 2 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 3+3 & 2-4 & 1-2 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 6 & -2 & -1 \end{bmatrix}$$

**Ex # 1.4****Now**

$$(A - B)^t = \begin{bmatrix} 6 \\ -2 \\ -1 \end{bmatrix}$$

**RHS:**  $A^t - B^t$ 

$$\text{As } A^t = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$$\text{And } B^t = \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix}$$

**Now**

$$A^t - B^t = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} - \begin{bmatrix} -3 \\ 4 \\ 2 \end{bmatrix}$$

$$A^t - B^t = \begin{bmatrix} 3+3 \\ 2-4 \\ 1-2 \end{bmatrix}$$

$$A^t - B^t = \begin{bmatrix} 6 \\ -2 \\ -1 \end{bmatrix}$$

**Hence**  $(A - B)^t = A^t - B^t$  **Proved:**

(ii) If  $C = \begin{bmatrix} 7 & -3 \\ 2 & -1 \end{bmatrix}$ ,  $D = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$  then prove that

$$(C + D)^t = C^t + D^t$$

**Solution:****LHS:**  $(C + D)^t$ 

$$C + D = \begin{bmatrix} 7 & -3 \\ 2 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$C + D = \begin{bmatrix} 7+1 & -3+1 \\ 2+2 & -1+2 \end{bmatrix}$$

$$C + D = \begin{bmatrix} 8 & -2 \\ 4 & 1 \end{bmatrix}$$

**Now**

$$(C + D)^t = \begin{bmatrix} 8 & 4 \\ -2 & 1 \end{bmatrix}$$

**RHS:**  $C^t + D^t$ 

$$\text{As } C^t = \begin{bmatrix} 7 & 2 \\ -3 & -1 \end{bmatrix}$$

$$\text{And } D^t = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

**Now**

$$C^t + D^t = \begin{bmatrix} 7 & 2 \\ -3 & -1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

$$C^t + D^t = \begin{bmatrix} 7+1 & 2+2 \\ -3+1 & -1+2 \end{bmatrix}$$

$$C^t + D^t = \begin{bmatrix} 8 & 4 \\ -2 & 1 \end{bmatrix}$$

**Hence**  $(C + D)^t = C^t + D^t$  **Proved:**

$$(C - D)^t = C^t - D^t$$

**Solution:****LHS:**  $(C - D)^t$ 

$$C - D = \begin{bmatrix} 7 & -3 \\ 2 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

$$C - D = \begin{bmatrix} 7-1 & -3-1 \\ 2-2 & -1-2 \end{bmatrix}$$

$$C - D = \begin{bmatrix} 6 & -4 \\ 0 & -3 \end{bmatrix}$$

**Now**

$$(C - D)^t = \begin{bmatrix} 6 & 0 \\ -4 & -3 \end{bmatrix}$$

**RHS:**  $C^t + D^t$ 

$$\text{As } C^t = \begin{bmatrix} 7 & 2 \\ -3 & -1 \end{bmatrix}$$

$$\text{And } D^t = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

**Now**

$$C^t - D^t = \begin{bmatrix} 7 & 2 \\ -3 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

**Ex # 1.4**

$$C^t - D^t = \begin{bmatrix} 7-1 & 2-2 \\ -3-1 & -1-2 \end{bmatrix}$$

$$C^t - D^t = \begin{bmatrix} 6 & 0 \\ -4 & -3 \end{bmatrix}$$

Hence  $(C - D)^t = C^t - D^t$  Proved:

**Q12:** If  $A = \begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix}$ ,  $B = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$  show that

$$(AB)^t = B^t A^t$$

**Solution:**

$$A = \begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix}, B = \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$$

$$(AB)^t = B^t A^t$$

LHS:  $(AB)^t$

$$AB = \begin{bmatrix} 2 & 5 \\ -3 & 4 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} (2)(-1) + (5)(2) & (2)(1) + (5)(3) \\ (-3)(-1) + (4)(2) & (-3)(1) + (4)(3) \end{bmatrix}$$

$$AB = \begin{bmatrix} -2+10 & 2+15 \\ 3+8 & -3+12 \end{bmatrix}$$

$$AB = \begin{bmatrix} 8 & 17 \\ 11 & 9 \end{bmatrix}$$

Now

$$(AB)^t = \begin{bmatrix} 8 & 11 \\ 17 & 9 \end{bmatrix}$$

RHS:  $B^t A^t$

$$\text{As } A^t = \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$$

$$\text{And } B^t = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}$$

Now

$$B^t A^t = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 5 & 4 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} (-1)(2) + (2)(5) & (-1)(-3) + (2)(4) \\ (1)(2) + (3)(5) & (1)(-3) + (3)(4) \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} -2+10 & 3+8 \\ 2+15 & -3+12 \end{bmatrix}$$

$$B^t A^t = \begin{bmatrix} 8 & 11 \\ 17 & 9 \end{bmatrix}$$

Hence  $(AB)^t = B^t A^t$  Proved:

(ii) If  $C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , show that  $(C^t)^t = C$

**Solution:**

$$C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

By taking transpose, we get

$$C^t = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Now again take transpose, so we get

$$(C^t)^t = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

(iii) If  $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 6 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 7 \\ -8 & 4 \\ 0 & 1 \end{bmatrix}$ , show that

$$(AB)^t = A^t B^t$$

**Solution:**

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 6 \end{bmatrix}, B = \begin{bmatrix} 1 & 7 \\ -8 & 4 \\ 0 & 1 \end{bmatrix}$$

$$(AB)^t = A^t B^t$$

**LHS:**  $(AB)^t$

$$AB = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & 7 \\ -8 & 4 \\ 0 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} (1)(1) + (0)(-8) + (-1)(0) & (1)(7) + (0)(4) + (-1)(1) \\ (2)(1) + (0)(-8) + (6)(0) & (2)(7) + (0)(4) + (6)(1) \end{bmatrix}$$

$$AB = \begin{bmatrix} 1+0+0 & 7+0-1 \\ 2+0+0 & 14+0+6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 6 \\ 2 & 20 \end{bmatrix}$$

**Ex # 1.4****Now**

$$(AB)^t = \begin{bmatrix} 1 & 2 \\ 6 & 20 \end{bmatrix}$$

**RHS:  $A^t B^t$** 

$$\text{As } A^t = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ -1 & 6 \end{bmatrix}$$

$$\text{And } B^t = \begin{bmatrix} 1 & -8 & 0 \\ 7 & 4 & 1 \end{bmatrix}$$

**Now**

$$A^t B^t = \begin{bmatrix} 1 & 2 \\ 0 & 0 \\ -1 & 6 \end{bmatrix} \begin{bmatrix} 1 & -8 & 0 \\ 7 & 4 & 1 \end{bmatrix}$$

$$A^t B^t = \begin{bmatrix} (1)(1)+(2)(7) & (1)(-8)+(2)(4) & (1)(0)+(2)(1) \\ (0)(1)+(0)(7) & (0)(-8)+(0)(4) & (0)(0)+(0)(1) \\ (-1)(1)+(6)(7) & (-1)(-8)+(6)(4) & (-1)(0)+(6)(1) \end{bmatrix}$$

$$A^t B^t = \begin{bmatrix} 1+14 & -8+8 & 0+2 \\ 0+0 & 0+0 & 0+0 \\ -1+42 & 8+24 & 0+6 \end{bmatrix}$$

$$A^t B^t = \begin{bmatrix} 15 & 0 & 2 \\ 0 & 0 & 0 \\ 41 & 32 & 6 \end{bmatrix}$$

**Hence  $(AB)^t \neq A^t B^t$** **Exercise # 1.5****Determinant of a Square Matrix****Determinant of A denoted by  $|A|$  or  $\det A$ .**

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$|A| = ad - cb$$

**Ex # 1.5****(i) Singular**If  $|A| = 0$  then A is Singular Matrix.**(ii) Non-Singular Matrix**If  $|A| \neq 0$  then A is Non-Singular Matrix.**Adjoint of Square Matrix**

$$\text{Let } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

As change the places of a and d with each other and change the size of b and c. So

$$\text{Adj}A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

**Multiplicative Inverse**If  $AB = BA = I$  then A is the multiplicative inverse of B**For Non-Singular matrix,**

$$A^{-1} = \frac{1}{|A|} \text{Adj } A$$

$$\text{Let } A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$$

**Now**

$$AB = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} (3)(3)+(2)(-4) & (3)(-2)+(2)(3) \\ (4)(3)+(3)(-4) & (4)(-2)+(3)(3) \end{bmatrix}$$

$$AB = \begin{bmatrix} 9-8 & -6+6 \\ 12-12 & -8+9 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

**Since  $AB = I = BA$** **Therefore, A is the inverse of B.**

**Ex # 1.5****Verification of  $AA^{-1} = I = A^{-1}A$** 

$$A = \begin{bmatrix} -2 & -1 \\ 3 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{4}{5} & -\frac{1}{5} \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} -2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -\frac{4}{5} & -\frac{1}{5} \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} (-2)\left(\frac{-4}{5}\right) + (-1)\left(\frac{3}{5}\right) & (-2)\left(-\frac{1}{5}\right) + (-1)\left(\frac{2}{5}\right) \\ (3)\left(\frac{-4}{5}\right) + (4)\left(\frac{3}{5}\right) & (3)\left(-\frac{1}{5}\right) + (4)\left(\frac{2}{5}\right) \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} -\frac{8}{5} - \frac{3}{5} & \frac{2}{5} - \frac{2}{5} \\ -\frac{12}{5} + \frac{12}{5} & -\frac{3}{5} + \frac{8}{5} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} \frac{5}{5} & 0 \\ 0 & \frac{5}{5} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now

$$A^{-1}A = \begin{bmatrix} -\frac{4}{5} & -\frac{1}{5} \\ \frac{3}{5} & \frac{2}{5} \end{bmatrix} \begin{bmatrix} -2 & -1 \\ 3 & 4 \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} \left(\frac{-4}{5}\right)(-2) + \left(\frac{-1}{5}\right)(3) & \left(\frac{-4}{5}\right)(-1) + \left(\frac{-1}{5}\right)(4) \\ \left(\frac{3}{5}\right)(-2) + \left(\frac{2}{5}\right)(3) & \left(\frac{3}{5}\right)(-1) + \left(\frac{2}{5}\right)(4) \end{bmatrix}$$

**Ex # 1.5**

$$A^{-1}A = \begin{bmatrix} -\frac{8}{5} - \frac{3}{5} & \frac{4}{5} - \frac{4}{5} \\ -\frac{6}{5} + \frac{6}{5} & -\frac{3}{5} + \frac{8}{5} \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} \frac{5}{5} & 0 \\ 0 & \frac{5}{5} \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

**Thus  $AA^{-1} = I = A^{-1}A$** **Exercise # 1.5****Page # 37, 38****Q1: Find the determinant of the following matrices and evaluate them.**

(i)  $A = \begin{bmatrix} 5 & 6 \\ -4 & 1 \end{bmatrix}$

**Solution:**

$$A = \begin{bmatrix} 5 & 6 \\ -4 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 6 \\ -4 & 1 \end{vmatrix}$$

$$|A| = (5)(1) - (-4)(6)$$

$$|A| = 5 - (-24)$$

$$|A| = 5 + 24$$

$$|A| = 29$$

(ii)  $B = \begin{bmatrix} 4 & -2 \\ 5 & 13 \end{bmatrix}$

**Solution:**

$$B = \begin{bmatrix} 4 & -2 \\ 5 & 13 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 4 & -2 \\ 5 & 13 \end{vmatrix}$$

$$|B| = (4)(13) - (5)(-2)$$

$$|B| = 52 - (-10)$$

$$|B| = 52 + 10$$

$$|B| = 62$$



(iii)  $C = \begin{bmatrix} 11 & 7 \\ -6 & 5 \end{bmatrix}$

**Solution:**

$$C = \begin{bmatrix} 11 & 7 \\ -6 & 5 \end{bmatrix}$$

$$|C| = \begin{vmatrix} 11 & 7 \\ -6 & 5 \end{vmatrix}$$

$$|C| = (11)(5) - (-6)(7)$$

$$|C| = 55 - (-42)$$

$$|C| = 55 + 42$$

$$|C| = 97$$

(iv)  $D = \begin{bmatrix} 5 & 6 \\ -8 & -9 \end{bmatrix}$

**Solution:**

$$D = \begin{bmatrix} 5 & 6 \\ -8 & -9 \end{bmatrix}$$

$$|D| = \begin{vmatrix} 5 & 6 \\ -8 & -9 \end{vmatrix}$$

$$|D| = (5)(-9) - (-8)(6)$$

$$|D| = -45 - (-48)$$

$$|D| = -45 + 48$$

$$|D| = 3$$

(v)  $E = \begin{bmatrix} 2p & -3q \\ r & -s \end{bmatrix}$

**Solution:**

$$E = \begin{bmatrix} 2p & -3q \\ r & -s \end{bmatrix}$$

$$|E| = \begin{vmatrix} 2p & -3q \\ r & -s \end{vmatrix}$$

$$|E| = (2p)(-s) - (r)(-3q)$$

$$|E| = -2ps - (-3qr)$$

$$|E| = -2ps + 3qr$$

$$|E| = 3qr - 2ps$$

### Ex # 1.5

(vi)  $F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

**Solution:**

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$|F| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expand by Row 1:

$$|F| = 1 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - 0 \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} + 0 \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

$$|F| = 1(1 - 0) - 0 + 0$$

$$|F| = 1(1)$$

$$|F| = 1$$

(vii)  $G = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ -2 & -3 & 4 \end{bmatrix}$

**Solution:**

$$G = \begin{bmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ -2 & -3 & 4 \end{bmatrix}$$

$$|G| = \begin{vmatrix} 1 & 2 & 2 \\ 3 & 2 & 3 \\ -2 & -3 & 4 \end{vmatrix}$$

$$|G| = 1 \begin{vmatrix} 2 & 3 \\ -3 & 4 \end{vmatrix} - 2 \begin{vmatrix} 3 & 3 \\ -2 & 4 \end{vmatrix} + 2 \begin{vmatrix} 3 & 2 \\ -2 & -3 \end{vmatrix}$$

$$|G| = 1(8 - (-9)) - 2(12 - (-6)) + 2(-9 - (-4))$$

$$|G| = 1(8 + 9) - 2(12 + 6) + 2(-9 + 4)$$

$$|G| = 1(17) - 2(18) + 2(-5)$$

$$|G| = 17 - 36 - 10$$

$$|G| = -29$$

**Ex # 1.5**

(Viii)  $H = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$

**Solution:**

$$H = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$|H| = \begin{vmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{vmatrix}$$

$$|H| = a \begin{vmatrix} b & 0 \\ 0 & c \end{vmatrix} - 0 \begin{vmatrix} 0 & 0 \\ 0 & c \end{vmatrix} + 0 \begin{vmatrix} 0 & b \\ 0 & 0 \end{vmatrix}$$

$$|H| = a(bc - 0) - 0 + 0$$

$$|H| = a(bc)$$

$$|H| = abc$$

**Q2:** Find which of the following matrices are singular ad which are non-singular.

(i)  $A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$

**Solution:**

$$A = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 3 \\ 2 & 1 \end{vmatrix}$$

$$|A| = 5 - 6$$

$$|A| = -1 \neq 0$$

Thus A is a non-singular matrix.

(ii)  $B = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$

**Solution:**

$$B = \begin{bmatrix} 3 & -6 \\ -2 & 4 \end{bmatrix}$$

**Ex # 1.5****Ex # 1.5**

$$|B| = \begin{vmatrix} 3 & -6 \\ -2 & 4 \end{vmatrix}$$

$$|B| = 12 - 12$$

$$|B| = 0$$

Thus B is a singular matrix.

(iii)  $C = \begin{bmatrix} 3a & -2b \\ 2a & b \end{bmatrix}$

**Solution:**

$$C = \begin{bmatrix} 3a & -2b \\ 2a & b \end{bmatrix}$$

$$|C| = \begin{vmatrix} 3a & -2b \\ 2a & b \end{vmatrix}$$

$$|C| = 3ab - (-4ab)$$

$$|C| = 3ab + 4ab$$

$$|C| = 7ab \neq 0$$

Thus C is a non-singular matrix.

(iv)  $D = \begin{bmatrix} -3 & 6 \\ 2 & -4 \end{bmatrix}$

**Solution:**

$$D = \begin{bmatrix} -3 & 6 \\ 2 & -4 \end{bmatrix}$$

$$|D| = \begin{vmatrix} -3 & 6 \\ 2 & -4 \end{vmatrix}$$

$$|D| = 12 - 12$$

$$|D| = 0$$

Thus D is a singular matrix.

**Q3:** Find the adjoint of the following matrices.

(i)  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

**Solution:**

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

**Ex # 1.5**

$$(ii) \quad B = \begin{bmatrix} -3 & -1 \\ 2 & 3 \end{bmatrix}$$

**Solution:**

$$B = \begin{bmatrix} -3 & -1 \\ 2 & 3 \end{bmatrix}$$

$$\text{Adj } B = \begin{bmatrix} 3 & 1 \\ -2 & -3 \end{bmatrix}$$

$$(iii) \quad C = \begin{bmatrix} 2 & -4 \\ 3 & 1 \end{bmatrix}$$

**Solution:**

$$C = \begin{bmatrix} 2 & -4 \\ 3 & 1 \end{bmatrix}$$

$$\text{Adj } C = \begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix}$$

$$(iv) \quad D = \begin{bmatrix} -3 & 6 \\ 2 & -4 \end{bmatrix}$$

**Solution:**

$$D = \begin{bmatrix} -3 & 6 \\ 2 & -4 \end{bmatrix}$$

$$\text{Adj } D = \begin{bmatrix} -4 & -6 \\ -2 & -3 \end{bmatrix}$$

**Q4:** Find the multiplicative inverse of the following matrices if they exist.

$$(i) \quad A = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$$

**Solution:**

$$A = \begin{bmatrix} 4 & 1 \\ 3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A \dots \text{Equ (i)}$$

$$|A| = \begin{vmatrix} 4 & 1 \\ 3 & 1 \end{vmatrix}$$

**Ex # 1.5**

$$|A| = 4 - 3$$

$$|A| = -1 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$$

**Put the values in equ (i)**

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix}$$

$$(ii) \quad B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

**Solution:**

$$B = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \text{Adj } B \dots \text{Equ (i)}$$

$$|B| = \begin{vmatrix} 3 & 4 \\ 1 & 2 \end{vmatrix}$$

$$|B| = 6 - 4$$

$$|B| = 2 \neq 0$$

$$\text{Adj } B = \begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}$$

**Put the values in equ (i)**

$$B^{-1} = \frac{1}{2} \begin{bmatrix} 2 & -4 \\ -1 & 3 \end{bmatrix}$$

$$(iii) \quad C = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

**Solution:**

$$C = \begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$$

$$C^{-1} = \frac{1}{|C|} \text{adj } C \dots \text{Equ (i)}$$

**Ex # 1.5**

$$|C| = \begin{vmatrix} 4 & -3 \\ -1 & 2 \end{vmatrix}$$

$$|C| = 8 - 3$$

$$|C| = 5 \neq 0$$

$$\text{Adj } C = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

**Put the values in equ (i)**

$$C^{-1} = \frac{1}{5} \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$$(iv) \quad D = \begin{bmatrix} 0 & -3 \\ 2 & 4 \end{bmatrix}$$

**Solution:**

$$D = \begin{bmatrix} 0 & -3 \\ 2 & 4 \end{bmatrix}$$

$$D^{-1} = \frac{1}{|D|} \text{adj } D \quad \dots \text{Equ (i)}$$

$$|D| = \begin{vmatrix} 0 & -3 \\ 2 & 4 \end{vmatrix}$$

$$|D| = 0 - (-6)$$

$$|D| = 0 + 6$$

$$|D| = 6 \neq 0$$

$$\text{Adj } D = \begin{bmatrix} 4 & 3 \\ -2 & 0 \end{bmatrix}$$

**Put the values in equ (i)**

$$D^{-1} = \frac{1}{6} \begin{bmatrix} 4 & 3 \\ -2 & 0 \end{bmatrix}$$

$$(v) \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

**Solution:**

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I^{-1} = \frac{1}{|I|} \text{adj } I \quad \dots \text{Equ (i)}$$

**Ex # 1.5**

$$|I| = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$|I| = 1 - 0$$

$$|I| = 1 \neq 0$$

$$\text{Adj } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

**Put the values in equ (i)**

$$I^{-1} = \frac{1}{1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

**Q5:** If  $A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$ , find

**(i)  $AB$**

**Solution:**

$$A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$$

**Now**

$$AB = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} (2)(1) + (0)(-1) & (2)(-1) + (0)(3) \\ (-3)(1) + (1)(-1) & (-3)(-1) + (1)(3) \end{bmatrix}$$

$$AB = \begin{bmatrix} 2+0 & -2+0 \\ -3-1 & 3+3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2 & -2 \\ -4 & 6 \end{bmatrix}$$

**(ii)  $BA$**

**Solution:**

$$A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$$

**Now**

$$BA = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$$

**Ex # 1.5**

$$BA = \begin{bmatrix} (1)(2) + (-1)(-3) & (1)(0) + (-1)(1) \\ (-1)(2) + (3)(-3) & (-1)(0) + (3)(1) \end{bmatrix}$$

$$BA = \begin{bmatrix} 2+3 & 0-1 \\ -2-9 & 0+3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & -1 \\ -11 & 3 \end{bmatrix}$$

(iii)  **$A^{-1}$  and  $B^{-1}$** **Solution:** **$A^{-1}$** 

$$A = \begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{adj } A \quad \dots \text{Equ (i)}$$

$$|A| = \begin{vmatrix} 2 & 0 \\ -3 & 1 \end{vmatrix}$$

$$|A| = 2 - 0$$

$$|A| = 2 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

**Put the values in equ (i)**

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

 **$B^{-1}$** 

$$B = \begin{bmatrix} 1 & -1 \\ -1 & 3 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \text{Adj } B \quad \dots \text{Equ (i)}$$

$$|B| = \begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix}$$

$$|B| = 3 - 1$$

$$|B| = 2 \neq 0$$

$$\text{Adj } B = \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

**Ex # 1.5****Put the values in equ (i)**

$$B^{-1} = \frac{1}{2} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

**Show that  $(AB)^{-1} = B^{-1}A^{-1}$** **Solution:**

$$(AB)^{-1} = B^{-1}A^{-1}$$

**LHS:  $(AB)^{-1}$** 

$$\text{As } AB = \begin{bmatrix} 2 & -2 \\ -4 & 6 \end{bmatrix}$$

**So**

$$(AB)^{-1} = \frac{1}{|AB|} \text{Adj } (AB) \quad \dots \text{Equ (i)}$$

**Now**

$$|AB| = \begin{vmatrix} 2 & -2 \\ -4 & 6 \end{vmatrix}$$

$$|AB| = 12 - 8$$

$$|AB| = 4 \neq 0$$

$$\text{Adj } (AB) = \begin{bmatrix} 6 & 2 \\ 4 & 2 \end{bmatrix}$$

**Put the values in equ (i)**

$$(AB)^{-1} = \frac{1}{4} \begin{bmatrix} 6 & 2 \\ 4 & 2 \end{bmatrix}$$

**RHS:  $B^{-1}A^{-1}$** 

$$\text{As } A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

$$\text{And } B^{-1} = \frac{1}{2} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

**Now**

$$B^{-1}A^{-1} = \frac{1}{2} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} \times \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{2} \times \frac{1}{2} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{4} \begin{bmatrix} 3+3 & 0+2 \\ 1+3 & 0+2 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{4} \begin{bmatrix} 6 & 2 \\ 4 & 2 \end{bmatrix}$$

**Hence  $(AB)^{-1} = B^{-1}A^{-1}$  Proved:**

**Ex # 1.5****Show that  $(BA)^{-1} = A^{-1}B^{-1}$** **Solution:**

$$(BA)^{-1} = A^{-1}B^{-1}$$

**LHS:  $(BA)^{-1}$** 

$$\text{As } BA = \begin{bmatrix} 5 & -1 \\ -11 & 3 \end{bmatrix}$$

**So**

$$(BA)^{-1} = \frac{1}{|BA|} \text{Adj}(BA) \dots\dots \text{Equ (i)}$$

$$|BA| = \begin{vmatrix} 5 & -1 \\ -11 & 3 \end{vmatrix}$$

$$|BA| = 15 - 11$$

$$|BA| = 4 \neq 0$$

$$\text{Adj}(BA) = \begin{bmatrix} 3 & 1 \\ 11 & 5 \end{bmatrix}$$

**Put the values in equ (i)**

$$(BA)^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 \\ 11 & 5 \end{bmatrix}$$

**RHS:  $A^{-1}B^{-1}$** 

$$\text{As } A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$$

$$\text{And } B^{-1} = \frac{1}{2} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

**Now**

$$A^{-1}B^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix} \times \frac{1}{2} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^{-1}B^{-1} = \frac{1}{2} \times \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^{-1}B^{-1} = \frac{1}{4} \begin{bmatrix} 3+0 & 1+0 \\ 9+2 & 3+2 \end{bmatrix}$$

$$A^{-1}B^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 \\ 11 & 5 \end{bmatrix}$$

**Hence  $(BA)^{-1} = A^{-1}B^{-1}$  Proved:****Ex # 1.5**

**Q6: If  $A = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$  then show that  
(i)  $(AB)^{-1} = B^{-1}A^{-1}$**

**Solution:**

$$A = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

**LHS:  $(AB)^{-1}$** **Now**

$$AB = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} (0)(2)+(-1)(1) & (0)(3)+(-1)(0) \\ (2)(2)+(1)(1) & (2)(3)+(1)(0) \end{bmatrix}$$

$$AB = \begin{bmatrix} 0-1 & 0+0 \\ 4+1 & 6+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & 0 \\ 5 & 6 \end{bmatrix}$$

**As we have:**

$$(AB)^{-1} = \frac{1}{|AB|} \text{Adj}(AB) \dots\dots \text{Equ (i)}$$

$$|AB| = \begin{vmatrix} -1 & 0 \\ 5 & 6 \end{vmatrix}$$

$$|AB| = -6 - 0$$

$$|AB| = -6 \neq 0$$

**So solution exists**

$$\text{Adj}(AB) = \begin{bmatrix} 6 & 0 \\ -5 & -1 \end{bmatrix}$$

**Put the values in equ (i)**

$$(AB)^{-1} = \frac{1}{-6} \begin{bmatrix} 6 & 0 \\ -5 & -1 \end{bmatrix}$$

**RHS:  $B^{-1}A^{-1}$** **First we find  $A^{-1}$** 

$$\text{As } A = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$$


**Ex # 1.5**

**As we have**

$$A^{-1} = \frac{1}{|A|} \text{Adj } A \dots\dots \text{Equ (i)}$$

$$|A| = \begin{vmatrix} 0 & -1 \\ 2 & 1 \end{vmatrix}$$

$$|A| = 0 - (-2)$$

$$|A| = 0 + 2$$

$$|A| = 2 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$$

**Put the values in equ (i)**

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$$

**Now we find  $B^{-1}$**

$$\text{As } B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

**As we have:**

$$B^{-1} = \frac{1}{|B|} \text{Adj } B \dots\dots \text{Equ (i)}$$

$$|B| = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix}$$

$$|B| = 0 - 3$$

$$|B| = -3 \neq 0$$

$$\text{Adj } B = \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix}$$

**Put the values in equ (i)**

$$B^{-1} = \frac{1}{-3} \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix}$$

**Now**

$$B^{-1}A^{-1} = \frac{1}{-3} \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix} \times \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{-3} \times \frac{1}{2} \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$$

**Ex # 1.5**

$$B^{-1}A^{-1} = \frac{1}{-6} \begin{bmatrix} 0+6 & 0+0 \\ -1-4 & -1+0 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{-6} \begin{bmatrix} 6 & 0 \\ -5 & -1 \end{bmatrix}$$

**Hence  $(AB)^{-1} = B^{-1}A^{-1}$  Proved:**

**Q6:** If  $A = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$ ,  $B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$  then show that  
**(ii)**  $(BA)^{-1} = A^{-1}B^{-1}$

**Solution:**

$$A = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

$$(BA)^{-1} = A^{-1}B^{-1}$$

LHS:  $(BA)^{-1}$

$$BA = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} (2)(0)+(3)(2) & (2)(-1)+(3)(1) \\ (1)(0)+(0)(2) & (1)(-1)+(0)(1) \end{bmatrix}$$

$$BA = \begin{bmatrix} 0+6 & -2+3 \\ 0+0 & -1+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 6 & 1 \\ 0 & -1 \end{bmatrix}$$

As we have

$$(BA)^{-1} = \frac{1}{|BA|} \text{Adj } (BA) \dots\dots \text{Equ (i)}$$

$$|BA| = \begin{vmatrix} 6 & 1 \\ 0 & -1 \end{vmatrix}$$

$$|BA| = -6 - 0$$

$$|BA| = -6 \neq 0$$

**So solution exists**

$$\text{Adj } (BA) = \begin{bmatrix} -1 & -1 \\ 0 & 6 \end{bmatrix}$$

**Put the values in equ (i)**

$$(BA)^{-1} = \frac{1}{-6} \begin{bmatrix} -1 & -1 \\ 0 & 6 \end{bmatrix}$$

**Ex # 1.5****RHS:  $A^{-1}B^{-1}$** **First we find  $A^{-1}$** 

$$\text{As } A = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A \dots\dots \text{Equ (i)}$$

$$|A| = \begin{vmatrix} 0 & -1 \\ 2 & 1 \end{vmatrix}$$

$$|A| = 0 - (-2)$$

$$|A| = 0 + 2$$

$$|A| = 2 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$$

**Put the values in equ (i)**

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix}$$

**Now we find  $B^{-1}$** 

$$\text{As } B = \begin{bmatrix} 2 & 3 \\ 1 & 0 \end{bmatrix}$$

**As we have:**

$$B^{-1} = \frac{1}{|B|} \text{Adj } B \dots\dots \text{Equ (i)}$$

$$|B| = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix}$$

$$|B| = 0 - 3$$

$$|B| = -3 \neq 0$$

$$\text{Adj } B = \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix}$$

**Put the values in equ (i)**

$$B^{-1} = \frac{1}{-3} \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix}$$

**Now**

$$A^{-1}B^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} \times \frac{1}{-3} \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix}$$

**Ex # 1.5**

$$A^{-1}B^{-1} = \frac{1}{2} \times \frac{1}{-3} \begin{bmatrix} 1 & 1 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 0 & -3 \\ -1 & 2 \end{bmatrix}$$

$$A^{-1}B^{-1} = \frac{1}{-6} \begin{bmatrix} 0-1 & -3+2 \\ 0+0 & 6+0 \end{bmatrix}$$

$$A^{-1}B^{-1} = \frac{1}{-6} \begin{bmatrix} -1 & -1 \\ 0 & 6 \end{bmatrix}$$

**Hence  $(BA)^{-1} = A^{-1}B^{-1}$  Proved:****Exercise # 1.6****Page # 45****Q1: Solve the following system of linear equation using Inversion Method.**

$$2x + 3y = -1, \quad x - y = 2$$

**(i) Solution:**

$$2x + 3y = -1$$

$$x - y = 2$$

**In matrix form:**

$$\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$$\text{As } AX = B$$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \text{Adj } A \times B \dots\dots \text{Equ (i)}$$

$$|A| = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix}$$

$$|A| = -2 - 3$$

$$|A| = -5 \neq 0$$

**Thus Solution exists**

$$\text{Adj } A = \begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix}$$

**Put the values in equ (i)**

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5} \begin{bmatrix} -1 & -3 \\ -1 & 2 \end{bmatrix} \times \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

**Ex # 1.6**

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5} \begin{bmatrix} (-1)(-1) + (-3)(2) \\ (-1)(-1) + (2)(2) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5} \begin{bmatrix} 1 - 6 \\ 1 + 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-5} \begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \times \frac{1}{-5} \\ 5 \times \frac{1}{-5} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$x = 1$$

$$y = -1$$

**Thus Solution Set = { (1, -1) }**

(ii)  $x + 2y = -13, \quad 3x + 6y = 11$

**Solution:**

$$x + 2y = -13$$

$$3x + 6y = 11$$

**In matrix form:**

$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -13 \\ 11 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} -13 \\ 11 \end{bmatrix}$$

**As  $AX = B$**

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \text{Adj } A \times B \quad \dots \text{Equ (i)}$$

**First we find  $|A|$**

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 6 \end{vmatrix}$$

$$|A| = 6 - 6$$

$$|A| = 0$$

$$\text{As } |A| = 0$$

**So Solution is not possible.**

**Ex # 1.6**

(iii)  $x + 2y = 1, \quad 2x + 3y = \frac{5}{2}$

**Solution:**

$$x + 2y = 1$$

$$2x + 3y = \frac{5}{2}$$

**In matrix form:**

$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{5}{2} \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 1 \\ \frac{5}{2} \end{bmatrix}$$

**As  $AX = B$**

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \text{Adj } A \times B \quad \dots \text{Equ (i)}$$

**First we find  $|A|$**

$$|A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix}$$

$$|A| = 3 - 4$$

$$|A| = -1 \neq 0$$

**Thus solution exists**

$$\text{Adj } A = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$$

**Put the values in equ (i)**

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ \frac{5}{2} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} (3)(1) + (-2)\left(\frac{5}{2}\right) \\ (-2)(1) + (1)\left(\frac{5}{2}\right) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 3 + (-1)(5) \\ -2 + \frac{5}{2} \end{bmatrix}$$

**Ex # 1.6**

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 3-5 \\ -4+5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -1 \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -\frac{1}{2} \end{bmatrix}$$

$$x = 2$$

$$y = -\frac{1}{2}$$

$$\text{Thus Solution Set} = \left\{ \left( 2, -\frac{1}{2} \right) \right\}$$

$$(iii) \quad x + 2y = 1, \quad 2x + 3y = \frac{5}{2}$$

**Solution:**

$$x + 2y = 1$$

$$2x + 3y = \frac{5}{2}$$

**Multiply B.S by 2**

$$2(2x + 3y) = 2 \times \frac{5}{2}$$

$$4x + 6y = 5$$

**So write in matrix form:**

$$\begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

**As**  $AX = B$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \text{Adj } A \times B \dots \text{Equ (i)}$$

**First we find  $|A|$**

$$|A| = \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix}$$

**Ex # 1.6**

$$|A| = 6 - 8$$

$$|A| = -2 \neq 0$$

**Thus solution exists**

$$\text{Adj } A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

**Put the values in equ (i)**

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} (6)(1) + (-2)(5) \\ (-4)(1) + (1)(5) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 6 - 10 \\ -4 + 5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} -4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \times \frac{1}{-2} \\ 1 \times \frac{1}{-2} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -\frac{1}{2} \end{bmatrix}$$

$$x = 2$$

$$y = -\frac{1}{2}$$

$$\text{Thus Solution Set} = \left\{ \left( 2, -\frac{1}{2} \right) \right\}$$

$$(iv) \quad x - 2y - 1 = 0, \quad 2x + y + 3 = 0$$

**Solution:**

$$x - 2y - 1 = 0$$

$$2x + y + 3 = 0$$

**Hence**

$$x - 2y = 1$$

$$2x + y = -3$$

**In matrix form:**

$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$



Let  $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$ ,  $X = \begin{bmatrix} x \\ y \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$

As  $AX = B$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \text{Adj } A \times B \dots \text{Equ (i)}$$

First we find  $|A|$

$$|A| = \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix}$$

$$|A| = 1 - (-4)$$

$$|A| = 1 + 5$$

$$|A| = 5 \neq 0$$

Thus solution exists

$$\text{Adj } A = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

Put the values in equ (i)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} (1)(1) + (2)(-3) \\ (-2)(1) + (1)(-3) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 1 + (-6) \\ -2 + (-3) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 1 - 6 \\ -2 - 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -5 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \times \frac{1}{5} \\ -5 \times \frac{1}{5} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$x = -1$$

$$y = -1$$

Thus Solution Set =  $\{(-1, -1)\}$

### Ex # 1.6

Q2: Solve the following system of linear equations using Cramer's Rule

(i)  $x - 2y = 5, \quad 2x - y = 6$

**Solution:**

$$x - 2y = 5$$

$$2x - y = 6$$

**In matrix form:**

$$\begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 5 \\ 6 \end{bmatrix}$$

First we find  $|A|$

$$|A| = \begin{vmatrix} 1 & -2 \\ 2 & -1 \end{vmatrix}$$

$$|A| = -1 - (-4)$$

$$|A| = -1 + 4$$

$$|A| = 3 \neq 0$$

Thus solution exists.

To find the value of  $x$ , Replace the coefficient of  $x$  in  $A$  by Matrix  $B$ .

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{\begin{vmatrix} 5 & -2 \\ 6 & -1 \end{vmatrix}}{3}$$

$$x = \frac{-5 - (-12)}{3}$$

$$x = \frac{-5 + 12}{3}$$

$$x = \frac{7}{3}$$

To find the value of  $y$ , Replace the coefficient of  $y$  in  $A$  by Matrix  $B$ .

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{\begin{vmatrix} 1 & 5 \\ 2 & 6 \end{vmatrix}}{3}$$

**Ex # 1.6**

$$y = \frac{6-10}{3}$$

$$y = \frac{-4}{3}$$

$$x = \frac{7}{3}$$

$$y = \frac{-4}{3}$$

**Thus Solution Set** =  $\left\{ \left( \frac{7}{3}, -\frac{4}{3} \right) \right\}$

(ii)  $4x + 3y = -2, \quad x - 2y = 5$

**Solution:**

$$4x + 3y = -2$$

$$x - 2y = 5$$

**In matrix form:**

$$\begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$$

Let  $A = \begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}$ ,  $X = \begin{bmatrix} x \\ y \end{bmatrix}$ ,  $B = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$

**First we find  $|A|$**

$$|A| = \begin{vmatrix} 4 & 3 \\ 1 & -2 \end{vmatrix}$$

$$|A| = -8 - 3$$

$$|A| = -11 \neq 0$$

**Thus solution exists.**

**To find the value of  $x$ , Replace the coefficient of  $x$  in A by Matrix B.**

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{\begin{vmatrix} -2 & 3 \\ 5 & -2 \end{vmatrix}}{-11}$$

$$x = \frac{4-15}{-11}$$

$$x = \frac{-11}{-11}$$

$$x = 1$$

**Ex # 1.6**

**To find the value of  $y$ , Replace the coefficient of  $y$  in A by Matrix B.**

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{\begin{vmatrix} 4 & -2 \\ 1 & 5 \end{vmatrix}}{-11}$$

$$y = \frac{20 - (-2)}{-11}$$

$$y = \frac{20+2}{-11}$$

$$y = \frac{22}{-11}$$

$$y = -2$$

$$x = 1$$

**Thus Solution Set** =  $\{(1, -2)\}$

(iii)  $5x + 7y = 3, \quad 3x + y = 5$

**Solution:**

$$5x + 7y = 3$$

$$3x + y = 5$$

**In matrix form**

$$\begin{bmatrix} 5 & 7 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

Let  $A = \begin{bmatrix} 5 & 7 \\ 3 & 1 \end{bmatrix}$ ,  $X = \begin{bmatrix} x \\ y \end{bmatrix}$ ,  $B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

**First we find  $|A|$**

$$|A| = \begin{vmatrix} 5 & 7 \\ 3 & 1 \end{vmatrix}$$

$$|A| = 5 - 21$$

$$|A| = -16 \neq 0$$

**Thus solution exists.**

**To find the value of  $x$ , Replace the coefficient of  $x$  in A by Matrix B.**

$$x = \frac{|A_x|}{|A|}$$

**Ex # 1.6**

$$x = \frac{\begin{vmatrix} 3 & 7 \\ 5 & 1 \end{vmatrix}}{-16}$$

$$x = \frac{3 - 35}{-16}$$

$$x = \frac{-32}{-16}$$

$$x = 2$$

**To find the value of y, Replace the coefficient of y in A by Matrix B.**

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{\begin{vmatrix} 5 & 3 \\ 3 & 5 \end{vmatrix}}{-16}$$

$$y = \frac{25 - 9}{-16}$$

$$y = \frac{16}{-16}$$

$$y = -1$$

$$x = 2$$

$$y = -1$$

**Thus Solution Set= { (2 , -1) }**

- Q3: Amjad thought of two numbers whose sum is 12 and whose difference is 4. Find the numbers.**

Solution:

Let the one number=  $x$

And second number=  $y$

According to given condition:

$$x + y = 12$$

$$x - y = 4$$

In matrix form

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 4 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 12 \\ 4 \end{bmatrix}$$

**Ex # 1.6**

By Cramer's Method:

First we find  $|A|$

$$|A| = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}$$

$$|A| = -1 - 1$$

$$|A| = -2 \neq 0$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{\begin{vmatrix} 12 & 1 \\ 4 & -1 \end{vmatrix}}{-2}$$

$$x = \frac{-12 - 4}{-2}$$

$$x = \frac{-16}{-2}$$

$$x = 8$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{\begin{vmatrix} 1 & 12 \\ 1 & 4 \end{vmatrix}}{-2}$$

$$y = \frac{4 - 12}{-2}$$

$$y = \frac{-8}{-2}$$

$$y = 4$$

So one number= 8

And second number= 4

**Ex # 1.6**

- Q4:** The length of a rectangular playground is twice its width. The perimeter is 30. Find its dimensions.

**Solution:**

Let the width =  $x$

And length =  $y$

**According to first condition:**

$$2x = y$$

$$2x - y = 0 \dots\dots \text{Equ (i)}$$

**As perimeter = 30**

As we have

$$2(x + y) = P$$

$$2(x + y) = 30$$

$$x + y = 15 \dots\dots \text{Equ (ii)}$$

Equ (i) and Equ (ii) in Matrix form

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 15 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 0 \\ 15 \end{bmatrix}$$

**First we find  $|A|$**

$$|A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix}$$

$$|A| = 2 - (-1)$$

$$|A| = 2 + 1$$

$$|A| = 3 \neq 0$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{\begin{vmatrix} 0 & -1 \\ 15 & 1 \end{vmatrix}}{3}$$

$$x = \frac{0 - (-15)}{3}$$

$$x = \frac{15}{3}$$

$$x = 5$$

$$y = \frac{|A_y|}{|A|}$$

**Ex # 1.6**

$$y = \frac{\begin{vmatrix} 2 & 0 \\ 1 & 15 \end{vmatrix}}{3}$$

$$y = \frac{30 - 0}{3}$$

$$y = \frac{30}{3}$$

$$y = 10$$

**So the width = 5**

**And length = 10**

- Q5:** 3 bags and 4 pens together cost 257 rupees whereas 4 bags and 3 pens together cost 324 rupees. Find the cost of a bag and 10 pens.

**Solution:**

**According to condition:**

Let the cost of bag =  $x$

And the cost of pen =  $y$

$$3x + 4y = 257$$

$$4x + 3y = 324$$

$$\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 257 \\ 324 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 257 \\ 324 \end{bmatrix}$$

**By Cramer's Rule**

**First we find  $|A|$**

$$|A| = \begin{vmatrix} 3 & 4 \\ 4 & 3 \end{vmatrix}$$

$$|A| = 9 - 16$$

$$|A| = -7 \neq 0$$

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{\begin{vmatrix} 257 & 4 \\ 324 & 3 \end{vmatrix}}{-7}$$

$$x = \frac{771 - 1296}{-7}$$

**Ex # 1.6**

$$x = \frac{-525}{-7}$$

$$x = 75$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{\begin{vmatrix} 3 & 257 \\ 4 & 324 \end{vmatrix}}{-7}$$

$$y = \frac{972 - 1028}{-7}$$

$$y = \frac{-56}{-7}$$

$$y = 8$$

**So the cost of bag = Rs. 75**

**And the cost of 10 pens=  $10 \times 8 = \text{Rs. } 80$**

- Q6:** If twice the son's age in years is added to the father's age, the sum is 70. But if the father's age is added to the son's age, the sum is 95.  
Find the ages of father and son.

**Solution:**

Let the age of Son =  $x$

And the age of father =  $y$

**According to condition:**

$$2x + y = 70$$

$$x + 2y = 95$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 70 \\ 95 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 70 \\ 95 \end{bmatrix}$$

**By Cramer's Rule**

**First we find  $|A|$**

$$|A| = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix}$$

$$|A| = 4 - 1$$

$$|A| = 3 \neq 0$$

**Ex # 1.6**

$$x = \frac{|A_x|}{|A|}$$

$$x = \frac{\begin{vmatrix} 70 & 1 \\ 95 & 2 \end{vmatrix}}{3}$$

$$x = \frac{140 - 95}{3}$$

$$x = \frac{45}{3}$$

$$x = 15$$

$$y = \frac{|A_y|}{|A|}$$

$$y = \frac{\begin{vmatrix} 2 & 70 \\ 1 & 95 \end{vmatrix}}{3}$$

$$y = \frac{190 - 70}{3}$$

$$y = \frac{120}{3}$$

$$y = 40$$

**The age of Son = 15**

**And the age of father = 40**



## REVIEW EXERSICE 1

Page # 47

**Q2: Find  $x$  and  $y$**

$$\begin{bmatrix} x-1 & 4 \\ y+3 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -2 & -7 \end{bmatrix}$$

**Solution:**

$$\begin{bmatrix} x-1 & 4 \\ y+3 & -7 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -2 & -7 \end{bmatrix}$$

**Compare the corresponding elements**

$$x - 1 = 0$$

$$x = 1$$

$$y + 3 = -2$$

$$y = -2 - 3$$

$$y = -5$$

**Q3: Find the product if possible**

$$\begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix} \begin{bmatrix} -6 & 5 & 8 \\ 0 & 4 & 1 \end{bmatrix}$$

As number of Columns in first matrix = 1

And number of Rows in second matrix = 2

**Thus these are not conformable for multiplication.**

**Q4: Find the inverse of the matrix  $A = \begin{bmatrix} 6 & -3 \\ 5 & -2 \end{bmatrix}$**

**Solution:**

$$A = \begin{bmatrix} 6 & -3 \\ 5 & -2 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \text{Adj } A \quad \text{Equ (i)}$$

**First we find  $|A|$ :**

$$|A| = \begin{vmatrix} 6 & -3 \\ 5 & -2 \end{vmatrix}$$

$$|A| = -12 - (-15)$$

$$|A| = -12 + 15$$

$$|A| = 3 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -2 & 3 \\ -5 & 6 \end{bmatrix}$$

Put the values in equ (i)

$$A^{-1} = \frac{1}{3} \begin{bmatrix} -2 & 3 \\ -5 & 6 \end{bmatrix}$$

**Q5: Solve the system:  $2x + 5y = 9, 5x - 2y = 8$**

**Solution:**

$$2x + 5y = 9$$

$$5x - 2y = 8$$

**In matrix form:**

$$\begin{bmatrix} 2 & 5 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 2 & 5 \\ 5 & -2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \text{Adj } A \times B \quad \text{Equ (i)}$$

$$|A| = \begin{vmatrix} 2 & 5 \\ 5 & -2 \end{vmatrix}$$

$$|A| = -4 - 25$$

$$|A| = -29 \neq 0$$

$$\text{Adj } A = \begin{bmatrix} -2 & -5 \\ -5 & 2 \end{bmatrix}$$

Put the values in equ (i)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-29} \begin{bmatrix} -2 & -5 \\ -5 & 2 \end{bmatrix} \times \begin{bmatrix} 9 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-29} \begin{bmatrix} (-2)(9) + (-5)(8) \\ (-5)(9) + (2)(8) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-29} \begin{bmatrix} -18 - 40 \\ -45 + 16 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-29} \begin{bmatrix} -58 \\ -29 \end{bmatrix}$$



$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -58 \times \frac{1}{-29} \\ -29 \times \frac{1}{-29} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$x = 2$$

$$y = 1$$

**Thus Solution Set = { (2, 1) }**

- Q6:** Qasim and Farzana are selling fruit for a school fundraiser. Customers can buy small boxes of oranges and large boxes of oranges. Qasim sold 3 small boxes of oranges and 14 large boxes of oranges for a total of Rs. 203. Farzana sold 11 small boxes of oranges ad 11 large boxes of oranges for a total of Rs. 220. Find the cost of one small box of oranges and one large box of oranges.

**Solution:**

Let small box of oranges =  $x$

And large box of oranges =  $y$

According to given condition:

$$3x + 14y = 203$$

$$11x + 11y = 220$$

In matrix form:

$$\begin{bmatrix} 3 & 14 \\ 11 & 11 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 203 \\ 220 \end{bmatrix}$$

$$\text{Let } A = \begin{bmatrix} 3 & 14 \\ 11 & 11 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 203 \\ 220 \end{bmatrix}$$

$$AX = B$$

$$X = A^{-1}B$$

$$X = \frac{1}{|A|} \text{Adj } A \times B \dots\dots \text{Equ (i)}$$

First find  $|A|$

$$|A| = \begin{vmatrix} 3 & 14 \\ 11 & 11 \end{vmatrix}$$

$$|A| = 33 - 154$$

$$|A| = -121$$

$$\text{Adj } A = \begin{bmatrix} 11 & -14 \\ -11 & 3 \end{bmatrix}$$

Put the values in equ (i)

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-121} \begin{bmatrix} 11 & -14 \\ -11 & 3 \end{bmatrix} \times \begin{bmatrix} 203 \\ 220 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-121} \begin{bmatrix} (11)(203) + (-14)(220) \\ (-11)(203) + (3)(220) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-121} \begin{bmatrix} 2233 - 3080 \\ -2233 + 660 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-121} \begin{bmatrix} -847 \\ -1573 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -847 \times \frac{1}{-121} \\ -1573 \times \frac{1}{-121} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$$

$$x = 7$$

$$y = 13$$

Thus small box of oranges = Rs. 7

And large box of oranges = Rs. 13