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ترجمہ: ''شروع اللہ کے نام سے جو بڑا مہربان نہایت رحم والا ہے۔''

Mathematics

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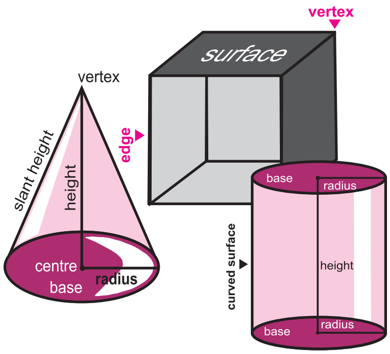

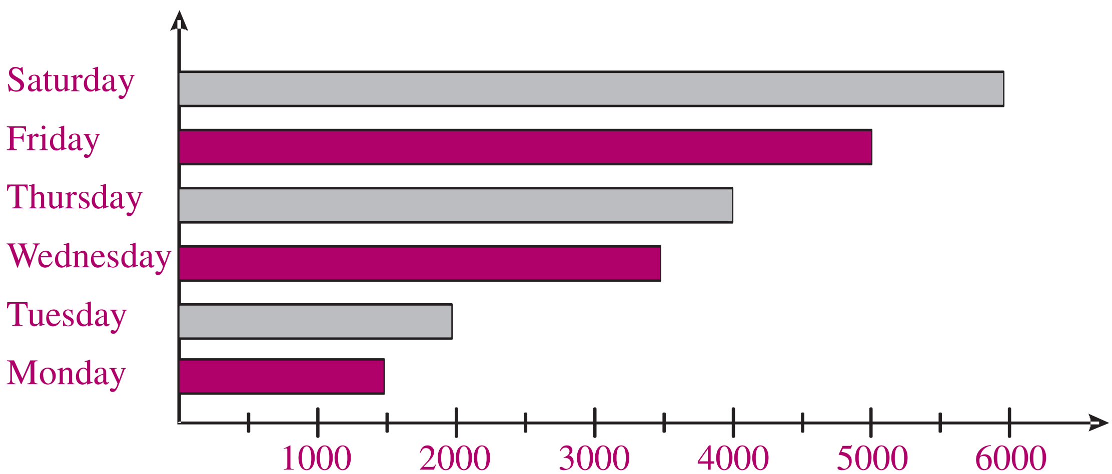

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Unit 1

SETS

Student Learning Outcomes

After studying this unit, students will be able to:

- Define set. Recognize notation of a set and its objects/elements.
- Describe tabular form of a set and demonstrate through examples.
- Define:
  - finite and infinite sets,
  - empty / void / null set,
  - singleton,
  - equal and equivalent sets,
  - subset and superset of a set,
  - proper and improper subsets of a set,

  and demonstrate through examples.

1.1 Introduction

In 19th century Geroge Cantor was the first mathematician who gave the proper idea of sets that we are using now in various branches of mathematics.

George Cantor

We often speak some words in our daily life which represent a collection of things, for example, a team of players, a bunch of flowers, a cluster of trees, a group of friends, a swarm of birds and so on. But in mathematics, we can use a single word **"set"** to denote all such type of words that show the collection of things such as, a set of players, a set of flowers, a set of trees, a set of friends and a set of birds, etc. Thus we can define a set as:

"A collection of distinct and well-defined objects is called a set."

The objects of a set are called its members or elements.

- **Well-defined**

Well-defined means a specific property of an object that enables it to be an element of a set or not. To make it clear consider the following examples of collections.

(i) The collection of good stories (ii) The collection of tasty foods

(iii) The collection of favourite poems

In the above examples, we can examine that the words good, tasty and favourite are not well-defined because a food may be favourite of one person but may not be for another. Similarly, a story may be good in view of one person but may not be for another. So, these are not suitable examples of sets as these are not well defined.

Distinct means the same objects should not appear more than once. For example, the set of letters of the word "*small*" is $\{s, m, a, l\}$. In this example, we can see that the letter '*l*' has been written only once. If it is written twice then it is not a set.

1.1.1 Set Notations

A set is represented by a capital letter A, B, C, ... , Z of English alphabets and its members or elements are written within brackets { } separated by commas, e.g.

- Set of pets: A={cow,horse, goat...}

Symbolically, we can write the members of the set A as,

cow $\in$ A is read as cow is an element of the set A

goat $\in$ A is read as goat is an element of the set A and so on.

Now examine whether a tree is the element of the set A. No! a tree is not an element of the set A. So, we can write this statement symbolically as:

tree $\notin$ A is read as tree is not the element of the set A.

The symbol $\in$ is a Greek letter which is used to tell that an object "is an element of" or "belongs to" or "is a member of" a set and the symbol $\notin$ means "does not belong to" or "is not the element of" the set.

Some important sets are given below:

- N = Set of natural numbers
- E = Set of even numbers
- O = Set of odd numbers
- W = Set of whole numbers
- P = Set of prime numbers

Example 1: Write the elements of the following sets.

$A = \{1, 2, 3, 4, 5\}$ $\quad B = \{\Delta, \square, \bigcirc\}$ $\quad C = \{\text{Lahore, Karachi, Sialkot, Islamabad, Faisalabad}\}$

Solution:

- The elements of the set A are 1,2,3,4 and 5. We can write them as,

$1 \in A$
$2 \in A$
$3 \in A$
$4 \in A$
and $5 \in A$

- The elements of the set B are $\Delta, \square$ and $\bigcirc$. We can write them as,

$\Delta \in B$
$\square \in B$
and $\bigcirc \in B$

- The elements of the set C are Lahore, Karachi, Sialkot, Islamabad and Faisalabad. We can write them as,

Lahore $\in$ C
Karachi $\in$ C
Sialkot $\in$ C
Islamabad $\in$ C
and Faisalabad $\in$ C

Example 2: Whether the following are sets or not.

$A = \{1,2,3,4\}$ $\quad B = \{1,2,2,3\}$ $\quad C = \{k,i,n,g\}$ $\quad D = \{b,a,1,1\}$

E = The set of brave boys $\quad$ F = The set of Pakistani singers

Solution:

(i) A, C and F are sets because all their objects are distinct and well-defined.
(ii) B and D are not sets because their objects are not distinct.
(iii) E is not a set because its objects are not well-defined.

EXERCISE 1.1

1. Which of the following statement is a set or not?

(i) The five provinces of Pakistan.
(ii) The difficult questions of a test.
(iii) The geometrical instruments.
(iv) The naughty boys of the street.
(v) The capital letters of the English alphabet.
(vi) The players of Pakistan cricket team.
(vii) The sharp boys of a school.
(viii) The natural numbers less than 50.
(ix) The whole numbers less than 9.

2. If A = $\{a,e,i,o,u\}$ and B = $\{a,b,c,...,z\}$ then, which of the following statements are true or false.

(i) $a \in A$ (ii) $b \in A$ (iii) $d \notin A$ (iv) $c \notin B$

(v) $i \in B$ (vi) $i \in A$ (vii) $f \in A$ (viii) $v \in A$

(ix) $x \notin A$ (x) $z \in B$ (xi) $m \notin B$ (xii) $a \in B$

3. Fill in the blanks by using either of the symbols $\in$ and $\notin$.

(i) 1 _____ {2, 3} (ii) b _____ $\{a, b, c\}$

(iii) i_____ The set of vowels (iv) Snake _____ The set of pets

(v) 15 _____ The set of counting numbers

(vi) 0 _____ The set of whole numbers

(vii) Goal keeper _____ The set of cricket team

(viii) B _____ The set of small letters of the English alphabet

(ix) Note-book _____ The set of articles of a school bag

(x) Lion _____ The set of jungle animals

4. Write the following statements into the symbolic form.

(i) 0 is an element of the set W. (ii) Lahore is a member of the set P.

(iii) 1 is not an element of the set E. (iv) Sindh does not belong to the set B.

(v) Potato is an element of the set V. (vi) o belongs to the set A.

(vii) c is not an element of the set C. (viii) Mango is not the member of the set F.

(ix) 5 is an element of the set N. (x) 4 is not an element of the set O.

5. Which of the following collections are not sets and why?

A = $\{b,a,n,k\}$ B = {2,4,6,8} C = {0,1,2,0} D = $\{k,i,l,l,e,r\}$ E = $\{l,e,g,a,l\}$

F = {9,3,5,1} G = The set of storybooks. H = The set of beautiful birds.

I = The set of rich people. J = The set of students in the 7^{th} class.

K = The set of fish in the river Ravi. L = The set of bad students in a school.

M = The set of wooden chairs.

6. List the elements of the following sets.

(i) The set of five countries. (ii) The set of three games.

(iii) The set of first ten natural numbers. (iv) The set of first eight even numbers.

(v) The set of vowels. (vi) The set of last four months.

(vii) The set of seven days of the week.

(viii) The set of the colours of Pakistan's flag.

(ix) The set of the five rivers of the Punjab.

(x) The set of three Islamic months.

1.1.2 Describing a Set

Descriptive form

In descriptive form, we describe the property of a set by a statement as given in the following examples.

A = The set of English books in the library. B = The set of natural numbers.

C = The set of animals in the zoo.

Tabular form

In tabular form, we list all elements within the brackets { } and separate each element by using a comma " , ". The elements of a set can be listed as:

(i) For a few number of elements:
$A = \{1,2,3,4,5,6,7,8,9\}$

(ii) For more but limited number of elements:
$B = \{1,2,3,\ldots,500\}$

(iii) For an un-limited number of elements.
$C = \{1,2,3, \ldots\}$

Note: Tabular form of a set is also known as roster form.

Example 1: Write the following sets into tabular form.

(i) A = The set of vowels of the English alphabet

(ii) B = The set of names of all games

(iii) C = The set of years of the 21^{st} century

Solution: (i) $A = \{a, e, i, o, u\}$ (ii) B = {hockey, football, cricket, …}

(iii) $C = \{2001, 2002, 2003, \ldots, 2100\}$

EXERCISE 1.2

1. Write the following sets into the descriptive form.

   $A = \{1,2,3,4,5,6\}$ $B = \{0,1,2,\ldots,99\}$
   C = {cricket, football, hockey, tennis} $E = \{2,4,6,\ldots\}$
   F = {potato, ladyfinger, carrot, brinjal} $N = \{1,2,3,\ldots\}$
   $O = \{1,3,5,\ldots\}$ $W = \{0, 1, 2, 3, \ldots\}$
   X = {father, mother, brother, sister}

2. Write the following sets into the tabular form.

   A = The set of names of 3 boys whose names start with letter "B".
   B = The set of names of 3 girls whose names start with letter "I".
   C = The set of 4 domestic animals.
   D = The set of 5 birds.
   E = The set of 3 prime ministers of Pakistan.
   F = The set of 5 famous cities of Pakistan.
   G = The set of letters of the word *banana*.
   I = The set of vowels of the word *naughty*.
   J = The set of years greater than 2006 and less than 2009.
   K = The set of 3 bakery items.

1.2 Types of Sets

There are three types of sets

(i) Finite sets (ii) Infinite sets (iii) Empty sets

1.2.1 Finite and Infinite Sets

Finite Set

"A set having a finite number of elements is called a finite set." Consider the following examples of sets.

A = The set of natural numbers less than 10
B = The set of vowels in the English alphabet

We can examine that it is very easy to count the elements of above two sets A and B. So, the set A and the set B are finite sets. Now we consider some more examples of sets which are given below,

C = The set of the population of Pakistan
D = The set of the hair of your head

Can we count the elements of the set C and D? Certainly, it is not an easy task, but sooner or later we can count the elements of these sets too. So, these are also finite sets.

Infinite Set

A set having unlimited number of elements is called an infinite set. For example,

(i) The set of counting numbers: $N = \{1, 2, 3, 4, 5, ...\}$
(ii) The set of odd numbers: $O = \{1, 3, 5, 7, 9, ...\}$
(iii) The set of whole numbers: $W = \{0, 1, 2, 3, 4, ...\}$

(i), (ii) and (iii) are examples of infinite sets because these sets have unlimited number of elements.

> It is possible to find the last element or member of a finite set. But it is impossible to find the last element or member of an infinite set.

Example 2: Separate the finite and infinite sets.

(i) The set of departmental store items. (ii) The set of the English alphabet.
(iii) $\{2, 4, 6, 8, \dots\}$ (iv) $\{0, 1, 2, 3, \dots\}$

Solution:

(i) and (ii) have limited number of elements. So, these are the examples of finite sets.

(iii) and (iv) have an unlimited number of elements. So, these are the examples of infinite sets.

1.2.2 Empty Set / Null Set

Everyone understands very well the meaning of the word empty. It means containing nothing, in the same sense we use it for set.

We make it clear with examples. When there is nothing in my pocket, it means my pocket is empty and if there is no any water in a glass, we say that the glass is empty. Similarly, when there is no element in a set, it means the set is empty. We can define it as,

"A set having no element is known as an empty set or null set".

An empty set is denoted by the Greek letter ϕ, which is read as phi or simply can be denoted as { }. Following are some examples of empty set.

(i) The set of 100 feet tall boys.
(ii) The set of days of 25 hours.
(iii) The set of horns of an ass.
(iv) The set of counting numbers between 1 and 2.

We can observe that while writing the elements of above examples of sets, we are not able to write a single element of any set, so these are empty sets.

1.2.3 Singleton Set

A set having a single element is called a singleton set. For example: {a}, {b}, $\{\frac{1}{2}\}$, etc. are singleton sets.

1.2.4 Equal and Equivalent Sets

From the previous classes, we are familiar with the concept of one-to-one correspondence. Here we use the same concept to define the equivalent and non-equivalent sets. Suppose, $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$ are any two sets, now we can check, whether the two sets A and B are establishing one-to-one correspondence or not by the following way.

$$\begin{array}{l} A = \{1,\ 2,\ 3,\ 4\} \\ \quad\ \ \ \updownarrow\ \ \updownarrow\ \ \updownarrow\ \ \updownarrow \\ B = \{a,\ b,\ c,\ d\} \end{array}$$

We can examine that corresponding to each element of set A, there is an element of set B. It means two sets are establishing one-to-one correspondence, which can be denoted as $A \longleftrightarrow B$. Such types of sets are called equivalent sets.

"Two sets are called equivalent if and only if one-to-one correspondence can be established between them".

Again consider two sets $C = \{1, 3, 5, 7\}$, $D = \{a, e, i, o, u\}$. We try to establish one-to-one correspondence between them as given below.

$$\begin{array}{l} C = \{1,\ 3,\ 5,\ 7\} \\ \quad\ \ \ \updownarrow\ \ \updownarrow\ \ \updownarrow\ \ \updownarrow \\ D = \{a,\ e,\ i,\ o,\ u\} \end{array}$$

From above, we can notice that corresponding to the element u of set D, there is no element of set C. It means two sets are not establishing the one-to-one correspondence which can be represented as $C \not\leftrightarrow D$. Such types of sets are called non-equivalent sets.

"Two sets are called non equivalent if one-to-one correspondence cannot be established between them".

Example 1: If, A={2,4,6,8} and B={1,3,5,7,9}. Are sets equivalent or not?

Solution:

$$A = \{2, 4, 6, 8\}$$

$$\updownarrow \ \updownarrow \ \updownarrow \ \updownarrow$$

$$B = \{1, 3, 5, 7, 9\}$$

It is impossible to establish the one-to-one correspondence between sets A and B. So set A and set B are not equivalent sets.

- **Equal Sets**

Suppose, set P and set Q are any two sets i.e.

$$P = \{1, 2, 3, 4, 5\} \text{ and } Q = \{3, 1, 5, 2, 4\}$$

From above, we can examine that each element of the set P is also an element of the set Q. Similarly, each element of set Q is also an element of the set P. Such types of sets are called equal sets which can be written as; $P = Q$

1.2.5 Subset and Superset of a Set

If each element of a set A is also an element of a set B then the set A is called the subset of the set B which is denoted by the symbol $\subseteq$, e.g.

$$A = \{2, 4, 6, 8, 10\}$$
$$B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

In above example, we can observe that each element of the set A is also an element of set B. So, $A \subseteq B$ that can be read as "set A is a subset of set B".

> Every set is also a subset of itself. e.g. $A \subseteq A$ and $B \subseteq B$.

- **Super Set**

If $A \subseteq B$, then the set B is called the super set of set A. i.e. $B \supseteq A$ that can be read as "the set B is the super set of set A".

1.2.6 Proper and Improper Subset of a Set

A = The set of natural numbers less than ten

B = The set of first nine natural numbers

From above it can be seen that the set A consisting of all the elements of set B, i.e, $A = B$.
Hence, set A is called the improper subset of set B, we write it as $A \subseteq B$.
Now we examine another example which is given below.

$$A = \{a, o, u\} \qquad B = \{a, e, i, o, u\}$$

In this example, we can see that all elements of the set A are also the elements of the set B but two elements of set B are not the elements of the set A. Here set A is called the proper subset of set B, we write it as $A \subset B$.

EXERCISE 1.3

1. Which of the following set is the empty set?

   (i) The set of whole numbers less than 1
   (ii) The set of the English alphabet between u and v
   (iii) The set of vowels other than a and o
   (iv) The set of natural numbers less than 1
   (v) The set of even numbers which are called odd numbers

2. Separate the finite and infinite set from the following.

   A = {letters of the word "*halla gulla*"}
   B = The set of natural numbers
   C = {number of days in a year}
   D = {Islamic months}
   E = {3,6,9,...}
   F = $\{c, f, i, m, o, r, u\}$
   G = $\left\{\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \ldots\right\}$

3. Find that the following pairs of sets are equivalent or non-equivalent.

   (i) A={1,2}, B = {1,3,7}
   (ii) A=$\{a,b\}$, B = $\{x,y\}$
   (iii) A ={-2,-1,0,1}, B ={3,5,7,8}
   (iv) A={0,1,2,3,4}, B ={1,2,3,4,5}
   (v) A = $\{a,e,i,o,u\}$, B=$\{l, m, n, o\}$
   (vi) A = The set of 4 natural numbers B =The set of 3 wild animal
   (vii) A = The set of 5 rivers of Punjab B =The set of colours in Pakistan's flag

4. Which of the following pairs of sets are equal sets?

   (i) A = $\{a, b, c, d, e\}$, B = $\{b, a, e, d, c\}$
   (ii) A = {1,2,3,4,5}, B = {1+0, 1+1, 1+2, 1+3, 1+4}
   (iii) A = {0, 1, 2, 3,4}, B = {6–2, 5–1, 4–2, 3–1, 2–2}
   (iv) A = The set of even numbers less than 9
   B = {0,2,4, 6, 8}

5. If, A = $\{a, b, c\}$, B = $\{b, c, d\}$, C = $\{c, d, e\}$ and D = $\{a, b, c, d\}$ then: which of the following statements are true?

   (i) $A \subset B$
   (ii) $B \subset D$
   (iii) $C \subset D$
   (iv) $A \subset D$
   (v) $B \subset C$
   (vi) $C \subset A$

Summary

- A collection of distinct and well-defined objects is called a set.
- Distinct means the same element should not appear more than once.
- If set A is subset of set B and set B is not subset of set A, then set A is called proper subset of set B.
- If two sets are equal, then they are improper subsets of each other.
- A set having a finite number of elements is called a finite set and a set having unlimited number of elements is called an infinite set.
- Two sets are called equivalent if and only if one-to-one correspondence can be established between them.
- Two sets with same elements are called equal sets.
- A set having a single element is called a singleton set.
- A set having no element is called an empty set.
- The method of establishing one to one correspondence between two sets helps us to find out whether the sets are equivalent or non-equivalent.
- If each element of a set A is also element of a set B then the set A is called the subset of the set B and set B is called the super set of the set A.

Review Exercise 1

1. List the elements of following sets.
   (i) The set of first four months. (ii) The set of last six capital letters
   (iii) The set of five odd numbers. (iv) The set of four colours.
   (v) The set of three planets in the Solar System.
2. Write the reason why following collections are not sets.
   (i) $\{1,1,2,2,3,3\}$ (ii) $\{b,o,o,k\}$
   (iii) $\{\triangle,\bigcirc,\triangle,\square\}$ (iv) The collection of dirty clothes.
   (v) The collection of fat boys in a school.
3. Write following sets into the tabular form.
   (i) The set of five even numbers.
   (ii) The set of numbers less than 23 and greater than 17.
   (iii) The set of letters of the word ORANGE.
   (iv) The set of whole numbers less than 5.
4. Separate finite, infinite and empty sets.
   (i) A = The set of rivers in Pakistan (ii) B = The set of all natural numbers
   (iii) C = The set of number of people on the moon

5. Separate equivalent and non-equivalent pair of sets.

(i) A = {1,3,5}, B = {2,4,6} (ii) C = {k,i,n,g}, D = {a,l,m}

(iii) E = The set of 5 whole numbers, F = The set of vowels in English alphabet

(iv) G = The set of days of a week, H = The set of counting numbers less than 7

6. Fill in the blanks in following pairs of sets to make them equal.

(i) A = {e,a,r,t,h}, B={t,__,r,h,__} (ii) A={0,2,4}, B={__,__,0}

(iii) A = {apple, mango, orange}, B={orange, ________, ________}

Objective Exercise 1

1. Answer the following questions.
   - (i) Define a set.
   - (ii) What is meant by the symbol $\in$.
   - (iii) Name two forms for describing a set.
   - (iv) What is meant by the word distinct in a set?
   - (v) If X = {a, b, c} and Y = {c, a, b}. Are these sets equal or not?
2. Fill in the blanks.
   - (i) One-to-one correspondence can not be established between _________ sets.
   - (ii) _________ means a specific property of an object that enables it to be an element of a set or not.
   - (iii) The symbol _________ means does not belong to the set.
   - (iv) _________ set is also known as null set.
   - (v) The symbol $\not\leftrightarrow$ is used for two _________ sets.
3. Tick (✓) the correct answer.
   - (i) To represent two equal sets, we use the symbol:
     (a) $\leftrightarrow$ (b) $\subset$ (c) $\subseteq$ (d) $=$
   - (ii) To write an empty set, we use the symbol:
     (a) $\in$ (b) $\subseteq$ (c) ϕ (d) $\leftrightarrow$
   - (iii) If A = set of vowels, then:
     (a) $i \notin A$ (b) $c \in A$ (c) $u \in A$ (d) $a \notin A$
   - (iv) If A = {1, 2, 3} and B = {0, 1, 2, 3, 4}, then:
     (a) $A \subset B$ (b) $A = B$ (c) $A \subseteq B$ (d) $A \leftrightarrow B$
   - (v) {11} is known as:
     (a) null set (b) singleton set (c) subset (d) power set
   - (vi) To represent the equivalent sets, we use the symbol:
     (a) $=$ (b) $\in$ (c) $\subset$ (d) $\leftrightarrow$

Unit 2

WHOLE NUMBERS

Student Learning Outcomes

After studying this unit, students will be able to:

- Differentiate between natural and whole numbers.
- Identify natural and whole numbers, and their notations.
- Represent
  - a given list of whole numbers
  - whole number > (or <) a given whole number,
  - whole number ≥ (or ≤) a given whole number,
  - whole number > but < a given whole number,
  - whole number ≥ but ≤ a given whole number,
  - sum of two or more given whole numbers,

  on the number line.
- Add and subtract two given whole numbers.
- Verify commutative and associative law (under addition) of whole numbers.
- Recognize '0' as additive identity.
- Multiply and divide two given whole numbers.
- Verify commutative and associative law (under multiplication) of whole numbers.
- Recognize '1' as multiplicative identity.
- Verify distributive law of multiplication over addition.
- Verify distributive law of multiplication over subtraction (with positive difference).

2.1 Introduction

In our day-to-day life, we often come across the situations where we have to count objects:

- How many apples are there in a box?
- How many toffees are there in a tin?
- How many eggs are there in a crate?
- How many books are there in a shelf?

To answer such questions, we associate different objects with numbers i.e. single object with number "one", two objects with number "two", three objects with number "three" and so on. Thus, we obtain the numbers one, two, three, four, five, etc which are denoted by the different symbols as shown below.

1, 2, 3, 4, 5, 6, 7, 8, 9,...

(one) (two) (three) (four) (five) (six) (seven) (eight) (nine)

These symbols are called numerals.

2.1.1 Natural and Whole Numbers

The numbers 1, 2, 3, 4,... that we use for counting the objects are called the counting numbers or natural numbers. From above it can be observed that the smallest natural number is 1. But which is the largest natural number? We think of a natural number and there is a number larger than it. so, we cannot answer the above question as largest number does not exist.

- **Whole Numbers**

The number **0** together with the natural or counting numbers gives us the whole numbers. The set of whole numbers is denoted by the letter W:

$$W = \{0, 1, 2, 3, 4, ...\}$$

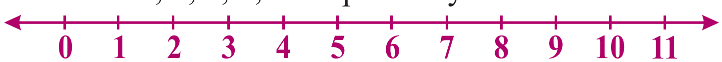
It can be noticed that the smallest number of the set W is 0 and we cannot tell about the largest number in the set because the elements of this set are infinite.

2.1.2 Whole Numbers and Number Line

Let us represent whole numbers on a number line as given below:

(i) Draw a line and mark a point with first whole number 0.

(ii) Mark the points at equal distance and label them with 1, 2, 3, 4, ... respectively as shown.



The arrow mark on the right side of the number line indicates that the list of whole numbers is increasing indefinitely.

Now let us discuss the properties of whole numbers with the help of given number line.

(i) There is no whole number on the left of "0". Thus, 0 is the smallest whole number.

(ii) Each number is one more than its previous number. Hence, each number is called the successor of its previous number and each number is one less than its next number. Hence, each number is called the predecessor of its next number. For example;

- 1 is the successor of 0 and 0 is the predecessor of 1.
- 2 is the successor of 1 and 1 is the predecessor of 2.
- 3 is the successor of 2 and 2 is the predecessor of 3.

(iii) Each whole number on the number line is greater than each whole number on its left, e.g., 5>3, 2>0, 9>8 etc.

(iv) Each whole number on the number line is less than each whole number on its right, e.g., 2<7, 3<5, 0<1 etc.

> The symbol $\geq$ is used to represent 'is greater than or equal to' and the symbol $\leq$ is used to represent 'is less than or equal to'. For example $x \geq 4$ means 'x is greater than or equal to 4 and $x \leq 4$ means 'x is less than or equal to 4'.

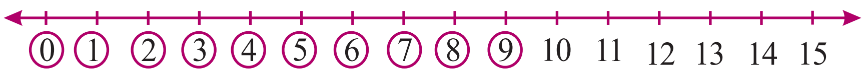
Example 1: Draw a number line to write the whole numbers:

(i) less than 10 (ii) greater than 6 (iii) ≥ 9

(iv) ≤ 5 (v) >7 but <15 (vi) ≥ 2 but ≤ 12

Solution: (i) less than 10

We use circles to represent the whole numbers less than 10.

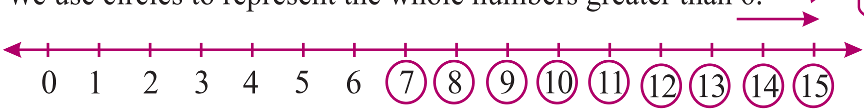


Thus, the whole numbers less than 10 are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

(ii) greater than 6

We use circles to represent the whole numbers greater than 6.

This arrow indicates that the list of whole numbers is increasing continuously.

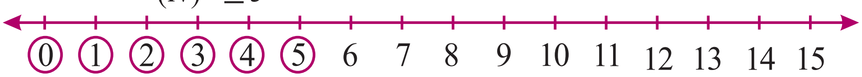


Thus, the whole numbers greater than 6 are 7, 8, 9, 10, ...

(iii) $>\underline{9}$

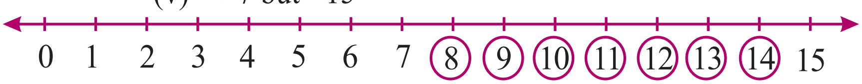
Thus, the required whole numbers are 9, 10, 11, 12, ...

(iv) ≤ 5



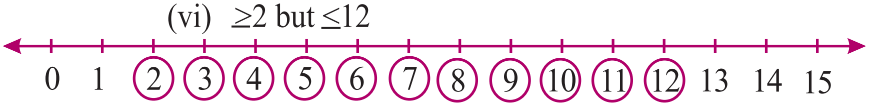
Thus, the required whole numbers are 0, 1, 2, 3, 4, 5.

(v) >7 but <15



Thus, the required whole numbers are 8, 9, 10, 11, 12, 13, 14.

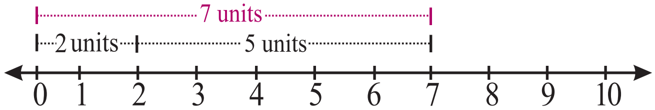
(vi) ≥ 2 but ≤ 12



Example 2: Find the sum of 2 and 5 by using a number line.

Solution:

Draw a number line and move 5 units to the right from 2 as shown below.



- The sum of 2 and 5 is 2 + 5 = 7

EXERCISE 2.1

1. Write the successor and predecessor of the following whole numbers.
   (i) 36 (ii) 74 (iii) 199 (iv) 350 (v) 789
2. Write three next consecutive whole numbers starting from 509.
3. Write the natural numbers smaller than 6.
4. In each of the following pair of numbers, state which number is greater.
   (i) 345, 435 (ii) 889, 989 (iii) 1010, 1001
   (iv) 5342, 3425 (v) 10100, 10010 (vi) 13791, 13971
5. Write the whole numbers:
   (i) < 15 but > 9 (ii) smaller than 7 (iii) greater than 15 (iv) > 73 but < 99
   (v) ≤ 11 (vi) ≥ 5 (vii) ≤ 64 but ≥ 57 (viii) ≥ 82 but < 94
   (ix) > 23 but ≤ 34 (x) ≥ 48 but ≤ 51
6. Draw the number line to represent the following whole numbers.
   (i) 0, 3, 5, and 9 (ii) 4, 8, 12 and 13 (iii) < 5
   (iv) ≥ 74 (v) > 4 but < 12 (vi) ≥ 6 but ≤ 15
7. Find the sum of following numbers by using a number line.
   (i) 1 and 3 (ii) 3 and 4 (iii) 2 and 9

2.2 Addition and Subtraction of Whole Numbers

Example 1: Find the sum of given whole numbers.

(i) 95 + 63 (ii) 634 + 179 + 358 (iii) 9056 + 8172

Solution: To find the sum of given numbers, write them in vertical columns while placing each digit according to its place value.

(i) 95 + 63

$$\begin{array}{ccc} & T & U \\ ① & 9 & 5 \\ + & 6 & 3 \\ \hline 1 & 5 & 8 \end{array}$$

$\because$ 5 + 3 = 8
9 + 6 = 15

Thus, 95 + 63 = 158

(ii) 634 + 179 + 358

$$\begin{array}{cccc} & H & T & U \\ ① & ① & ② & \\ & 6 & 3 & 4 \\ & 1 & 7 & 9 \\ + & 3 & 5 & 8 \\ \hline 1 & 1 & 7 & 1 \end{array}$$

$\because$ 4 + 9 + 8 = 21
2 + 3 + 7 + 5 = 17
1 + 6 + 1 + 3 = 11

Thus, 634 + 179 + 358 = 1171

(iii) 9056 + 8172

$$\begin{array}{ccccc} & \text{Th} & \text{H} & \text{T} & \text{U} \\ 1 & & 1 & & \\ & 9 & 0 & 5 & 6 \\ + & 8 & 1 & 7 & 2 \\ \hline 1 & 7 & 2 & 2 & 8 \\ \hline \end{array}$$

$\because$ $6 + 2 = 8$
$5 + 7 = 12$
$1 + 1 = 2$
$9 + 8 = 17$

Thus, 9056 + 8172 = 17228

Example 2: Perform the following subtractions.

(i) 842 – 391 (ii) 2105 – 1726

Solution: To perform the subtractions, write them in vertical columns while placing each digit according to its place value.

(i) 842 – 391

$$\begin{array}{cccc} & \text{H} & \text{T} & \text{U} \\ & 7 & 14 & \\ & \cancel{8} & \cancel{4} & 2 \\ - & 3 & 9 & 1 \\ \hline & 4 & 5 & 1 \\ \hline \end{array}$$

$\because$ $2 - 1 = 1$
$14 - 9 = 5$
$7 - 3 = 4$

Thus, 842 – 391 = 451

(ii) 2105 – 1726

$$\begin{array}{ccccc} & \text{Th} & \text{H} & \text{T} & \text{U} \\ & 1 & 10 & 9 & 15 \\ & \cancel{2} & \cancel{1} & \cancel{0} & \cancel{5} \\ - & 1 & 7 & 2 & 6 \\ \hline & & 3 & 7 & 9 \\ \hline \end{array}$$

$\because$ $15 - 6 = 9$
$9 - 2 = 7$
$10 - 7 = 3$
$1 - 1 = 0$

Thus, 2105 – 1726 = 379

EXERCISE 2.2

1. Evaluate:

(i) 486 + 732 (ii) 654 + 198 (iii) 811 – 356
(iv) 923 – 437 (v) 1096 + 4833 (vi) 4001 – 809
(vii) 5121 – 2674 (viii) 815 + 186 + 334 (ix) 650 + 809 + 97

2. Write the correct digit within each box.

(i)
$$\begin{array}{cccc} & \square & 4 & 3 \\ + & 2 & 5 & \square \\ \hline & 9 & \square & 7 \\ \hline \end{array}$$

(ii)
$$\begin{array}{cccc} & 4 & 9 & 1 \\ + & \square & 8 & \square \\ \hline & 6 & \square & 6 \\ \hline \end{array}$$

(iii)
$$\begin{array}{cccc} & \square & 2 & 3 \\ + & 2 & \square & 4 \\ & 3 & 4 & \square \\ \hline & 7 & 5 & 2 \\ \hline \end{array}$$

(iv)
$$\begin{array}{cccc} & 6 & 9 & 7 \\ - & \square & 5 & \square \\ \hline & 1 & \square & 4 \\ \hline \end{array}$$

(v)
$$\begin{array}{cccc} & 2 & \square & 6 \\ - & \square & 6 & \square \\ \hline & 0 & 7 & 6 \\ \hline \end{array}$$

(vi)
$$\begin{array}{ccccc} & 6 & 3 & 2 & \square \\ - & \square & \square & \square & 9 \\ \hline & 2 & 0 & 1 & 4 \\ \hline \end{array}$$

3. Find the sum of first five whole numbers.

4. Find the sum of smallest three-digit number and largest two-digit number.

5. Find the difference between the largest three-digit whole number and the smallest four-digit whole number.

6. Subtract the smallest whole number from the smallest natural number.

2.2.1 Laws of Addition

Commutative Law

In addition of two whole numbers, the result remains unchanged by changing their order. For example, consider two whole numbers 4, 6 and find the result by adding them respectively.

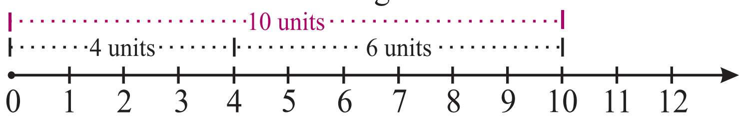
$$4 + 6 = 10$$

Now find the result by changing their order.

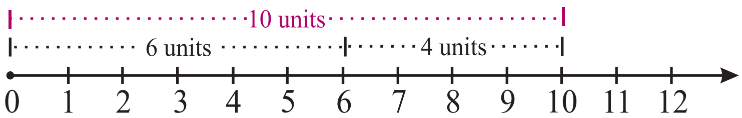
$$6 + 4 = 10$$

Hence, the result is not changed by changing their order. This can be checked by using the number ray as shown below.

- Start from 4 and move 6 units to the right.



- Now start from 6 and move 4 units to the right.



It can be seen that: $4 + 6 = 6 + 4 = 10$. This is called the commutative law of addition.

Associative Law

In addition of three whole numbers, any two whole numbers can be added first. For example, consider three whole numbers 2, 3 and 4. Now according to the above statement, we have to check:

$$(2 + 3) + 4 = 2 + (3 + 4)$$
$$5 + 4 = 2 + 7$$
$$9 = 9$$

> Associative and commutative laws with respect to subtraction do not hold.
> - $6 - 4 \neq 4 - 6$
> - $(4-3) - 2 \neq 4-(3-2)$

Hence, $(2 + 3) + 4 = 2 + (3 + 4)$. This law is called associative law of addition.

2.2.2 Additive Identity

Look at the following examples.

(i) $1 + 0 = 1$ (ii) $0 + 9 = 9$

From the above, it can be seen that the sum of a whole number and zero is always the whole number itself. Hence, "0" is known as the additive identity.

EXERCISE 2.3

1. Fill in the boxes with the help of commutative and associative laws.

(i) $14 + \square = 9 + 14$

(ii) $(1 + \square) + 2 = \square + (4 + 2)$

(iii) $(\square + \square) + 5 = 1 + (3 + \square)$

(iv) $4 + 11 = \square + \square$

(v) $(5 + 7) + 9 = \square + (\square + \square)$

(vi) $\square + 7 = \square + 6$

(vii) $11 + \square = 13 + \square$

(viii) $(5 + \square) + \square = \square + (10 + 15)$

2. Prove and identify the law used in each of the following.

(i) $5 + 3 = 3 + 5$ (ii) $11 + 14 = 14 + 11$

(iii) $26 + 49 = 49 + 26$ (iv) $6 + (9 + 15) = (6 + 9) + 15$

(v) $1 + (2 + 3) = (1 + 2) + 3$ (vi) $65 + 105 = 105 + 65$

(vii) $(44 + 66) + 55 = 44 + (66 + 55)$ (viii) $(10 + 100) + 1000 = 10 + (100 + 1000)$

(ix) $123 + (231 + 321) = (123 + 231) + 321$

2.3 Multiplication and Division of Whole Numbers

We know that the multiplication is repeated addition. If we have 4 boxes, each box having 12 pencils, then the total number of pencils is 12 + 12 + 12 + 12 = 48. Instead of adding 12 four times, we get the same result by multiplying 4 by 12. i.e.

$$\mathbf{12 + 12 + 12 + 12 = 4 \times 12 = 48}$$

From above we can also get $48 \div 4 = 12$. This shows that division is the inverse operation of multiplication.

Consider the following examples:

Example 1: Find the product of given whole numbers.

(i) 74, 23 (ii) 407 , 115 (iii) 888 , 56

Solution:

(i) 74×23

$$\begin{array}{r} 7\ 4 \\ \times 2\ 3 \\ \hline 2\ 2\ 2 \\ +1\ 4\ 8\ \times \\ \hline 1\ 7\ 0\ 2 \\ \hline \end{array}$$

Thus, $74 \times 23 = 1702$

(ii) 407×115

$$\begin{array}{r} 4\ 0\ 7 \\ \times 1\ 1\ 5 \\ \hline 2\ 0\ 3\ 5 \\ 4\ 0\ 7\ \times \\ +4\ 0\ 7\ \times\ \times \\ \hline 4\ 6\ 8\ 0\ 5 \\ \hline \end{array}$$

Thus, $407 \times 115 = 46805$

(iii) 888×56

$$\begin{array}{r} 8\ 8\ 8 \\ \times\ 5\ 6 \\ \hline 5\ 3\ 2\ 8 \\ +4\ 4\ 4\ 0\ \times \\ \hline 4\ 9\ 7\ 2\ 8 \\ \hline \end{array}$$

Thus, $888 \times 56 = 49728$

Example 2: Divide 27552 by 112.

Solution:

Use the long division method.

$$\begin{array}{r|l} & 2\ 4\ 6 \\ \hline 112 & 2\ 7\ 5\ 5\ 2 \\ & 2\ 2\ 4 \\ \hline & \ \ 5\ 1\ 5 \\ & \ \ 4\ 4\ 8 \\ \hline & \ \ \ \ 6\ 7\ 2 \\ & \ \ \ \ 6\ 7\ 2 \\ \hline & \ \ \ \ \ \ 0 \\ \hline \end{array}$$

Thus, $27552 \div 112 = 246$

Example 3: Find the largest 5-digit number which is exactly divisible by 145.

Solution: We know that the largest 5-digit number is 99999.

$$\begin{array}{r|l l} & \ \ 6\ 8\ 9 & \leftarrow \text{(quotient)} \\ \hline \text{(divisor)}\ 145 & 9\ 9\ 9\ 9\ 9 & \leftarrow \text{(dividend)} \\ & 8\ 7\ 0 & \\ \hline & 1\ 2\ 9\ 9 & \\ & 1\ 1\ 6\ 0 & \\ \hline & \ \ 1\ 3\ 9\ 9 & \\ & \ \ 1\ 3\ 0\ 5 & \\ \hline & \ \ \ \ \ \ 9\ 4 & \leftarrow \text{(remainder)} \\ \hline \end{array}$$

The required number $= 99999 - 94$

$= 99905$

Example 4: Find the smallest 4-digit number which is exactly divisible by 135.

Solution: We know that the smallest 4- digit number is 1000. So,

$$\begin{array}{r|l} & \,7 \\ \hline 135 & 1000 \\ & 945 \\ \hline & 55 \end{array}$$

To find the required number subtract 55 from 135, i.e., 135–55 = 80. Thus, the required number is 1000+80=1080

EXERCISE 2.4

1. Find the product of following whole numbers.

(i) 87 , 62 (ii) 59 , 91 (iii) 101 , 77 (iv) 456 , 150 (v) 372 , 84

(vi) 762 , 309 (vii) 2468 , 111 (viii) 1357 , 123 (ix) 1572 , 241

2. Solve.

(i) $748 \div 11$ (ii) $1125 \div 9$ (iii) $3345 \div 15$ (iv) $7854 \div 7$

(v) $6136 \div 52$ (vi) $9801 \div 81$ (vii) $6216 \div 111$ (viii) $54756 \div 234$

(ix) $14985 \div 135$

3. Find the product of the smallest 4-digit number and the greatest 2-digit number.

4. Find the smallest 3-digit number which is exactly divisible by 16.

5. Find the largest 3-digit number which is exactly divisible by 24.

2.3.1 Laws of Multiplication

We have studied the laws of addition of whole numbers. Now we discuss the laws of multiplication which are given as follows:

- **Commutative Law**

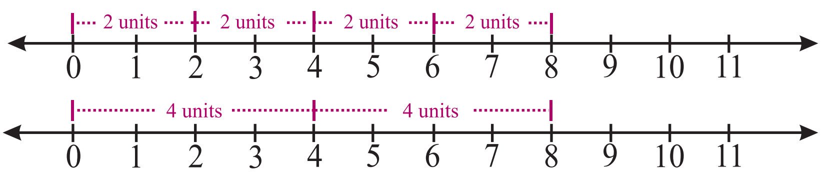
Consider any two whole numbers, say 2, 3 and multiply them in the given order: $\mathbf{2 \times 3 = 6}$

Now change their order and perform the multiplication again: $\mathbf{3 \times 2 = 6}$

Hence, the result is the same after changing their order. So,

$$a \times b = b \times a \text{ for all } a, b \in W$$

This result can also be verified by using a number line.



- **Associative Law**

Consider any three whole numbers say, 2, 3 and 5. Now according to the associative law we have to check that:

$$(2 \times 3) \times 5 = 2 \times (3 \times 5)$$
$$6 \times 5 = 2 \times 15$$
$$30 = 30$$

Thus, the multiplication of whole numbers also obeys the associative law. i.e. $(a \times b) \times c = a \times (b \times c)$ for all $a, b, c \in W$. We can also define it as: by changing the position of brackets while multiplying three whole numbers, the result remains unchanged.

Commutative and associative laws with respect to division do not hold.

2.4 Multiplication over Addition (Subtraction) of Whole Numbers

- **Distributive Laws**

Consider the whole numbers 2, 3 and 4. We multiply one whole number by the sum of other two as shown below.

$$2 \times (3 + 4) = 2 \times (7) = 14$$

Now multiply first 2 by 3, then 2 by 4 and put the addition sign between the two products.

$$2 \times 3 + 2 \times 4 = 6 + 8 = 14$$

Thus, we have $$2 \times (3 + 4) = 2 \times 3 + 2 \times 4$$

This is called distributive law of multiplication over addition.

Similarly, we can prove the distributive law of multiplication over subtraction, i.e.,

$$3 \times (4 - 2) = 3 \times 4 - 3 \times 2$$

Thus, according to the distributive law, we have;

(i) $a \times (b + c) = ab + ac$

(ii) $a \times (b - c) = ab - ac$ For all $a, b, c \in W$.

2.4.1 Multiplicative Identity

Look at the following examples.

(i) $1 \times 8 = 8$ (ii) $11 \times 1 = 11$

From the above, it can be seen that the product of any whole number with 1 is always the whole number itself. Hence, "1" is known as the multiplicative identity.

EXERCISE 2.5

1. Fill in the boxes with the help of laws of multiplication.

(i) $\square \times 2 = \square \times 4$

(ii) $7 \times 9 = \square \times \square$

(iii) $3 \times (9 - 6) = 3 \times 9 - 3 \times \square$

(iv) $5 \times \square = 6 \times \square$

(v) $2 \times (1 + 2) = \square \times 1 + 2 \times 2$

(vi) $7 + (1 + 6) = (7 + 1) + \square$

(vii) $3 \times (2 \times 5) = (\square \times \square) \times \square$

(viii) $1 \times (\square - \square) = \square \times 11 - \square \times 12$

(ix) $\square \times (11 \times 9) = (2 \times 11) \times \square$

(x) $9 \times (\square + 4) = \square \times 5 + \square \times \square$

2. Prove and identify the law used in each of the following.

(i) $3 + 2 = 2 + 3$

(ii) $1 \times (3 \times 2) = (1 \times 3) \times 2$

(iii) $9 + (11 + 13) = (9 + 11) + 13$

(iv) $8 \times 7 = 7 \times 8$

(v) $2 \times (1 + 5) = (2 \times 1) + (2 \times 5)$

(vi) $100 + (99 + 50) = (100 + 99) + 50$

(vii) $3 \times (4 - 1) = (3 \times 4) - (3 \times 1)$

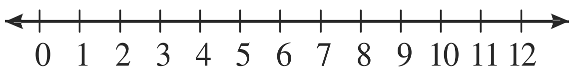
(viii) $10 \times (5 - 7) = (10 \times 5) - (10 \times 7)$

(ix) $25\times(10+8) = (25\times10)+(25\times8)$

(x) $100\times(11+13)=(100\times11)+(100\times13)$

Summary

- The numbers that we use for counting objects are called the counting or natural numbers.
- The number 0 together with the natural numbers gives us the whole numbers.
- According to the commutative law while adding or multiplying the two whole numbers, the result remains unchanged by changing their orders.
- According to the associative law while adding or multiplying three whole numbers, any two whole numbers can be added or multiplied first.
- The whole numbers can be represented by using a number line.



Review Exercise 2

1. Write the whole numbers smaller than 4.
2. Draw the number line to represent the whole numbers.

(i) 1,5,10 (ii) < 7 (iii) >100

3. Use the number line to find the given sums.

(i) 2 and 3 (ii) 1 and 6 (iii) 4 and 5

4. Evaluate

(i) $678+322$ (ii) $1234+2345$ (iii) $6565+1144$ (iv) $1000-789$

(v) $7350-1846$ (vi) $9999-999$ (vii) 999×111 (viii) 123×45

(ix) 1122×786 (x) $5782\div49$ (xi) $6655\div55$ (xii) $15129\div123$

5. Prove and identify the law.

(i) $11\times(28+72)=(11\times28)+(11\times72)$

(ii) $842+248=248+842$

(iii) $333\times111=111\times333$

(iv) $100\times(45-21)=(100\times45)-(100\times21)$

(v) $48+(37+55)=(48+37)+55$

(vi) $12\times(13\times14)=(12\times13)\times14$

Objective Exercise 2

1. Answer the following questions.
   (i) What is a number?
   (ii) What are the numerals?
   (iii) Why 0 is called the additive identity?
   (iv) Which whole number is the successor of 0.
   (v) What is meant by the commutative law of addition?

2. Fill in the blanks.
   (i) The number __________ together with the natural numbers give us the whole numbers.
   (ii) The numbers that we use for counting objects are called the _________ numbers.
   (iii) The numbers that can be divided by 2 are called _________ numbers.
   (iv) To represent the set of _________ numbers we use the capital letter W.
   (v) The product of 1 and a whole number is always the whole number itself. Hence 1 is called the _________ identity.

3. Tick (✓) the correct answer.
   (i) The numbers that cannot be divided by 2 completely are called:
   (a) even numbers (b) natural numbers (c) whole numbers (d) odd numbers
   (ii) The smallest natural number is:
   (a) 0 (b) 1 (c) 2 (d) 3
   (iii) To represent the set of natural numbers, we use the capital letter.
   (a) E (b) O (c) N (d) W
   (iv) To represent "is greater than or equal to" we use the symbol:
   (a) $<$ (b) $\leq$ (c) $>$ (d) $\geq$
   (v) The sum of two whole numbers is always:
   (a) a prime number (b) an odd number (c) an even number (d) a whole number

Unit 3

FACTORS AND MULTIPLES

Student Learning Outcomes

After studying this unit, students will be able to:

- Define a factor as a number which divides the dividend completely leaving zero remainder.
- Define a multiple as a dividend into which a factor can divide.
- Define even numbers as the numbers which are multiples of 2.
- Define odd numbers as the numbers which are not multiples of 2.
- Define prime numbers as numbers which have only two factors (i.e., 1 and itself).
- Define composite numbers as numbers which have more than two factors.
- Know that 1 is neither prime nor composite as it has only one factor which is 1 itself.
- Know that 1 is a factor of every number.
- Know that 2 is the only even prime number whereas all other prime numbers are odd.
- Test by inspection whether the numbers 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 15 and 25 can divide a given number.
- Define prime factorization as the process of factorizing a number into its prime factors.
- Recognize index notation.
- Factorize a given number and express its factors in the index notation.
- Define HCF as the greatest number which is a common factor of two or more numbers.
- Find HCF of two or more than two numbers by
  - prime factorization,
  - long division method.
- Define LCM as the smallest number which is a common multiple of two or more numbers.
- Find LCM of two or more number by
  - prime factorization,
  - long division method.
- Solve real life problems related to HCF and LCM.

3.1 Factors and Multiples

3.1.1 Factors

We know that if a number is divided by another number and the remainder is "0" then the 1st number is said to be divisible by the 2nd number. For example,

$$\begin{array}{r} 18 \\ 1\overline{)\,18} \\ -18 \\ \hline 0 \end{array} \qquad \begin{array}{r} 9 \\ 2\overline{)\,18} \\ -18 \\ \hline 0 \end{array} \qquad \begin{array}{r} 6 \\ 3\overline{)\,18} \\ -18 \\ \hline 0 \end{array}$$

$$\begin{array}{r} 3 \\ 6\overline{)\,18} \\ -18 \\ \hline 0 \end{array} \qquad \begin{array}{r} 2 \\ 9\overline{)\,18} \\ -18 \\ \hline 0 \end{array} \qquad \begin{array}{r} 1 \\ 18\overline{)\,18} \\ -18 \\ \hline 0 \end{array}$$

From the above it can be seen that the number 18 is divisible by 1, 2, 3, 6, 9 and 18. These numbers are known as the factors of the number 18, i.e, the factors of 18 are 1,2,3,6,9 and 18.

Similarly, we can find the factors of any other number as given below.

The factors of 12 are 1, 2, 3, 4, 6, 12.

The factors of 15 are 1, 3, 5, 15.

The factors of 42 are 1, 2, 3, 6, 7, 14, 21, 42.

Thus, a factor of a number can be defined as: "A number that divides the given number completely is called a factor of the given number".

NOTE: Every number greater than 1 has at least two factors.

3.1.2 Multiples

All the numbers which are divisible by another number are called the multiples of that number, for example:

Multiples of 2 are 2, 4, 6, 8, ...
Multiples of 3 are 3, 6, 9, 12, ...

In the given example, as we say that the factors of 18 are 1, 2, 3, 6, 9 and 18, we can also say that 18 is a multiple of each of the number 1, 2, 3, 6, 9 and 18.

The multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, ...

The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, ...

The multiples of 9 are 9, 18, 27, ...

Thus, a number is said to be a multiple of each of its factors. It can also be noticed that the multiple of a number is either greater than or equal to the number itself.

3.1.3 Types of Natural Numbers

Natural numbers are classified in two ways, either even and odd or prime and composite.

Even Numbers: The numbers which are divisible by 2 are called even numbers. We can also say that all multiples of 2 are even numbers. It means 2, 4, 6, 8,... are all even numbers because all numbers are multiple of 2. The set of even numbers is denoted by the capital letter E.

$$E = \{2, 4, 6, 8, \dots\}$$

Odd Numbers: The numbers which are not divisible by 2 are called odd numbers. It can also be said that the numbers which are not multiples of 2 are odd number, i.e. 1, 3, 5, 7.... are all odd numbers. The set of odd numbers is denoted by the capital letter O.

$$O = \{1, 3, 5\ \ 7, \dots\}$$

Prime Number: A number having exactly two factors, 1 and the number itself, is called the prime number, i.e. 2, 3, 5, 7, 11, 13, 17, 19 … are all prime numbers. The set of prime numbers is denoted by the capital letter P.

$$P = \{2, 3, 5, 7, 11, 13, \dots\}$$

The number 2 is the only even prime number whereas all other prime numbers are odd.

Composite Numbers: A number having factors other than 1 and itself is called a composite number or we can say that the numbers having more than two factors are composite numbers, i.e. 4, 6, 8, 9, 10 … are all composite numbers because each of these numbers has more than two factors. The set of composite numbers is denoted by the capital letter C.

$$C = \{4, 6, 8, 9, 10, 12, 14, \dots\}$$

Note: The number 1 is neither prime nor composite number because it has only one factor which is 1 itself.

Example 1: Write all the factors of the following numbers.

(i) 56 (ii) 121

Solution: (i) 56

To find the factors of 56, start from 1 and proceed as given below.

$$\begin{aligned} 56 &= 1 \times 56 \\ &= 2 \times 28 \\ &= 4 \times 14 \\ &= 7 \times 8 \end{aligned}$$

Thus, the factors of 56 are 1, 2, 4, 7, 8, 14, 28 and 56.

(ii) 121

$$\begin{aligned} 121 &= 1 \times 121 \\ &= 11 \times 11 \end{aligned}$$

Thus, the factors of 121 are 1, 11, and 121.

EXERCISE 3.1

1. Write all factors of each of the following numbers.

   (i) 21 (ii) 36 (iii) 48 (iv) 99

2. Write first five multiples of each of the following numbers.

   (i) 3 (ii) 5 (iii) 9 (iv) 12

3. Separate the odd and even numbers.

   (i) 135 (ii) 342 (iii) 1112 (iv) 5008

   (v) 9427 (vi) 8134 (vii) 10006 (viii) 78965

4. List all prime numbers between:

   (i) 10 and 50 (ii) 25 and 60 (iii) 32 and 48 (iv) 76 and 90

5. List all numbers less than 100 which are multiples of 5 and 10.
6. List all numbers less than 50 which are multiples of 3 and 4.
7. Write all composite numbers less than 20.
8. Write five consecutive composite numbers just below 50.
9. Write all prime numbers less than 15.

3.2 Tests of Divisibility

In order to test the divisibility of a number by another number, we actually divide the number, and find if the remainder is zero. But this method can be very difficult and time consuming. Here we shall learn some easier methods by which we can say whether the number is divisible by a given number or not. These methods are known as "tests of divisibility".

(i) A number is divisible by 2, if the digit at the units place is 0, 2, 4, 6 or 8.

- 670 is divisible by 2 since the digit at the unit place is 0, i.e., $670 \div 2 = 335$
- 138 is divisible by 2 since the last digit is an even number, i.e., $138 \div 2 = 69$

(ii) A number is divisible by 3, if the sum of its digits is divisible by 3.

- 531 is divisible by 3 since the sum of the digits is 9 (5+3+1) and 9 is divisible by 3, i.e., $9 \div 3 = 3$
- 6396 is divisible by 3 since the sum of the digits is 24 (6+3+9+6) and 24 is divisible by 3, i.e., $24 \div 3 = 8$

(iii) A number is divisible by 4, if the digits at the units and tens places are 0's or a number formed by the tens and units digits can be divided by 4.

- 4500 is divisible by 4 since the digits at the units and tens places are 0's, i.e., $4500 \div 4 = 1125$
- 7632 is divisible by 4 since the number formed by the tens and units digits (32) can be divided by 4, i.e., $7632 \div 4 = 1908$

(iv) A number is divisible by 5, if the digit at the units places is 0 or 5.

- 2360 is divisible by 5 since the digit at the units places is 0, i.e., $2360 \div 5 = 472$
- 2385 is divisible by 5 since the number at the units place is 5, i.e., $2385 \div 5 = 477$

(v) A number is divisible by 6, if it has even number at the units place and the sum of its digits is divisible by 3.

- 642 is divisible by 6 since the digit at the units place is an even number and sum of its digits (6+4+2=12) is divisible by 3, i.e., $642 \div 6 = 107$
- 2472 is divisible by 6 since the digit at the units place is an even number and sum of its digits (2+4+7+2=15) is divisible by 3, i.e., $2472 \div 6 = 412$.

(vi) A number is divisible by 8, if the digits at the units, tens and hundreds places are 0's or they comprise a number divisible by 8.

- 89000 is divisible by 8 since the digits at the units, tens and hundreds places are 0's, i.e., $89000 \div 8 = 11125$
- 7424 is divisible by 8 since the number formed by the digits, units, tens and hundreds places (424) is divisible by 8, i.e., $7424 \div 8 = 928$

(vii) A number is divisible by 9, if the sum of its digits is divisible by 9.

- 531 is divisible by 9 since the sum of the digits is 9 (5+3+1) and 9 is divisible by 9, i.e., $9 \div 9 = 1$, i.e. $531 \div 9 = 59$
- 8496 is divisible by 9 since the sum of the digits is 27 $(8 + 4 + 9 + 6)$ and 27 is divisible by 9, i.e., $8496 \div 9 = 944$

(viii) A number is divisible by 10, if the digits at the units place are 0.

- 330 is divisible by 10 since the digit at the unit place is 0, i.e., $330 \div 10 = 33$
- 12340 is divisible by 10 since the digits at the unit place is 0, i.e., $12340 \div 10 = 1234$

(ix) A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places is either 0 or divisible by 11.

- 2574 is divisible by 11 since the difference of the sum of its digits at the odd places and the sum of its digits at even places is 0, i.e.,
  Sum of digits at odd places $= 2 + 7 = 9$
  Sum of digits at even places $= 5 + 4 = 9$
  The difference of these sums $= 9 - 9 = 0$
- 1749 is divisible by 11 since the difference of the sum of its digits at the odd places and the sum of its digits at even places is 11, i.e.,
  Sum of digits at odd places $= 1 + 4 = 5$
  Sum of digits at even places $= 7 + 9 = 16$
  The difference of these sums $= 16-5 = 11$

(x) A number is divisible by 12 (a) if the sum of its digits is divisible by 3 and (b) the number formed by its tens and units is divisible by 4.

- 1476 is divisible by 12 since the number formed by its tens and units (76) is divisible by 4 and the sum of its digits (1+4+7+6=18) is divisible by 3, i.e., $1476 \div 12 = 123$
- 37548 is divisible by 12 since the number formed by its tens and units (48) is divisible by 4 and the sum of its digits (3+7+5+4+8=27) is divisible by 3, i.e., $37548 \div 12 = 3129$

(xi) A number is divisible by 15 (a) if the sum of its digits is divisible by 3 and (b) the digits at the units place is 0 or 5.

- 5175 is divisible by 15 since the sum of its digits (5+1+7+5=18) is divisible by 3 and the digit at the units place is 5, i.e., $5175 \div 15 = 345$
- 5940 is divisible by 15 since the sum of its digits (5+9+4+0=18) is divisible by 3 and the digit at the units place is 0, i.e., $5940 \div 15 = 396$

(xii) A number is divisible by 25, if the digits at the units and tens places are 0's or a number formed by the tens and units digits can be divided by 25

- 12300 is divisible by 25 since the digits at the units and tens places are 0's, i.e., $12300 \div 25 = 492$
- 9175 is divisible by 25 since the number formed by the tens and units digits (75) can be divided by 25, i.e., $9175 \div 25 = 367$

EXERCISE 3.2

1. Separate the following into even and odd numbers without carrying division.

(i) 6423	(ii) 8321	(iii) 6254	(iv) 989
(v) 810	(vi) 8394	(vii) 1234	(viii) 1357
(ix) 54321	(x) 86420	(xi) 99880	(xii) 30005

2. Which of the following numbers are divisible by 3; by 4 and by 5.

(i) 762	(ii) 512	(iii) 110	(iv) 968
(v) 3692	(vi) 5361	(vii) 1215	(viii) 7310
(ix) 1010	(x) 12345	(xi) 4952	(xii) 45678

3. Using the divisibility tests, determine which of the following numbers are divisible by 8 or 9.

(i) 512	(ii) 333	(iii) 440	(iv) 904
(v) 56565	(vi) 2968	(vii) 6669	(viii) 11241
(ix) 16920	(x) 11088	(xi) 9144	(xii) 6312

4. Find the number which is divisible by 11.

(i) 2550	(ii) 3673	(iii) 8415	(iv) 5155
(v) 135795	(vi) 21211212	(vii) 7654321	(viii) 654313

5. Which of the following numbers are divisible by 12 or 15.

(i) 312	(ii) 576	(iii) 729	(iv) 1140
(v) 1335	(vi) 4428	(vii) 3150	(viii) 612
(ix) 11112	(x) 12345	(xi) 23448	(xii) 70350

6. Find the numbers which can be divided by 25.

(i) 142300	(ii) 5412625	(iii) 810235
(iv) 1111150	(v) 626205	(vi) 100200300

3.3 Factorization

"The process of writing a number into its factors is called factorization".

We know that a natural number can be expressed as the product of its factors as given below.

- $$\begin{aligned} 24 &= 1 \times 24 \\ &= 2 \times 12 \\ &= 3 \times 8 \\ &= 4 \times 6 \end{aligned}$$
  $\boxed{24 = 2 \times 2 \times 2 \times 3}$

- $$\begin{aligned} 42 &= 1 \times 42 \\ &= 2 \times 21 \\ &= 3 \times 14 \\ &= 6 \times 7 \end{aligned}$$
  $\boxed{42 = 2 \times 3 \times 7}$

It can be observed that the number 24 has many factors, but out of them 2, 2, 2 and 3 are the only factors which are prime numbers. Similarly, 42 has many factors, but out of them2, 3 and 7 are the prime factors. such factorization in which a number is expressed as the product of prime factors is called the prime factorization.

Now look at the prime factors of the number 42 which are given below.

- $$\begin{aligned} \mathbf{42} &= \mathbf{2 \times 21 = 2 \times 3 \times 7} \\ &= \mathbf{3 \times 14 = 3 \times 2 \times 7} \\ &= \mathbf{7 \times 6 = 7 \times 2 \times 3} \end{aligned}$$

In all the above cases, we can observe that the order of factorization may be different but prime factors remain the same. Thus, the prime factors of a number can be written in any order but we often write them in ascending order.

The prime factors of a number can also be expressed by using a tree like diagram is called factor tree.

24
2 × 12
2 × 2 × 6
2 × 2 × 2 × 3

3.3.1 Index Notation

Look at the prime factors of following numbers.

- $49 = 7 \times 7$
- $81 = 3 \times 3 \times 3 \times 3$
- $125 = 5 \times 5 \times 5$
- $32 = 2 \times 2 \times 2 \times 2 \times 2$

In short, we can write the prime factors of above given numbers as:

- $7 \times 7 = 7^2$ *(square of 7)*
- $5 \times 5 \times 5 = 5^3$ *(cube of 5)*
- $3 \times 3 \times 3 \times 3 = 3^4$ *(3 to the power of 4)*
- $2 \times 2 \times 2 \times 2 \times 2 = 2^5$ *(2 to the power of 5)*

Example 1: Express the following factors by using the index notation.

(i) 11×11 (ii) $3 \times 3 \times 7 \times 7$ (iii) $2 \times 2 \times 5 \times 5 \times 5$

Solution:

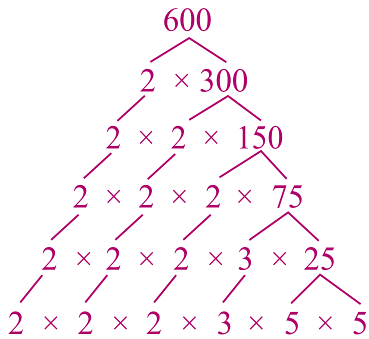
(i) $11 \times 11 = 11^2$ (ii) $3 \times 3 \times 7 \times 7 = 3^2 \times 7^2$ (iii) $2 \times 2 \times 5 \times 5 \times 5 = 2^2 \times 5^3$

Example 2: Find the prime factors of the following numbers.

(i) 600 (ii) 8820

Solution: There are two methods for factorization of 600.

Factors tree method



Division method

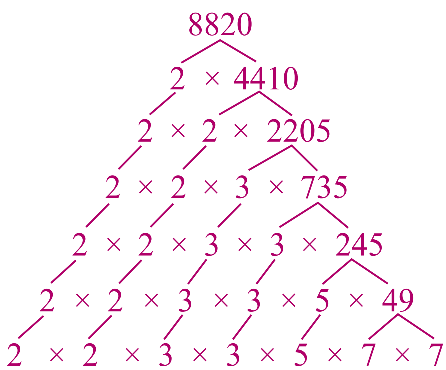
2	600
2	300
2	150
3	75
5	25
	5

Hence the prime factors of 600 are:

$2 \times 2 \times 2 \times 3 \times 5 \times 5 = 2^3 \times 3 \times 5^2$

(ii) 8820

Factors tree method



Hence, the prime factors of 8820 are:

$2 \times 2 \times 3 \times 3 \times 5 \times 7 \times 7 = 2^2 \times 3^2 \times 5 \times 7^2$

Division method

2	8820
2	4410
3	2205
3	735
5	245
7	49
7	7
	1

EXERCISE 3.3

1. Express the following factors by using the index notation:

(i) $13 \times 13 \times 13$

(ii) $7 \times 7 \times 7 \times 7$

(iii) 29×29

(iv) $5 \times 5 \times 5 \times 5 \times 5 \times 5$

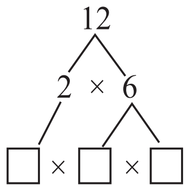
(v) $11 \times 11 \times 11 \times 11$

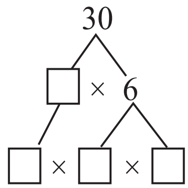
(vi) $3 \times 3 \times 3 \times 3 \times 3 \times 3$

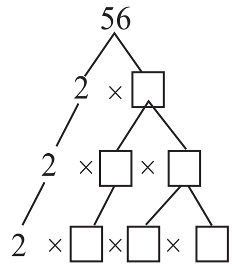
(vii) $2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5$

(viii) $7 \times 7 \times 11 \times 23 \times 23$

2. Fill in the boxes to complete the factor tree.

(i) 

(ii) 

(iii) 

3. Find the prime factors of the following numbers by using division method.

(i) 20	(ii) 36	(iii) 98	(iv) 225
(v) 216	(vi) 441	(vii) 256	(viii) 392
(ix) 5250	(x) 2310	(xi) 2058	(xii) 1248

4. Factorize the following numbers into prime factors by using the factor tree method.

(i) 24	(ii) 36	(iii) 60	(iv) 72
(v) 108	(vi) 462	(vii) 390	(viii) 770

3.4 Highest Common Factor (HCF)

Let us find the factors of 24 and 30.

- The factors of 24 are 1 , 2 , 3 , 4 , 6 , 8 , 12 , 24
- The factors of 30 are 1 , 2 , 3 , 5 , 6 , 10 , 15 , 30

It can be observed that 1,2,3 and 6 are the common factors of 24 and 30 but 6 is the largest factor among them. Hence, 6 is known as the highest common factor (HCF) of 24 and 30.

Similarly, we can find the highest common factor of more than two numbers as shown below.

- The factors of 12 are 1 , 2 , 3 , 4 , 6 , 12
- The factors of 18 are 1 , 2 , 3 , 6 , 9 , 18
- The factors of 42 are 1 , 2 , 3 , 6 , 7 , 14 , 21 , 42

The common factors of 12, 18 and 42 are 1, 2, 3 and 6 and largest of these common factors is 6. Thus, the highest common factor is 6.

Sometimes it becomes very difficult to write all possible factors of a number. Hence, to solve this problem we use the following two methods for finding the HCF of given numbers.

(i) Prime factorization method. (ii) Long division method.

3.4.1 HCF by Prime Factorization Method

We can find the highest common factor (HCF) of two or more than two numbers by prime factorization. To understand this method, let us consider numbers 72, 48 and 132 and find their HCF.

Step 1: Find the prime factors of each number.

$$\begin{array}{r|l} 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 132 \\ \hline 2 & 66 \\ \hline 3 & 33 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

Step 2: Express the prime factors of each number. Choose the common numbers from all sets.

- The prime factors of 72 $= 2 \times 2 \times 2 \times 3 \times 3$
- The prime factors of 48 $= 2 \times 2 \times 2 \times 2 \times 3$
- The prime factors of 132 $= 2 \times 2 \times 3 \times 11$

Step 3: Get the product of these chosen common factors to find the HCF.

$$\mathbf{2 \times 2 \times 3 = 4 \times 3 = 12}$$

Thus, the HCF of 72, 48 and 132 is 12

3.4.2 HCF by Long Division Method

Sometimes we have very large numbers. To find the highest common factors of such numbers through prime factorization is very difficult. So, here we can find the HCF of the given numbers by long division method. Let us make it clear by finding the HCF of 928 and 324.

Step 1: Make the larger number the dividend and the smaller number the divisor.

$$\begin{array}{rl} & \;\;\;\;\;\;2 \leftarrow \text{(quotient)} \\ 324 \,\big) & \overline{9\;\;2\;\;8} \leftarrow \text{(dividend)} \\ \nearrow & \underline{6\;\;4\;\;8} \\ \text{(divisor)} & 2\;\;8\;\;0 \leftarrow \text{(remainder)} \end{array}$$

Step 2: Make remainder the new divisor, and old divisor the dividend. Repeat the process until you get 0 as remainder.

$$\begin{array}{r|l|l} \text{divisor} & \text{dividend} & \text{quotient} \\ \hline 324 & 928 & 2 \\ & \underline{648} & \\ 280 & 324 & 1 \\ & \underline{280} & \\ 44 & 280 & 6 \\ & \underline{264} & \\ 16 & 44 & 2 \\ & \underline{32} & \\ 12 & 16 & 1 \\ & \underline{12} & \\ 4 & 12 & 3 \\ & \underline{12} & \\ & 0 & \end{array}$$

Step 3: The last divisor is the highest common divisor of two numbers.

Thus, the HCF of 928 and 324 is 4.

Example 1: Use the prime factorization method to find the HCF of 96, 108 and 420.

Solution:

The prime factors of 96, 108 and 420 are given below.

$$\begin{array}{r|l} 2 & 96 \\ \hline 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 108 \\ \hline 2 & 54 \\ \hline 3 & 27 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 420 \\ \hline 2 & 210 \\ \hline 3 & 105 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

Thus,

The prime factors of 96 = $2 \times 2 \times 2 \times 2 \times 2 \times 3$

The prime factors of 108 = $2 \times 2 \times 3 \times 3 \times 3$

The prime factors of 420 = $2 \times 2 \times 3 \times 5 \times 7$

The HCF of 96, 108 and 420 = $2 \times 2 \times 3 = 4 \times 3 = 12$

Example 2: Find the HCF of 1353, 979 and 1078, using the long division method.

Solution:

Make the largest of three numbers as dividend and the second larger number as divisor.

$$\begin{array}{r|l|l} 1078 & 1353 & 1 \\ & 1078 & \\ \hline 275 & 1078 & 3 \\ & 825 & \\ \hline 253 & 275 & 1 \\ & 253 & \\ \hline 22 & 253 & 11 \\ & 242 & \\ \hline 11 & 22 & 2 \\ & 22 & \\ \hline & 0 & \end{array}$$

- It can be seen the HCF of 1078 and 1353 is 11. Consider this HCF of two numbers as divisor and the smallest of three numbers as dividend.

$$\begin{array}{r|l} & 89 \\ \hline 11 & 979 \\ & 88 \\ \hline & 99 \\ & 99 \\ \hline & 0 \end{array}$$

Thus, HCF of 1353, 979 and 1078 is 11.

EXERCISE 3.4

1. Find all the common factors.

(i)	6 and 10	(ii)	8 and 12	(iii)	10 and 15
(iv)	12 and 18	(v)	20 and 30	(vi)	28 and 36

2. Find HCF by writing the common factors of each number.

(i)	24, 36	(ii)	25, 45	(iii)	21,49
(iv)	12, 33	(v)	39, 52	(vi)	16, 20
(vii)	4,6,10	(viii)	22, 44, 66	(ix)	35, 20, 45

3. Find the HCF of following numbers, using the prime factorization method.

(i)	12, 18	(ii)	22, 55	(iii)	36, 54
(iv)	24, 48	(v)	22, 132	(vi)	60, 72
(vii)	16, 54, 84	(viii)	22, 55, 110	(ix)	56, 189, 175

4. Find the HCF of the following numbers, using the long division method.

(i)	72, 184	(ii)	63, 112	(iii)	276, 161
(iv)	314, 334	(v)	405, 513	(vi)	128, 340
(vii)	234, 538, 678	(viii)	155, 341, 1302	(ix)	399, 665, 1463

3.5 Least Common Multiple (LCM)

The least common multiple of two or more numbers is the smallest number which is a multiple of each of the given numbers.

Look at the multiples of 2 and 3 which are given below.

Multiples of 2: 2 , 4 , 6 , 8 , 10 , 12 , 14 , 16 , 18 , ...

Multiples of 3: 3 , 6 , 9 , 12 , 15 , 18 , 21 , 24 , ...

It can be noticed that 6, 12, 18 etc are the common multiples of 2 and 3 but the smallest among then is 6. Hence, 6 is known as the least common multiple (LCM) of 2 and 3. i.e.

Common multiples are 6, 12, 18, ...

Thus, LCM = 6

Similarly, we can find the least common multiple of more than two numbers that can be seen in the following example.

Multiples of 3: 3 , 6 , 9 , 12 , 15 , 18 , 21 , 24 , 27 , 30 , 33 , 36 ...

Multiples of 4: 4 , 8 , 12 , 16 , 20 , 24 , 28 , 32 , 36 , 40 , ...

Multiples of 6: 6 , 12 , 18 , 24 , 30 , 36 , 42 , ...

Common multiples are 12, 24, 36, ...

Thus, LCM = 12

In case of large numbers, it becomes difficult to find the least common multiple (LCM) by finding their multiples. So, here we can use the following two methods.

1. Prime factorization method. 2. Division method

Let us discuss them one by one.

3.5.1 LCM by Prime Factorization Method

The LCM of given numbers can be obtained by finding their prime factors. To make it clear, let us find the LCM of 36, 48 and 56 with the help of prime factorization method which is given below.

Step 1: Find the prime factors of each of the numbers.

$$\begin{array}{r|l} 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 48 \\ \hline 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 56 \\ \hline 2 & 28 \\ \hline 2 & 14 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

Step 2: Express the prime factors of each number, using the index notation. Choose the largest number from each set.

Since,

$$\begin{aligned} 36 &= 2 \times 2 \times 3 \times 3 &&= 2^2 \times 3^2 \\ 48 &= 2 \times 2 \times 2 \times 2 \times 3 &&= 2^4 \times 3 \\ 56 &= 2 \times 2 \times 2 \times 7 &&= 2^3 \times 7 \end{aligned}$$

$$\boxed{2^4} \quad \boxed{3^2} \quad \boxed{7}$$

Step 3: Get the product of these chosen numbers to find the LCM.

Thus, the LCM of 36, 48 and 56 is $2^4 \times 3^2 \times 7$.

$= 16 \times 9 \times 7 = 1008$

3.5.2 LCM by Division Method

In case of more numbers, it becomes laborious to find the LCM. Hence, we use the division method which is simpler than the prime factorization method. To understand it, let us consider four numbers 24, 36, 54 and 81 for finding their LCM as given below.

Step 1: Arrange all the numbers in a row.

Step 2: Choose a common factor of at least two of the numbers and divide the numbers by it (carry forward which are not divisible).

$$\begin{array}{r|l} 2 & 24,\ 36,\ 54,\ 81 \\ \hline & 12,\ 18,\ 27,\ 81 \end{array}$$

Step 3: Repeat the process till no two numbers have common divisor.

Step 4: The product of the divisors and remainders is the LCM.

$$\begin{array}{r|l} 2 & 24,\ 36,\ 54,\ 81 \\ \hline 2 & 12,\ 18,\ 27,\ 81 \\ \hline 3 & 6,\ 9,\ 27,\ 81 \\ \hline 3 & 2,\ 3,\ 9,\ 27 \\ \hline 3 & 2,\ 1,\ 3,\ 9 \\ \hline & 2,\ 1,\ 1,\ 3 \end{array}$$

Thus, the LCM of 24, 36, 54 and 81 is $2\times2\times3\times3\times3\times2\times3=648$

Example 1: Determine the LCM of 9, 12 and 18:

(i) by finding their common multiples (ii) by prime factorization method.

(iii) by division method.

Solution: (i) LCM by finding the common multiples of 9, 12 and 18:

The Multiples of 9 : 9 , 18 , 27 , 36 , 45 , 54 , 63 , 72 , 81 , 90 , 99 , 108 ...

The Multiples of 12 : 12 , 24 , 36 , 48 , 60 , 72 , 84 , 96 , 108 ...

The Multiples of 18 : 18 , 36 , 54 , 72 , 90 , 108 , 126 , 144 ...

Common multiples of 9, 12 and 18 are 36, 72, 108

Least common multiple = 36

Thus, the LCM of 9, 12 and 18 is 36.

(ii) By prime factorization method.

$$\begin{array}{r|l} 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Thus,

$$\begin{array}{rlcccc} 9 & = 3 \times 3 & = & & & 3^2 \\ 12 & = 2 \times 2 \times 3 & = & 2^2 & \times & 3 \\ 18 & = 2 \times 3 \times 3 & = & 2 & \times & 3^2 \\ & & & \boxed{2^2} & & \boxed{3^2} \end{array}$$

Thus, the LCM of 9, 12 and 18 is $2^2 \times 3^2 = 4 \times 9 = 36$

(iii) By division method.

$$\begin{array}{r|l} 2 & 9,\ 12,\ 18 \\ \hline 3 & 9,\ \ 6,\ \ 9 \\ \hline 3 & 3,\ \ 2,\ \ 3 \\ \hline & 1,\ \ 2,\ \ 1 \end{array}$$

Thus, the LCM of 9, 12 and 18 is $2 \times 3 \times 3 \times 2 = 36$

3.5.3 Relation between HCF and LCM of two numbers

Consider any two numbers, says 12 and 16. Their HCF is 4 and LCM is 48.

- Product of HCF and LCM is $4 \times 48 = 192$
- Product of the two numbers is $12 \times 16 = 192$

We can see that; $12 \times 16 = 4 \times 48$

Again consider another pair of numbers say 15 and 25.

Their HCF is 5 and LCM is 75.

- Product of HCF and LCM is $5 \times 75 = 375$
- Product of the two numbers is $15 \times 25 = 375$

Again we can see that; $15 \times 25 = 5 \times 75$

From the above examples, we conclude that:

First Number $\times$ Second Number = HCF $\times$ LCM

i.e, Product of the two numbers = Product of their HCF and LCM

Example 2: Find the HCF and LCM of 27 and 45 find the product of HCF and LCM also find the product of the numbers. What relation do you observe between the two?

Solution: Using Factorization Method

Factors of 27 = $3 \times 3 \times 3$
Factors of 45 = $3 \times 3 \times 5$
Common factors = $3 \times 3 = 9$
Thus, the HCF of 27 and 45 is 9

Factors of 27 are $3 \times 3 \times 3 = 3^3$

Factors of 45 are $3 \times 3 \times 5 = 3^2 \times 5$

$\boxed{3^2 \times 3} \times \boxed{5}$

Thus, the LCM of 27 and 45 is $3^3 \times 5$.
$= 27 \times 5 = 135$

Product of two numbers $= 27 \times 45 = 1215$
LCM $\times$ HCF $= 135 \times 9 = 1215$
Hence, it can be observed:
Product of the two numbers = LCM $\times$ HCF

Example 3: The HCF and LCM of two numbers are 33 and 13860 respectively. If one of the numbers is 693, find the other.

Solution: We know that

First number $\times$ Second number = HCF $\times$ LCM

$693 \times$ Second number $= 33 \times 13860$

$$\text{Second number} = \frac{33 \times 13860}{693} = 660$$

Hence, the other number is 660.

EXERCISE 3.5

1. Find the LCM of given numbers by finding their common multiples.

(i) 2, 4	(ii) 5, 6	(iii) 3, 4	(iv) 7, 8
(v) 6, 9	(vi) 8, 12	(vii) 7, 14	(viii) 10, 15
(ix) 3, 6, 9	(x) 2, 6, 9	(xi) 4, 8, 12	(xii) 2, 6, 11

2. Find the LCM of the following numbers by prime factorization method.

(i)	18, 24	(ii)	16, 40	(iii)	30, 36	(iv)	28, 44
(v)	20, 32	(vi)	20, 135	(vii)	45, 75	(viii)	36, 84
(ix)	12, 18, 24	(x)	25, 35, 45	(xi)	9, 15, 21	(xii)	25, 50, 75

3. Find the LCM by using the division method.

(i)	27, 81, 54	(ii)	18, 45, 63	(iii)	35, 55,100
(iv)	210,140,315	(v)	112, 120, 150	(vi)	144, 180, 300

4. The HCF of two numbers 525 and 1155 is 105. Find their LCM.
5. The LCM of two numbers 660 and 2100 is 23100. Find their HCF.
6. The HCF and LCM of two numbers are 29 and 3045. If one of the numbers is 435, find the other.
7. The HCF of two numbers is 16 and their product is 3328. Find their LCM.

3.6 Application of HCF and LCM

The process of finding HCF and LCM has a role in our real life that can be observed in the following examples.

Example 1: Find the greatest length of scale that can measure the 18m, 24m and 42m long ropes exactly.

Solution:

To determine the length of the scale we find the HCF of given lengths.

$$\begin{array}{r|l} 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 24 \\ \hline 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

Thus,

$$18 = 2 \times 3 \times 3$$
$$24 = 2 \times 2 \times 2 \times 3$$
$$42 = 2 \times 3 \times 7$$

Hence, the HCF of 18, 24 and 42 is $2 \times 3 = 6$

Thus, the required length of the scale is 6m

Example 2: The floor of a hall is 1550cm × 1050 cm. Square shaped tiles of same size are to be fixed on it. Find

(i) The largest possible size of each tile so that all the tiles are used.

(ii) The least number of tiles required to cover the floor of hall completely.

Solution: The largest size of each tile will be the HCF of 1550 cm and 1050 cm.
Hence, the HCF of 1550 and 1050 is 50
Thus, the largest size of the square tile = 50cm × 50cm
$= 2500\text{cm}^2$

```
           1
1050 ) 1 5 5 0
       1 0 5 0      2
        5 0 0 ) 1 0 5 0
                1 0 0 0    1 0
                    5 0 ) 5 0 0
                          5 0 0
                            0
```

(ii) Area of the floor $= 1550\text{cm} \times 1050\text{cm}$
$= 1627500\text{cm}^2$

Area of the tile $= 2500\text{cm}^2$

Least number of tiles $= \dfrac{\text{Area of the floor}}{\text{Area of the tile}}$

$= \dfrac{1627500}{2500} = 651$

Thus, the least number of tiles required is 651 tiles.

Example 3: What should be the minimum length of a steel rod that can exactly be measured with 200cm, 150cm and 250cm long measures of tapes.

Solution: The length of the steel rod can be calculated by finding the LCM of 200, 150 and 250.

2	200, 150, 250
5	100, 75, 125
5	20, 15, 25
	4, 3, 5

Hence, the LCM of 200, 150 and 250 are $2\times5\times5\times4\times3\times5$
$= 3000$
Thus, the required length of the steel rod is 3000cm

Example 4: Four bells ring at intervals of 10,15,24 and 30 minutes respectively. At what time will they ring together if they start ringing simultaneously at 8 a.m?

Solution: We can tell the required time by finding the LCM of 10, 15, 24 and 30.

2	10
5	5
	1

3	15
5	5
	1

2	24
2	12
2	6
3	3
	1

2	30
3	15
5	5
	1

$$\begin{aligned}
10 &= 2 \times 5 &&= 2 \times 5\\
15 &= 3 \times 5 &&= 3 \times 5\\
24 &= 2 \times 2 \times 2 \times 3 &&= 2^3 \times 3\\
30 &= 2 \times 3 \times 5 &&= 2 \times 3 \times 5
\end{aligned}$$

$\boxed{2^3}$ $\boxed{3}$ $\boxed{5}$

Hence, the LCM of 10, 15, 24 and 30 is $2^3 \times 3 \times 5$
$= 8 \times 3 \times 5 = 120$
Thus, the required time is: 120 minutes = 2 hours
Thus, the bells will ring together at: (8 + 2) a.m. = 10 a.m.

EXERCISE 3.6

1. Find the greatest number that can exactly divide the numbers 108, 180 and 216.
2. Find the smallest number that is exactly divisible by 5, 15 and 25.
3. Find the greatest measure of a string that can measure exactly 27m, 45m and 63 m long wooden border exactly.
4. Find the shortest length of a pipe that can be measured exactly with 4m, 6m and 9m long measuring tapes respectively.
5. The paper of a note-book is 18cm by 24cm. Ali wants to cover the paper completely with square patterns of the same size. Find:
   (i) The largest possible area of each square pattern.
   (ii) The number of square patterns that Ali can draw to cover the paper completely.
6. In a morning walk, three friends step off together. Their steps measure 70cm, 76cm and 90cm respectively. At what distance from the starting point will they step off again together.
7. Two containers have 850 litres and 680 litres of milk respectively. Find the capacity of a largest container which can measure the milk in each container in exact number of times.
8. There are 416, 364 and 312 students in three classes respectively. Buses are to be hired to take these students for a school trip. Find the maximum number of students who can sit in a bus if each bus carries an equal number of students.
9. Three light houses flash their lights every 16 seconds, 24 seconds and 40 seconds respectively. If they flash together at 2p.m, at what time will they next flash together?
10. Manahil wants to prepare some handkerchiefs of same size from a piece of cloth 9m long and 1.25m wide. What will be the largest size of the handkerchiefs when no wastage is allowed.

Summary

- A number that divides a given number exactly is called a factor of the given number.
- The multiples of a number are obtained by multiplying the number with natural numbers.
- The numbers which are divisible by 2 are called even numbers and which are not divisible by 2 are called odd numbers.

- A number having exactly two factors, 1 and the number itself is called a prime number and a number having more than two factors is called a composite number.
- The number 1 is neither prime nor composite number.
- The process of writing a number into prime factors is called prime factorization.
- The largest common factor of two or more than two numbers is called highest common factor.
- The least common multiple is the smallest number which is a multiple of given two or more numbers.

Review Exercise 3

1. Write all numbers less than 40 which are

   (i) multiples of 2 (ii) multiples of 5

   (iii) multiples of 7 (iv) multiples of 9

2. Write all even, odd, prime and composite numbers less than 20.
3. Tell which of the following numbers are divisible by 2,3 and 5 without carrying division.

   (i) 6420 (ii) 7125 (iii) 5030 (iv) 4132

   (v) 11115 (vi) 20004 (vii) 45678 (viii) 32124

4. Write the prime factors of the following numbers using index notation.

   (i) 900 (ii) 1296 (iii) 7056 (iv) 39204

5. Use prime factorization method to find the HCF.

   (i) 48,72 (ii) 70,105 (iii) 33,44,77

6. Use division method to find the HCF.

   (i) 924,1045 (ii) 1505,2982 (iii) 710,1815,945

7. Use prime factorization method to find LCM.

   (i) 75,120 (ii) 234, 702 (iii) 75, 125, 350

8. Use long division method to find LCM.

   (i) 324, 1053 (ii) 385,1050,1155 (iii) 52, 56, 112, 156

9. The HCF of two numbers 2952 and 2256 is 24. Find their LCM.
10. The HCF and LCM of two numbers are 23 and 345. If one number is 115, find the other.

Objective Exercise 3

1. Answer the following questions.

   (i) What is meant by the factor of a number?

   (ii) Define the prime numbers.

   (iii) Which number has only one factor?

   (iv) How do we indicate if a number is divisible by 3?

   (v) What is meant by the prime factorization?

   (vi) Show the relation between HCF and LCM of two numbers by using a formula.

2. Fill in the blanks.

   (i) The numbers having no common factor other than 1 are called __________ numbers.

   (ii) A number having a factor other than 1 and itself is called __________ number.

   (iii) __________ is the only even prime number.

   (iv) A number is divisible by __________ if the digit at the units place is 0 or even number.

   (v) The process of writing a number into its factors is called __________ .

3. Tick (✓) the correct answer.

   (i) The factor of every number is:

   (a) 0 (b) 1 (c) 2 (d) 3

   (ii) Every number greater than 1 has at least factors:

   (a) one (b) two (c) three (d) four

   (iii) A number is divisible by 6, if it has even number at the unit place and the sum of its digits is divisible by:

   (a) 2 (b) 3 (c) 6 (d) 9

   (iv) The LCM of 2 and 3 is:

   (a) 2 (b) 3 (c) 4 (d) 6

   (v) If the LCM of two numbers 4 and 9 is 36, then its HCF will be:

   (a) 1 (b) 2 (c) 9 (d) 12

Unit 4

INTEGERS

Student Learning Outcomes

After studying this unit, students will be able to:

- Know that
  - the natural numbers 1, 2, 3, ..., are also called positive integers and the corresponding negative numbers –1, –2, –3, ..., are called negative integers,
  - '0' is an integer which is neither positive nor negative.
- Recognize integers.
- Represent integers on a number line.
- Know that on the number line any number lying
  - to the right of zero is positive,
  - to the left of zero is negative,
  - to the right of another number is greater,
  - to the left of another number is smaller.
- Know that every positive integer is greater than a negative integer.
- Know that every negative integer is less than a positive integer.
- Arrange a given list of integers in ascending and descending order.
- Define absolute or numerical value of a number as its distance from zero on the number line which is always positive.
- Arrange the absolute or numerical values of the given integers in ascending and descending order.
- Use number line to display:
  - sum of two or more given negative integers,
  - difference of two given positive integers,
  - sum of two given integers.
- Add two integers (with like signs) in the following three steps:
  a) Take absolute values of given integers, b) Add the absolute values,
  c) Give the result with the common sign,
- Add two integers (with unlike signs) in the following three steps:
  a) Take absolute values of given integers,
  b) Subtract the smaller absolute value from the larger,
  c) Give the result with the sign of the integer with the larger absolute value.
- Recognize subtraction as the inverse process of addition.
- Subtract one integer from the other by changing the sign of the integer being subtracted and adding according to the rules for addition of integers.
- Recognize that
  - the product of two integers of like signs is a positive integer.
  - the product of two integers of unlike signs is a negative integer.

  Recognize that division is the inverse process of multiplication.
  Recognize that on dividing one integer by another
  - if both the integers have like signs the quotient is positive.
  - if both the integers have unlike signs the quotient is negative.

  Know that division of an integer by '0' is not possible.

4.1 Integers

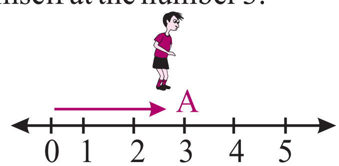
4.1.1 Positive Integers

We are familiar with the numbers of type 1, 2, 3, ... These numbers can be added, subtracted, multiplied and divided as the rules expressed in the previous chapter. We call them natural numbers. These are also called the positive integers.

4.1.2 Negative Integers

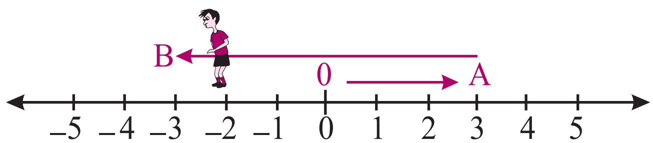
We know that all natural numbers 1, 2, 3, ... are greater than zero. Is there any number which is less than zero? We can subtract 3 from 5 such as $5-3=2$. But what is the result of $3-5$?

Let us try to solve these problems with an example. Waleed decided to play a game. He drew a line on the floor by marking some numbers after every step on the right of his starting point. He closed his eyes and moved three steps from zero to any point A. He opened his eyes and found himself at the number 3.



He again closed his eyes and returned back 5 steps from point A to any point B. But when he opened his eyes he found himself to the left of his starting point.

To find his position, he extended line on the left of his starting point and marked the same counting numbers but with a negative sign to show that he is on the left of starting point. He observed that he was at the number –2.



It means $3-5=-2$

(–2 is read as negative or minus 2)

This line is called the number line which is extending in both directions without ever ending. Here zero is known as starting point. Similarly we can get

$5-8=-3$, $3-4=-1$ and so on

These numbers $-1, -2, -3, \ldots$ are called negative integers.

We know that 1 is less than 2 and 0 is less than 1. In the same way –1 is less than 0 and –2 is less than –1.

The whole numbers 0,1,2, ... together with the negative numbers –1,–2,–3, ... are called integers.

We can write them as;

$$\ldots,-5,-4,-3,-2,-1,0,1,2,3,4,5,\ldots$$

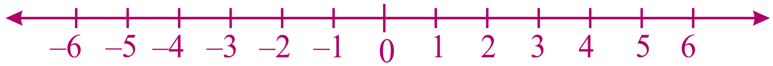
or

$$0,\pm1,\pm2,\pm3,\pm4,\ldots$$

Zero is also an integer but it is neither a positive integer nor a negative integer.

Integers are also known as directed numbers because these numbers also represent the direction, as well as the measurement.

4.2 Ordering of Integers



We can observe that on the number line, the integer that lies to the left of another integer is always smaller and the integer that lies to the right of the same integer is always greater. For example, zero lies to the left of all positive integers on the number line. So,

$0 < 1, 2, 3, \ldots$

and the integer 0 lies to the right of all negative integers.

So, $0 > -1, -2, -3, \ldots$

or we can write the above statements as: $\ldots, -3 < -2 < -1 < 0 < +1 < +2 < +3, \ldots$

Example 1: Put the appropriate sign > or < between the given pairs.

(i) 0, 1 (ii) –10, –15 (iii) –100, 10

Solution:

(i) 0, 1 [0 lies to the left of 1 on a number line]

$0 < 1$

(ii) –10, –15 [–10 lies to the right of –15 on a number line]

$-10 > -15$

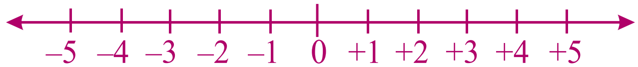
(iii) –100, 10 [–100 lies to the left of 10 on a number line]

$-100 < 10$

Example 2: Write two next integers which are greater than the integers: –5,–4,–3.

Solution: –5,–4,–3,…

Draw a number line which shows the given and the next integers.

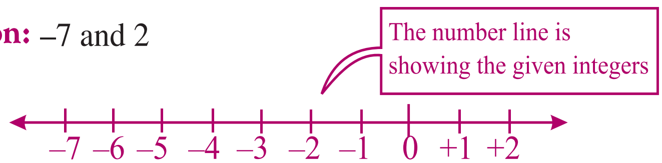


From above, we can observe that $-5 < -4 < -3 < -2 < -1$

Thus, the next two integers are –2 and –1.

Example 3: Write the integers between –7 and 2 drawing the number line.

Solution: –7 and 2

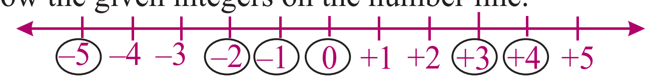


From the number line, we can observe that $-7 < -6 < -5 < -4 < -3 < -2 < -1 < 0 < 1 < 2$

Thus, the integers are –6,–5,–4,–3,–2,–1, 0, 1

Example 4: Arrange the given integers in ascending order and in descending order. 0, –5, –2, 4, –1, 3

Solution: Show the given integers on the number line.

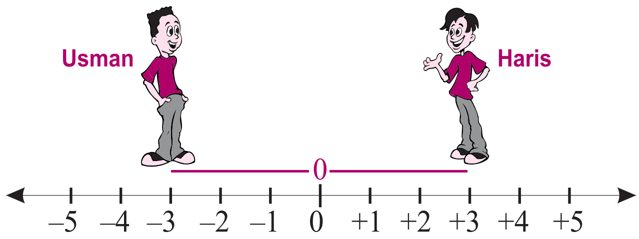


Ascending order : –5, –2, –1, 0, 3, 4 **Descending order:** 4, 3, 0, –1, –2, –5

4.3 Absolute or Numerical Value of an Integer

Numerical or absolute value of a directed number is a distance from zero to that number on the number line. We shall make it clear with an example.

Usman and Haris walked away 3 meters from the same place but in the opposite directions. Who covered more distance? We display this distance on the number line.



From the above number line we can observe that Usman and Haris are at the same distance from the starting point 0. So, we can say that the numerical or absolute value of 3 and –3 is the same because it gives us only distance but not direction.

We can write it as:

$$|+3| = 3$$
$$\text{and } |-3| = -(-3) = 3$$

Example 1: Find the numerical values of the following integers.

(i) – 4 (ii) 22 (iii) – 9

Solution:

(i) $|-4| = -(-4) = 4$ $\therefore$ Numerical value of – 4 is 4.

(ii) $|22| = 22$ $\therefore$ Numerical value of 22 is 22.

(iii) $|-9| = -(-9) = 9$ $\therefore$ Numerical value of – 9 is 9.

EXERCISE 4.1

1. Draw the number line and mark the following numbers.

   (i) –5 to 0 (ii) 0 to +5 (iii) –2 to +4 (iv) –4 to +1

2. Fill in the boxes with > or <.

   (i) 6 $\square$ 5 (ii) –6 $\square$ –5 (iii) –2 $\square$ 0

   (iv) 0 $\square$ 4 (v) 8 $\square$ –10 (vi) –9 $\square$ 1

3. Which is greater, –101 or –111?

4. Which is smaller, –99 or –199?

5. Write the integers between:

   (i) 2 and 6 (ii) –2 and 3 (iii) –6 and –1 (iv) –3 and 4

6. Give two possible integers in following cases.

   (i) 0 < (ii) < 0 (iii) > – 3

7. Write three integers smaller than 2.

8. Write four integers greater than –2.

9. Find the numerical values of each of the following.

   (i) 3 (ii) –8 (iii) 5 (iv) –9 (v) –6 (vi) –2

10. Write the integer whose numerical value is 0.

11. Arrange the given integers in ascending and descending order.

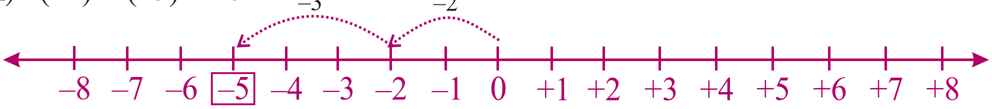
   (i) –4, 1, –2, 0 (ii) 1, –3, –4, 0 (iii) –2, –3, 3, 2

4.4 Addition of Integers

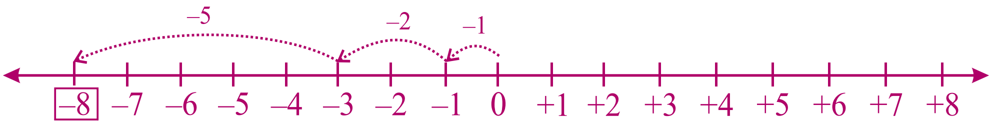
A number line can also be used to display the sum or difference of given integers as shown below.

- **Sum of two or more given integers**

(i) (–2) + (–3) = –5

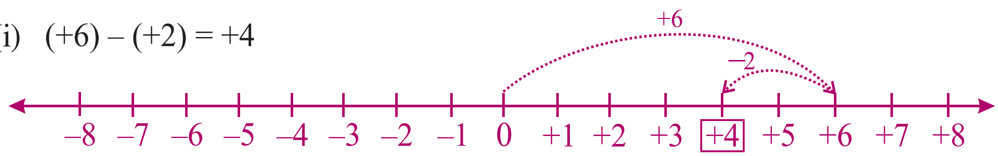


(ii) (–1) + (–2) + (–5) = –8



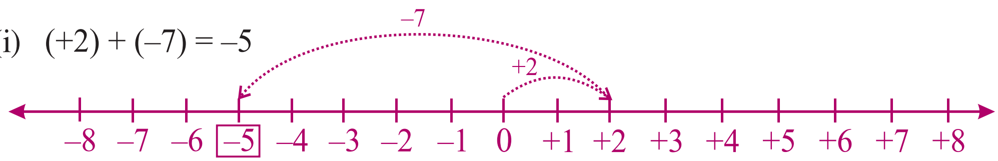
- **Difference of two given positive integers**

(i) (+6) – (+2) = +4

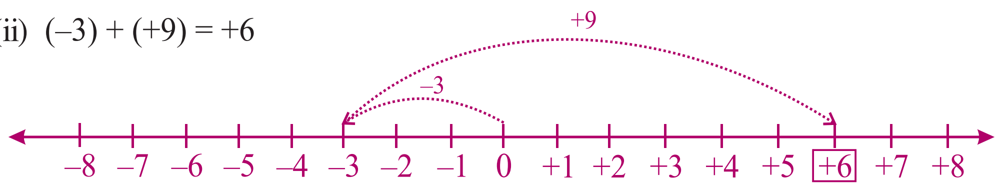


- **Sum of two integers with unlike signs**

(i) (+2) + (–7) = –5



(ii) (–3) + (+9) = +6



We are familiar with the addition methods but the addition of integers is little different from the addition of natural numbers or positive numbers. Some rules that we use in the addition of integers are given below.

Rule 1: Integers with like signs

Two integers with like signs are added in the following three steps.

(a) Take absolute values of given integers.
(b) Add the absolute values.
(c) Give the result with the common sign.

Example 1: Solve the following.

(i) (+16) + (+13) (ii) (–16) + (–13)

Solution:

(i) $(+16) + (+13)$ $\because |+16| = 16$
$= +(16 + 13)$ $|+13| = 13$
$= +29$ or 29

(ii) $(-16) + (-13)$ $\because |-16| = 16$
$= -(16+13)$ $|-13| = 13$
$= -29$

Rule 2: Integers with unlike signs

Two integers with unlike signs are added in the following three steps.

(a) Take absolute values of given integers.
(b) Subtract the smaller absolute value from the larger.
(c) Give the sign of the integer with the larger absolute value to the result.

Example 2: Add the following.

(i) (+9) + (–4) (ii) (–9) + (+4) (iii) (–16) + (+13)

Solution:

(i) $(+9) + (-4)$ $\because |+9| = 9$
$= +(9 - 4)$ $|-4| = 4$
$= +5$ or 5

(ii) $(-9) + (+4)$ $\because |-9| = 9$
$= -(9 - 4)$ $|+4| = 4$
$= -5$

(iii) $(-16) + (+13)$ $\because |-16| = 16$
$= -(16 - 13)$ $|+13| = 13$
$= -3$

Example 3: Simplify the following.

$[(-4) + (+6)] + (-9)$ $\because |+6| = 6$
$|-4| = 4$

Solution: First solve brackets.

$= [+(6 - 4)] + (-9)$ $\because |+2| = 2$
$= (+2) + (-9)$ $|-9| = 9$
$= -(9 - 2)$ $= -7$

which is the required answer.

EXERCISE 4.2

1. Use the number line to write the sum.

(i) $(-7) + (+3)$ (ii) $(-2) + (-4)$ (iii) $(+5) - (+1)$

(iv) $(+2) + (-3)$ (v) $(-1) + (-2) + (-3)$ (vi) $(-3) + (-4) + (-2)$

2. Find the sum of the following

(i) $(+5) + (+2)$ (ii) $(+9) + (+7)$ (iii) $(-4) + (-6)$

(iv) $(-8) + (-8)$ (v) $(+10) + (-2)$ (vi) $(-7) + (+6)$

(vii) $(-11) + (+7)$ (viii) $(+3) + (-9)$ (ix) $(+5) + (-8)$

(x) $(-13) + (-11)$ (xi) $(+12) + (+23)$ (xii) $(-27) + (-19)$

3. Fill in the boxes.

(i) $(+3) + (-6) = \boxed{}$ (ii) $(+7) + (+3) = \boxed{}$

(iii) $(-6) + (-9) = \boxed{}$ (iv) $(+5) + \boxed{} = (+7)$

(v) $\boxed{} + (-5) = (-16)$ (vi) $\boxed{} + (-17) = (+2)$

(vii) $(+100) + \boxed{} = (+50)$ (viii) $(-11) + \boxed{} = (-111)$

4. Solve the following.

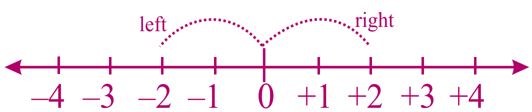
(i) $[(+2) + (+3)] + (+4)$ (ii) $[(-1) + (-1)] + (-5)$

(iii) $[(+3) + (+5)] + (-1)$ (iv) $[(-2) + (-6)] + (+4)$

(v) $(+25) + [(+25) + (+50)]$ (vi) $(-18) + [(25) + (-30)]$

4.5 Subtraction of Integers

We know that the subtraction is an opposite process of the addition. In integers we add 2 when we move 2 steps to the right side on a number line but we subtract 2 when we move 2 steps on the left side.



To subtract integers, we use the following rules.

(i) Change the sign of the integer which is being subtracted.

(ii) Add them according to the addition rules of integers.

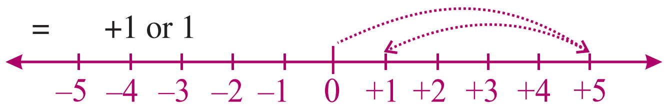
For example:

(i) $(+5) - (+4)$

$= (+5) + (-4)$ $\quad \because -(+4) = +(-4)$

$= +(5 - 4)$ $\quad \because |+5| = 5,\ |-4| = 4$

$= +1$ or 1

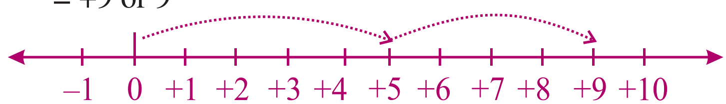


(ii) $(+5) - (-4)$

$= (+5) + (+4)$ $\quad \because -(-4) = +(+4)$

$= (5 + 4)$ $\quad \because |+5| = 5,\ |+4| = 4$

$= +9$ or 9



Example 1: Solve the following

(i) $(+19) - (+17)$ (ii) $(+23) - (-6)$ (iii) $(-13) - (+18)$ (iv) $(-18) - (-11)$

Solution: (i) $(+19) - (+17)$

$= (+19) + (-17)$

$= (19 - 17)$ $\quad \because |+19| = 19$

$= +2$ or 2 $\quad |-17| = 17$

(ii) $(+23) - (-6)$

$= (+23) + (+6)$

$= +(23 + 6)$ $\quad \because |+23| = 23$

$= +29$ or 29 $\quad |+6| = 6$

(iii) $(-13) - (+18)$

$= (-13) + (-18)$

$= -(13 + 18)$ $\quad \because |-13| = 13$

$= -31$ $\quad |-18| = 18$

(iv) $(-18) - (-11)$

$= (-18) + (+11)$

$= -(18 - 11)$ $\quad \because |-18| = 18$

$= -7$ $\quad |+11| = 11$

Example 2: Simplify the following.

(i) $(-195) - (-203)$ (ii) $[(+10) - (+6)] - (-8)$ (iii) $[(-13) - (-17)] - (+12)$

Solution: (i) $(-195) - (-203)$

$= (-195) + (+203)$

$= +(203 - 195)$ $\quad \because |-195| = 195$

$= +8$ or 8 $\quad |+203| = 203$

(ii) $[(+10) - (+6)] - (-8)$

$= [(+10) + (-6)] + (+8)$

$= [+(10 - 6)] + (+8)$ $\quad \because |+10| = 10$

$= (+4) + (+8)$ $\quad |-6| = 6$

$= +(4 + 8)$ $\quad \because |+4| = 4$

$= +12$ or 12 $\quad |+8| = 8$

(iii) $[(-13) - (-17)] - (+12)$

$= [(-13) + (+17)] + (-12)$ $\quad \because |-13| = 13$

$= [+(17 - 13)] + (-12)$ $\quad |+17| = 17$

$= (+4) + (-12)$ $\quad \because |+4| = 4$

$= -(12 - 4)$ $\quad |-12| = 12$

$= -8$

EXERCISE 4.3

1. Simplify the following.

(i)	(+4) – (+1)	(ii)	(+8) – (+5)	(iii)	(–6) – (–2)
(iv)	(–7) – (–9)	(v)	(+15) – (–4)	(vi)	(–18) – (+7)
(vii)	(+23) – (+15)	(viii)	(–42) – (–21)	(ix)	(+69) – (–21)
(x)	(+49)–(+81)	(xi)	(+102)–(–133)	(xii)	(–195)–(–165)

2. Fill in the blanks.

(i)	(+2) – (____) = (–7)	(ii)	(–8) – (____) = (–12)
(iii)	(–11) –(–13) = (____)	(iv)	(+16) – (____) = (+11)
(v)	(____) – (– 3) = (+9)	(vi)	(____) –(+13) = (– 29)

3. Simplify the following.

(i)	[(–8) – (–6)] – (–4)	(ii)	[(+ 11)– (+5)] –(+19)
(iii)	[(–13)–(–18)] – (–17)	(iv)	[(–18)–(+12)]–(–19)
(v)	[(+23)–(–9)] – (+29)	(vi)	[(+100)–(+50)] – (+25)

4. Subtract –111 from +111

5. The sum of two integers is –99. One integer is –66, find the other.

4.6 Multiplication of Integers

We know that multiplication is an operation of repeated addition or we can say that multiplication is a short method of adding the same numbers. For example:

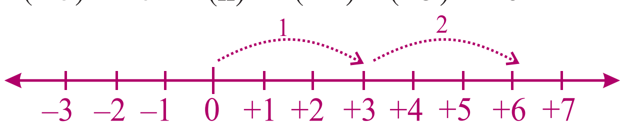
$$2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 8 \times 2 = 16$$

The rules for multiplication of integers are given below:

Rule 1: Integers with like signs

The product of two integers having same signs is always positive. For example:

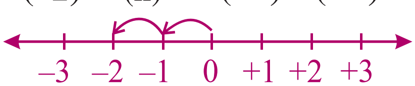
(i) $(+2) \times (+3) = +6$ (ii) $(-2) \times (-3) = +6$



Rule 2: Integers with unlike signs

The product of two integers having opposite signs is always negative. For example:

(i) $(+1) \times (-2) = (-2)$ (ii) $(-1) \times (+2) = (-2)$



Example: Find the product of the following pairs.

(i) +13, +9 (ii) –11, +11 (iii) –8, –7 (iv) +25, –6

Solution:

(i) +13, +9 (Like signs rule)

$(+13) \times (+9)$

$= +(13 \times 9) = +117$

(ii) −11, +11 (Unlike signs rule)

$(-11) \times (+11)$

$= -(11 \times 11) = -121$

(iii) −8, −7 (Like signs rule)

$(-8) \times (-7)$

$= +(8 \times 7) = +56$

(iv) +25, −6 (Unlike signs rule)

$(+25) \times (-6)$

$= -(25 \times 6) = -150$

EXERCISE 4.4

1. Fill in the boxes.

(i) $(+6) \times (-3) = \square$ (ii) $(-9) \times \square = 81$ (iii) $(-2) \times (+8) = \square$

(iv) $\square \times (+11) = 121$ (v) $\square \times (-7) = 56$ (vi) $-25 \times \square = -75$

(vii) $(-) \times (-) = \square$ (viii) $(+) \times (-) = \square$

2. Find the product of the following.

(i) +3, +4 (ii) −6, −2 (iii) +5, −5 (iv) −7, +8

(v) −9, −4 (vi) +3, −8 (vii) −10, −5 (viii) +11, −7

(ix) −9, −8 (x) +6, +12 (xi) −3, +50 (xii) −7, +7

(xiii) −4, −9 (xiv) −5, −13 (xv) +110, −8

3. Simplify each of the following.

(i) $(-1)\times(-1)\times(-1)\times(-1)$ (ii) $(+1)\times(-2)\times(+3)\times(+4)$ (iii) $[(+2)\times(+9)]\times(-4)$

(iv) $[(-18)\times(3)]\times(2)$ (v) $[(25)\times(-8)]\times(-16)$ (vi) $[(-100)\times(-15)]\times(3)$

4.7 Division of Integers

Division is a reverse process of multiplication. To make it clear consider the following statement.

$$2 \times 3 = \square \Rightarrow \frac{\square}{3} = 2$$

Try to guess the number to fill the boxes. Obviously 6 is the required number in both cases. So our problem can be solved by putting 6 in the box, i.e.,

$$2 \times 3 = \boxed{6} \Rightarrow \frac{\boxed{6}}{3} = 2$$

It means 2×3 =6 can be written as $2 = \frac{6}{3}$.

Now we discuss the rules for division.

Rule 1: Integers with like signs

When an integer is divided by another integer of same sign, the result is always positive.

(i) $(+4) \div (+2) = +\left(\frac{4}{2}\right) = (+2)$ (ii) $(-4) \div (-2) = +\left(\frac{4}{2}\right) = (+2)$

Rule 2: Integers with unlike signs

When an integer is divided by another integer of opposite sign, the result is always negative.

(i) $(+4) \div (-2) = -\left(\frac{4}{2}\right) = (-2)$ (ii) $(-4) \div (+2) = -\left(\frac{4}{2}\right) = (-2)$

4.7.1 Division of an Integer by '0' is not Possible

Division by zero is an operation for which we cannot find the answer as shown below.

$$\frac{4}{0} = \square \quad \text{or} \quad 4 = \square \times 0$$

But no value would work for $\square$ because 0 time any number is 0. So, division of an integer by '0' is not possible.

Example 1: Find the quotient of the following.

(i) $(-121) \div (-11)$ (ii) $(+169) \div (-13)$ (iii) $(-72) \div (+8)$ (iv) $(+144) \div (+16)$

Solution:

(i) $(-121) \div (-11)$

$= +(121 \div 11)$ (Like signs rule)

$= +\left(\frac{121}{11}\right) = +11$

(ii) $(+169) \div (-13)$

$= -(169 \div 13)$ (Unlike signs rule)

$= -\left(\frac{169}{13}\right) = -13$

(iii) $(-72) \div (+8)$

$= -(72 \div 8)$ (Unlike signs rule)

$= -\left(\frac{72}{8}\right) = -9$

(iv) $(+144) \div (+16)$

$= +(144 \div 16)$ (Like signs rule)

$= +\left(\frac{144}{16}\right) = +9$

EXERCISE 4.5

1. Solve.

(i) $(-42) \div (-7)$ (ii) $(+36) \div (+9)$ (iii) $(+65) \div (+5)$

(iv) $(-27) \div (-3)$ (v) $(-126) \div (+14)$ (vi) $(+34) \div (-17)$

(vii) $(+260) \div (-13)$ (viii) $(-189) \div (-21)$ (ix) $(-155) \div (+31)$

(x) $(+372) \div (+124)$

2. Fill in the following boxes.

(i) $\frac{12}{3} = \square$ (ii) $\frac{-16}{\square} = -2$ (iii) $\frac{\square}{5} = -4$

(iv) $\frac{30}{\square} = -6$ (v) $\frac{\square}{-8} = 9$ (vi) $\frac{169}{13} = \square$

(vii) $\frac{8}{2} = 2 \times \square$ (viii) $\frac{-16}{2} = 2 \times \square$ (ix) $\frac{-27}{-3} = 3 \times \square$

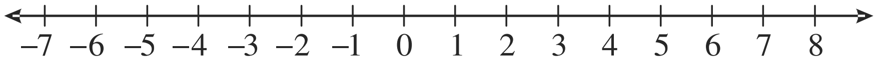
3. Find the quotient of the following.

(i) $(+252) \div (+18)$ (ii) $(-195) \div (+15)$ (iii) $(-480) \div (-120)$

(iv) $(+196) \div (-28)$ (v) $(-99) \div (+11)$ (vi) $(+2000) \div (-40)$

Summary

- The whole numbers 0,1,2,… together with the negative numbers –1,–2,–3… are called integers.
- Integers are also known as directed numbers.
- Integers can be represented by using a number line as shown below.



- The numerical value of an integer is a distance from zero to that number on the number line.
- Numerical value of an integer is also known as its absolute value.
- Addition of integers
  - (i) In case of like signs, add the absolute values of integers and common sign is written with the sum.
  - (ii) In case of unlike signs, subtract the smaller absolute value from the larger absolute value and the sign of integer of larger absolute value is written with the resulting value.
- Subtraction of integers

  Subtract one integer from the other by changing the sign of the integer being subtracted and adding according to the rules for addition of integers.
- Multiplication of integers
  - (i) The product of two integers of like signs is a positive integer.
  - (ii) The product of two integers of unlike signs is a negative integer.
- Division of integers
  - (i) If both the integers have like signs the quotient is positive.
  - (ii) If both the integers have unlike signs the quotient is negative.

Review Exercise 4

1. Draw the number line to represent the following numbers.

   (i) –4 to +3 (ii) –1 to +6 (iii) –5 to +5
2. Write the numerical values of given integers.

   (i) –6 (ii) –28 (iii) +43
3. Arrange the given integers in descending order.

   (i) –10,–1,+1,–6,0 (ii) +3,–3,+4,–4,+2,–2

4. Prove that

(i) $(+11)+(-6) = (-6)+(+11)$ (ii) $[(+1)\times(-2)]\times(-3) = (+1)\times[(-2)\times(-3)]$

(iii) $(-7)\times(-8) = (-8)\times(-7)$

(iv) $[(-24)+(-13)]+(+27) = (-24)+[(-13)+(+27)]$

5. Fill in the boxes

(i) $(+43)-(-18) = (\square)$ (ii) $(\square)-(+11) = (-31)$

(iii) $(-52)-(\square) = (-24)$ (iv) $(+123)-(+87) = (\square)$

6. Simplify the following

(i) $(-182)\div(+14)$ (ii) $(+345)\div(+23)$

(iii) $(+1221)\div(-111)$ (iv) $(-4140)\div(345)$

Objective Exercise 4

1. Answer the following questions.

(i) Define integers.

(ii) What is the absolute value of –9?

(iii) Which operation is known as the inverse process of addition?

(iv) Write two integers less than 1?

2. Fill in the blanks.

(i) In routine, we do not use sign with _________ integers.

(ii) _________ is neither a positive integer nor a negative integer.

(iii) The product of two integers of opposite signs is a _________ integer.

(iv) Integers are also known as _________ .

3. Tick (✓) the correct answer.

(i) The numerical value of –55 is:

(a) 55 (b) 5 (c) –5 (d) –55

(ii) Division of an integer is not possible by:

(a) positive integer (b) negative integer (c) zero (d) its absolute value

(iii) $(+7) + (-3) = ?$

(a) +10 (b) –4 (c) –10 (d) +4

(iv) $[(-1) + (-1)] - (-1) = ?$

(a) +1 (b) –1 (c) –2 (d) +2

(v) $(-1) \div (-1) = ?$

(a) +1 (b) –1 (c) –2 (d) 0

Unit 5

SIMPLIFICATION

Student Learning Outcomes

After studying this unit, students will be able to:

- Know that the following four kinds of brackets
  - ___ vinculum,
  - ( ) parentheses or curved brackets or round brackets,
  - { } braces or curly brackets,
  - [ ] square brackets or box brackets,

  are used to group two or more numbers together with operations.
- Know the order of preference as, ___, ( ), { } and [ ], to remove (simplify) them from an expression.
- Recognize BODMAS rule to follow the order in which the operations, to simplify mathematical expressions, are performed.
- Simplify mathematical expressions involving fractions and decimals grouped with brackets using BODMAS rule.
- Solve real life problems involving fractions and decimals.

5.1 Introduction

We have already learnt simplification of an expression involving two or more of four operations but sometimes it becomes more important to simplify some operations before the others in an expression. For this purpose, we use brackets which indicate the order of simplification. There are four types of brackets.

(i) "—" is called a bar or viniculum
(ii) "( )" is called a round or curved brackets or parentheses.
(iii) "{ }" is called a curly brackets or braces.
(iv) "[ ]" is called box brackets or square brackets.

Some Rules

There are some rules to simplify an expression containing brackets.

Rule 1: When an expression contains only addition and Subtraction: work from left to right within the brackets.

Rule 2: When an expression contains only multiplication and division: work from left to right within the brackets.

Rule 3: When an expression contains any three or all four operations: use the BODMAS rules.

5.1.1 BODMAS Rule

We know that addition, subtraction, multiplication and division are four basic operations of mathematics. To simplify an expression which contains all these operations, we use a rule, called simplification rule. According to this rule the four operations must be performed in the following order but after removing the brackets.

- BO **for** Brackets Of
- D **for** Division $\div$
- M **for** Multiplication $\times$
- A **for** Addition $+$
- S **for** Subtraction $-$

In short, this simplification rule is called the BODMAS rule.

Example 1: Simplify

$$\left[1\frac{7}{13}\times\left\{1\frac{2}{5}-\left(1\frac{5}{11}\div7\frac{1}{2}\times\overline{1\frac{1}{4}+1\frac{1}{2}}\right)\right\}\right]$$

Solution:

$$\left[1\frac{7}{13}\times\left\{1\frac{2}{5}-\left(1\frac{5}{11}\div7\frac{1}{2}\times\overline{1\frac{1}{4}+1\frac{1}{2}}\right)\right\}\right]$$

$$=\left[\frac{20}{13}\times\left\{\frac{7}{5}-\left(\frac{16}{11}\div\frac{15}{2}\times\overline{\frac{5}{4}+\frac{3}{2}}\right)\right\}\right]$$

$$=\left[\frac{20}{13}\times\left\{\frac{7}{5}-\left(\frac{16}{11}\div\frac{15}{2}\times\frac{5+6}{4}\right)\right\}\right]$$

$$=\left[\frac{20}{13}\times\left\{\frac{7}{5}-\left(\frac{16}{11}\div\frac{15}{2}\times\frac{11}{4}\right)\right\}\right]$$

$$=\left[\frac{20}{13}\times\left\{\frac{7}{5}-\left(\frac{\cancel{16}^{4}}{\cancel{11}}\times\frac{2}{15}\times\frac{\cancel{11}}{\cancel{4}}\right)\right\}\right]$$

$$=\left[\frac{20}{13}\times\left\{\frac{7}{5}-\frac{8}{15}\right\}\right]\quad=\left[\frac{20}{13}\times\left\{\frac{21-8}{15}\right\}\right]$$

$$=\left[\frac{\cancel{20}^{4}}{\cancel{13}}\times\frac{\cancel{13}}{\cancel{15}_{3}}\right]\quad=\frac{4}{3}\quad=1\frac{1}{3}$$

Hence, $1\frac{1}{3}$ is the required answer.

Example 2: Simplify the following.

$$\left[5.17+\left\{3.2\times\left(4.4\div\overline{3.3-1.1}\right)\right\}\right]$$

Solution:

$$=[5.17+\{3.2\times(4.4\div\overline{3.3-1.1})\}]$$

$$=[5.17+\{3.2\times(4.4\div2.2)\}]$$

$$=[5.17+\{3.2\times2\}]$$

$$=[5.17+6.4]$$

$$=11.57$$

Hence, 11.57 is the required answer.

EXERCISE 5.1

Simplify the following.

1. $\left[1\frac{1}{24}\div\left\{1\frac{1}{4}\times\left(1\frac{1}{10}+1\frac{2}{5}-1\frac{1}{4}\right)\right\}\right]$

2. $\frac{8}{9}+\left[\frac{5}{3}+\left\{\frac{4}{39}\times\left(\frac{3}{4}+\frac{2}{3}\times\frac{1}{2}\right)\right\}\right]$

3. $\left[1\frac{1}{4}+1\frac{1}{10}\times\left\{8\frac{1}{2}-\left(6\frac{1}{2}\times1\frac{5}{39}\right)\right\}\right]$

4. $2\frac{8}{14}\div\left[1\frac{4}{5}\times\left\{1\frac{1}{3}+\left(2\frac{1}{2}+1\frac{1}{3}-2\frac{1}{6}\right)\right\}\times1\frac{2}{3}\right]$

5. $\frac{5}{2}\times\left[\frac{7}{6}+\left\{\frac{245}{2}-\left(\frac{4}{3}\times121\div\frac{11}{8}\right)\right\}\right]$

6. $\left[2\frac{2}{3}\times\left\{2\frac{1}{4}\div\left(1\frac{1}{8}+\overline{2\frac{1}{4}-1\frac{1}{2}}\right)\right\}\right]-1\frac{2}{3}$

7. $1\frac{4}{5}\div\left[\frac{1}{25}\times\left\{1\frac{1}{4}+\left(3\frac{1}{3}\div2\frac{1}{2}\times1\frac{5}{16}\right)\right\}\right]\times\frac{1}{2}$

8. $\left[2\frac{1}{3}\div\left\{1\frac{1}{3}+\left(1\frac{1}{3}\times3\frac{1}{5}-3\frac{1}{5}\right)\right\}\times1\frac{4}{5}\right]$

9. $[2 + \{1.25 \times 3.85 \div (5.64 - \overline{2.9+1.2})\}]$

10. $[1.25 + \{12.099 \div (1.45 + 2.1 \times 1.23)\}]$

11. $2.25 \times [1.005 + \{0.5 \times (2.75 \div 2.2 \times 4.12)\}]$

12. $13.311 \div [3.251 + \{2.045 - (1.9\times\overline{1.06-1.02})\}]$

13. $0.6 \times [3.9 \times \{0.5328 \div (0.1 + 0.01 + 0.001)\}]$

14. $4.4238 \div [1.047 + \{1.111 \times (9.261 \div \overline{5.432+2.345})\}] \times 1.01$

15. $100.014 - [2.3584 \div \{0.044 \div (8.25 - \overline{5.235+1.255})\}]$

5.1.2 Word Problems

We discuss some problems of everyday life involving fractions. While solving any problem we need to concentrate on three points.

(i) What do you know? (ii) What do you want to know?
(iii) What is the proper operation?

Example 1: Iram bought $\frac{3}{4}$kg tomatoes, $\frac{1}{2}$kg potatoes, $\frac{1}{4}$ kg carrots and 2kg apples from a market. How much weight of vegetables and fruits did she carry from the market?

Solution: What do we know?

Weight of tomatoes $= \frac{3}{4}$kg Weight of potatoes $= \frac{1}{2}$kg

Weight of carrots $= \frac{1}{4}$kg Weight of apples $=$ 2kg

2. What do we want to know?
Total weight of the vegetables and fruit =?

3. What is the proper operation?
By adding weight of vegetables and fruit we can get the required weight. So,

$$\text{Total weight} = \left(\frac{3}{4}+\frac{1}{2}+\frac{1}{4}+2\right)\text{kg}$$

$$= \frac{3+2+1+8}{4} = \frac{14}{4}\text{kg} \quad = \frac{7}{2}\text{kg} = 3\frac{1}{2}\text{kg}$$

Hence, Iram carried $3\frac{1}{2}$kg weight from the market.

Example 2: A showroom owner bought 30 kg mangoes for his workers. He gave $\frac{3}{2}$ kg to each of 14 senior workers and every junior worker got $\frac{3}{7}$ kg from the remaining mangoes. Find the total number of workers of the showroom.

Solution:

Share of 14 senior workers $= \left(\frac{3}{2}\times 14\right)\text{kg} = 21\text{kg}$

Remaining mangoes $= (30 - 21)\text{ kg} = 9\text{kg}$

Every junior worker got share $= \frac{3}{7}\text{kg}$

Number of junior workers got share from 9kg mangoes $= 9 \div \frac{3}{7}$

$= 9 \times \frac{7}{3} = 21$ workers

Total workers = Senior workers + Junior workers
= 14 + 21
= 35 workers

Example 3: Sheraz purchased 24 notebooks at the rate of Rs. 12.45 per notebook, 48 lead pencils at the rate of Rs.3.25 per pencil and 25 ball point pens at the rate of Rs.4.15 per pen. Find what remaining amount he has out of Rs. 1000.

Solution:

Price of 24 notebooks	$= 24 \times 12.45$	= Rs.298.80	
Price of 48 lead pencils	$= 48 \times 3.25$	= Rs.156	
Price of 25 ball point pens	$= 25 \times 4.15$	= Rs.103.75	
Total purchase	= Rs.298.80 + Rs.156 + Rs.103.75		= Rs.558.55
Remaining amount	= Total amount – Total purchase		
	= 1000 – 558.55	= Rs.441.45	

EXERCISE 5.2

1. Three families live together in a house. The daily use of milk of one family is $5\frac{1}{2}$ litres and the other two families use $1\frac{1}{6}$ litres and $2\frac{1}{3}$ litres respectively. How much milk would a milkman supply them?

2. Nosheen bought 12 metres cloth from the market. She used half of cloth for her suit and $\frac{2}{3}$ rd of remaining for her daughter's suit. How much cloth was left with her?

3. Ahmed required $18\frac{1}{2}$ feet long cable wire for a connection. He joined two lengths of wires of $9\frac{3}{4}$ feet and $11\frac{1}{6}$ feet. Now how much length does he have more than the required length?

4. Saleem's salary is Rs. 12000. He gave $\frac{1}{12}$th of his salary as alms, half of the remaining for house expenditures and $\frac{2}{5}$ th of the remaining as debt that was due upon him. What is the remaining salary with him?

5. A person is walking with the speed of $1\frac{1}{8}$ km per hour. How much time does he require to reach a goal at the distance of $5\frac{1}{16}$ km?

6. The rate of a piece of gift paper is Rs.0.40 per paper. How many pieces of paper can we purchase for Rs.78.40.

7. The price of a book is Rs.650. Two friends have Rs.325 and Rs.296 respectively. Find how much more money the two friends need to buy that book?

8. Baber is a paying guest in a house, where he shares all utility bills equally with the landlord. What amount will Baber pay if the electricity bill is Rs.1240.50, sui gas bill is Rs.435.60 and water bill is Rs.278.90?

9. The price of a chemical of 16kg weight is Rs.1429.60. What is the price of 11.4kg chemical?

10. Sadaf bought 2.25kg beef at the rate of Rs.160 per kg, 0.75 kg mutton at the rate of Rs.350 per kg and 2.35kg chicken at the rate of Rs.170 per kg. What amount is she left without Rs.1500?

Summary

- Brackets indicate us the order of solving of the expression.
- Brackets are solved in the following order.

1. $\overline{\quad\quad}$	Bar or viniculum	2. ( )	Parenthesis
3. { }	Braces	4. [ ]	Square brackets

- Simplification rule is also called BODMAS rule. Which means to perform the four operations as?

(i)	BO	for	Brackets Of	(ii)	D	for	Division
(iii)	M	for	Multiplication	(iv)	A	for	Addition
(v)	S	for	Subtraction				

- While solving a problem we concentrate on following points.
  (i) What do you know? (ii) What do you want to know?
  (iii) What is the proper operation?

Review Exercise 5

Simplify:

1. $\left[1\frac{3}{8}-\left\{\frac{2}{3}+\frac{1}{2}\left(\frac{3}{4}\div\overline{\frac{5}{7}\times1\frac{1}{20}}\right)\right\}\right]$

2. $1\frac{1}{2}\div\left[5\frac{2}{5}-\left\{2\frac{3}{5}+\left(2\frac{1}{12}\div\overline{\frac{1}{2}+\frac{1}{3}}\right)\right\}\right]$

3. $4\frac{2}{3} \div \left[3\frac{8}{9} \times \left\{1\frac{3}{4} - \left(3\frac{1}{2} \div \overline{7\frac{1}{4} + 1\frac{1}{2}}\right)\right\}\right]$ 4. $\left[0.5 \times \left\{4.25 - \left(5.1 \div \overline{2.35 + 1.05}\right)\right\}\right]$

5. $\left[2.95 + \left\{3.02 \times \left(6.125 \div \overline{5.196 - 2.746}\right)\right\}\right]$

6. $11.34 \times \left[3.42 + \left\{11.075 - \left(3.045 + 2.064 \div 1.032\right)\right\}\right]$

7. What was the total rainfall in a week when it rained $1\frac{1}{2}$ cm on Thursday $\frac{2}{5}$ cm on Friday and $\frac{3}{10}$ cm on Sunday, the rest of week was dry?

8. From a $4\frac{3}{4}$ m long tin sheet. Ahmad cut two lengths, one was $2\frac{1}{2}$ m long and other was $1\frac{1}{3}$ m long. How much of the sheet was left?

Objective Exercise 5

1. Answer the following questions.
   (i) Write the order in which brackets are solved.
   (ii) What is BODMAS rule?
   (iii) What are three points needed to concentrate while solving a word problem?
   (iv) What is the other name of square brackets?

2. Fill in the blanks.
   (i) In short the simplification rule is called the __________.
   (ii) Addition, subtraction, multiplication and division are the four _________ of mathematics.
   (iii) _________ is called a curly bracket or braces.
   (iv) "( )" is called a round bracket or _____________.
   (v) "—" is called a bar or ___________.

3. Tick (✓) the correct answer.
   (i) According to the BODMAS rule, first basic operation is performed:
   (a) addition (b) division (c) subtraction (d) multiplication
   (ii) The bracket vinculum is denoted by:
   (a) ( ) (b) { } (c) [ ] (d) ——
   (iii) [ ] is called:
   (a) perentheses (b) braces (c) vinculum (d) box brackets
   (iv) After simplifying {1 + (2 +4 ÷ 2 × 1 – 3)}, we get:
   (a) 1 (b) 2 (c) 3 (d) –1
   (v) After simplify $\left[1 \div \left\{2 \times \left\{5 - \left(1 + \overline{6 \div 2}\right)\right\}\right\}\right]$, we got:
   (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $\frac{1}{4}$

Unit 6

RATIO AND PROPORTION

Student Learning Outcomes

After studying this unit, students will be able to:

- Define ratio as a relation which one quantity bears to another quantity of the same kind with regard to their magnitudes.
- Know that of the two quantities forming a ratio, the first one is called antecedent and the second one consequent.
- Know that a ratio has no units.
- Calculate ratio of two numbers.
- Reduce given ratio into lowest (equivalent) form.
- Describe the relationship between ratio and fraction.
- Know that an equality of two ratios constitutes a proportion, e.g., $a : b :: c : d$, where a, d are known as extremes and b, c are called the means.
- Find proportion (direct and inverse).
- Solve real life problems involving direct and inverse proportion.

6.1 Ratio

The numerical comparison between the two quantities of the same kind is called ratio.

We make it clear with an example: Azeem and Nabeel bought a clock for Rs.60. Azeem paid Rs.40 and Nabeel paid Rs.20. Now we can compare the amounts paid by Azeem and Nabeel to find the relation between the two amounts. This comparison can be done by the following two ways:

(i) By finding their difference.

Rs.40 – Rs.20 = Rs.20

The difference of Rs. 20 does not state the comparison with the other amount. Therefore, it is not the proper way to show the comparison between the two quantities.

(ii) By writing them into a fraction.

$$\frac{\text{Amount paid by Nabeel}}{\text{Amount paid by Azeem}} = \frac{\cancel{20}^{1}}{\cancel{40}^{2}} = \frac{1}{2}$$

This fraction is showing that for every 1 rupee of Nabeel, Azeem paid 2 rupees. This fraction shows the clear comparison between the two amounts. Therefore, it is a good way to show the comparison between the two amounts. It can be written by putting a colon (:) between the two quantities.

Nabeel Azeem
1 : 2

And read as "1 is to 2" but it cannot be read as "2 is to 1" because by changing the order of elements of a ratio, the value is also changed. Therefore, 1:2 and 2:1 are two different ratios. In general, we can write it as $a : b$ is not equal to $b{:}a$. A ratio can also be written for more than two quantities, i.e., 2:3:4 or $a : b : c$, etc.

Antecedent and Consequent

In a ratio the first element is called antecedent and the second element is called consequent.

Antecedent → 1 : 2 ← Consequent

Example1 : Write the following quantities in the form of $a : b : c$

(i) 500 grams, 800 grams and 1kg. (ii) 1 week, 2weeks and 9 days.

Solution:

(i) 500grams, 800grams and 1kg

500 : 800 : 1000 1kg = 1000grams.

5 : 8 : 10

(ii) 1 week, 2 weeks and 9 days.

7 : 14 : 9 1week = 7days.

Reduced form of a ratio

We have noticed that a ratio is another form of a common fraction. Therefore, we can simplify a ratio, as well as, a common fraction, by dividing them by their HCF. For instance, consider the students of a class, in which 25 are girls and 30 are boys. To find the ratio of girls is to boys, we can write it as.

Girls Boys

25 : 30

(To reduce it, divide by their HCF.)

Girls Boys

$25 \div 5 : 30 \div 5$

$5 : 6$

In this example, we can observe that the ratio 5 : 6 has no common factor and both elements are natural numbers. This is called the reduced form or the lowest form of a ratio.

When the two elements of a ratio are fractions, then first multiply them by the LCM of their denominators to change them into the integers. For example,

$$\frac{1}{2} : \frac{2}{3} = \frac{1}{2} \times 6 : \frac{2}{3} \times 6 = 3 : 4$$

Equivalent Ratios

We know that a ratio is another form of a common fraction. Therefore, the same rule for finding the equivalent fractions can be used to get equivalent ratios.

Rule: When both the elements of a ratio are multiplied or divided by same number, the value of a ratio is not changed. For example, we want to find some equivalent ratios of any ratio 1:2. We can do it as:

$1:2,\ 1\times2:2\times2 = 2:4,\ 1\times3:2\times3 = 3:6,\ 1\times4:2\times4 = 4:8$ and so on.

$1:2 = 2:4 = 3:6 = 4:8 = \ldots$

Similarly, we can prove that $4:8 = 3:6 = 2:4 = 1:2$

Relationship between ratio and fraction

Actually, ratio is the simplest form of a common fraction, in which the numerator denotes antecedent and the denominator denotes consequent.

Numerator → $\frac{1}{2} = 1 : 2$ ← Antecedent

Denominator ← Consequent

A ratio has no unit. It is just a number which indicates how many times one quantity is greater than the other.

Example 2: Find the reduced form of the following ratios.

(i) 16 : 20 (ii) $\frac{2}{3} : \frac{4}{7}$

Solution:

(i) 16 : 20

Divide by HCF 4.

$16\div4 : 20\div4$

$= 4 : 5$

(ii) $\frac{2}{3} : \frac{4}{7}$

Multiply by LCM 21 of 3 and 7.

$\frac{2}{\cancel{3}} \times \cancel{21}^{7} : \frac{4}{\cancel{7}} \times \cancel{21}^{3}$

$= 14 : 12$

Divide by HCF 2.

$14\div2 : 12\div2 = 7 : 6$

Example 3: Simplify the following.

$$\frac{1}{3}:\frac{1}{2}:\frac{5}{6}$$

Solution: $\frac{1}{3}:\frac{1}{2}:\frac{5}{6}$

Multiply by LCM 6 of 3,2 and 6.

$$\frac{1}{\cancel{3}}\times \cancel{6}^{2} : \frac{1}{\cancel{2}}\times \cancel{6}^{3} : \frac{5}{\cancel{6}}\times \cancel{6}^{1} = 2 : 3 : 5$$

EXERCISE 6.1

1. Write each of the following into ratio form.

(i) $\frac{3}{4}$ (ii) $\frac{2}{7}$ (iii) $\frac{9}{11}$ (iv) $\frac{1}{13}$ (v) $\frac{5}{6}$

(vi) $\frac{8}{13}$ (vii) $\frac{14}{23}$ (viii) $\frac{10}{99}$ (ix) $\frac{a}{b}$ (x) $\frac{x}{y}$

2. Write each of the following into fraction form.

(i) 2:3 (ii) 7:4 (iii) 19:20 (iv) 99:100

(v) 1:10 (vi) 4.1:5.2 (vii) a:b (viii) x:y

3. Simplify the following ratios.

(i) 3 : 9 (ii) 25 : 40 (iii) $\frac{1}{4}:\frac{1}{6}$ (iv) $\frac{2}{3}:\frac{1}{9}$

(v) $1:\frac{1}{7}$ (vi) $5:\frac{2}{3}$ (vii) 1.3:3.9 (viii) .02:0.4

(ix) $\frac{1}{4}:\frac{1}{6}:\frac{1}{8}$ (x) 75: 100: 125 (xi) 0.2 : 0.4 : 0.6 (xii) $\frac{1}{10}:\frac{1}{100}:\frac{1}{1000}$

4. Write each of the following quantities into ratios and reduce into the simplest form.

(i) Rs.100 and Rs.250 (ii) 2kg and 800grams (iii) 1m and 500cm

(iv) 1 year and 240 days. (v) 1day, 1week and 15days

5. Simplify:

(i) 12 is to 120 (ii) 25 is to 50 (iii) 80 is to 100

(iv) 72 is to 48 (v) 4000 is to 40 (vi) $\frac{1}{99}$ is to $\frac{2}{33}$

6.2 Proportion

We have learnt in equivalent ratios that:

$$1:2 = 2:4$$

or $$2:4 = 3:6$$

and so on.

"This relation of equality of two ratios is called proportion".

We make it clear with the following example:

Khalid bought two caps at the rate of Rs.25 per cap. How much did Khalid pay for two caps?

The cost of 1 cap = Rs.25
The cost of 2 caps = Rs.25×2
= Rs.50

We can write it as:

Caps		Cost
1:2	=	25:50

From the above, two equivalent ratios are indicating the relation between two different quantities of caps and their prices. We can say that the two ratios are proportional to each other, which are denoted by a symbol :: and can be written as 1 : 2 :: 25 : 50

(1 is to 2 is proportional to 25 is to 50)

In a proportion, the second and the third elements are called "means of a proportion" and the first and the fourth elements are called "extremes of a proportion" that can be shown from the following figure.

Means

1 : 2 :: 25 : 50

Extremes

To verify the equality of the given two ratios, we use the following formula.

Product of means = Product of extremes

$$2 \times 25 = 1 \times 50$$
$$50 = 50$$

When Second and third elements of a proportion have same value such as; $a : b :: b : c$ Then b is called the "mean proportion" and such proportion is called the continued proportion.

Consider the following example.

(i) Suppose that 2:5 and 6:15 are any two ratios. To verify their equality we can write them in a proportional form such as;

$$2:5 = 6:15$$
$$2\times15 = 6\times5$$
$$30 = 30$$

Therefore, above two ratios are in proportion.

$$2 : 5 :: 6 : 15$$

Example 1: Find the fourth proportional of 1, 3 and 6.

Solution:

Suppose that the fourth proportional is x. Then we can write them as:

$$1 : 3 = 6 : x$$
$$1\times x = 6 \times 3$$
$$x = 18$$

Hence, 18 is the fourth proportional of 1,3 and 6.

Example 2: Find the mean proportional of 1 and 9 when it is a continued proportion.

Solution:

Suppose that p is the mean proportional of 1 and 9 then

$$1 : p = p : 9$$
$$p \times p = 1 \times 9$$
$$p^2 = 9$$
$$p = 3$$

Hence, 3 is the mean proportional of 1 and 9.

Example 3: Which ratio is greater, 4:5 or 7:10?

Solution:

We can write them as:

$$4:5 = 7:10$$
$$4 \times 10 = 7 \times 5$$

But $40 > 35$

So, $4:5 > 7:10$ $\therefore$ 4:5 is greater than 7:10

6.2.1 Direct Proportion

Direct proportion is a relation in which one quantity increases or decreases in a same proportion by increasing or decreasing the other quantity.

The price of one chewing-gum is Rs.5. What is the price of two, three and more chewing-gums? We can find the prices of different quantities at the same rate that can be shown by a table given below.

Chewing gums	1	2	3	4	5
Cost (Rs.)	5	10	15	20	25

From above table, we can observe that the price is increasing in the same proportion as the quantity is increasing. Then it is said that the price is directly proportional to the quantity and such proportion is called the direct proportion.

Example 4: A washerman irons 2 shirts in 10 minutes. How many shirts can he iron in one hour? (Assume that every 2 shirts require an equal time).

Solution: (Time is directly proportional to the quantity.)

We can write above situation as:

Shirts	Time
2 ↓	↓ 10
x	60

1hour = 60 minutes

$$\frac{2}{x} = \frac{10}{60}$$
$$\frac{2}{x} = \frac{1}{6}$$
$$1 \times x = 2 \times 6 \quad \text{(By cross multiplication)}$$
$$x = 12$$

Hence, a washerman can iron 12 shirts in one hour.

6.2.2 Inverse Proportion

Inverse proportion is a relation in which one quantity increases by decreasing the other quantity and same quantity decreases by increasing the other quantity.

Saleem's factory is 100 km away from his house. He can travel this distance in different times by changing the speed of his car.

Following table is showing different speeds and time of a car.

Speed (km/hr)	40	50	60	80	100
Time (minutes)	150	120	100	75	60

From the above table, we can observe that by increasing the speed of a car, the travelling time is decreased. The time is inversely proportional to the speed and such proportion is called an inverse proportion.

Example 5: A project can be completed by 150 workers in 40 days. But project manager brought 30 more workers after 16 days. In how many days will the remaining work be finished?

Solution: (Number of workers is inversely proportional to the working days.)

Number of workers = 150 workers

150 workers can finish work in = 40 days.

Remaining days = 40 days – 16days = 24 days.

Total workers = 150 + 30 = 180 workers.

Suppose that 180 workers will finish work in x days.

Workers	Days
150 ↓	↑ 24
180	x

$$\frac{150}{180} = \frac{x}{24}$$

$$x \times 180 = 150 \times 24 \quad \text{(Cross multiplication)}$$

$$x = \frac{\cancel{150}^{5} \times \cancel{24}^{4}}{\cancel{180}_{6}} = 20 \text{ days.}$$

Hence, 180 workers will finish work in 20 days.

EXERCISE 6.2

1. Find the value of P in each of the following.

   (i) $\frac{2}{5} = \frac{P}{20}$ (ii) $\frac{P}{5} = \frac{3}{10}$ (iii) $\frac{0.1}{0.4} = \frac{6}{P}$

2. Find the value of x in each of the following proportions.

   (i) $2 : 7 :: x : 49$ (ii) $8 : 12 :: 6 : x$

   (iii) $1.2 : 3.6 :: x : 3$ (iv) $x : 2 :: 150 : 100$

3. 5:9 is a ratio, if we increase first element of the ratio up to 40, what will be the second element?
4. What is the fourth proportional of 1,3 and 4?
5. Find mean proportional of 4 and 9.
6. If 150 shirts can be stitched on 6 sewing machines in a day, how many machines are required to stitch 225 shirts in a day?
7. If 7 buffaloes give 56 litres milk, how much milk can we get from 12 buffaloes?
8. A farmer has 8 days' food for 33 cows. He bought 11 more cows. For how many days will the food be enough?
9. If 40 workers do a work in 35 days, in how many days will the same work be done by increasing 10 more workers?
10. Raheem paid his servant Rs.750 for 1 week and 3 days. What amount will he pay him for a month of 30 days?
11. A machine starts working in 45 minutes at the temperature of 60 °C. How much time is required to work it at the temperature of 75 °C?
12. 72 persons have enough food for 7 days. But after 1 day they decided to finish the food in 3 remaining days. For it they invited more persons. How many persons did they invite?

Summary

- The numerical comparison between two quantities of same kind is called ratio.
- To find the ratio of two quantities, their units must be the same. A ratio is another form of a common fraction.
- When both the elements of a ratio are multiplied or divided by a number, the value of ratio remains unchanged.
- The relation of equality of two ratios is called proportion.
- In a proportion, the 2nd and 3rd element are called means of a proportion and 1st and 4th elements are called extremes of a proportion.
- One ratio is proportional to the other ratio if and only if the product of extremes is equal to the product of means.
- Direct proportion is a relation in which one quantity increases or decreases in same proportion by increasing or decreasing the other quantity.
- Inverse proportion is a relation in which one quantity increases by decreasing the other quantity and vice versa.

Review Exercise 6

1. Write the following ratios into simplest form.

(i)	Rs. 105 and Rs. 150	(ii)	35m and 119m
(iii)	0.76m and 1.9m	(iv)	26 litres and 39litres

2. Out of 150 eggs in a basket, 25 eggs were found rotten. Find the ratio of:
   (i) rotten eggs to the good eggs. (ii) rotten eggs to the total eggs.
   (iii) good eggs to the total eggs.
3. Out of 75 passengers in a bus, 35 are males, 30 are females and remaining are children. Find the ratio of:
   (i) male passengers to the total passengers.
   (ii) female passengers to the male passengers.
   (iii) children to the total passengers.
4. Ali, Usman and Waleed distribute an amount in the ratio of 2:5:3. Find the amount of Usman and Waleed if Ali gets Rs. 170. Also find the total amount.
5. Aliha takes 200 steps for walking a distance of 160m. Find the distance covered by her in 350 steps.
6. If a car needs 9 litres of petrol for a journey of 162km. Find how many litres of petrol is required for 306 km.
7. An army camp of 200 men has enough food for 60 days. How long will the food last if the number of men in the camp is reduced to 160?
8. 45 goats can graze a field in 13 days. How many goats will graze the same field in a day?

Objective Exercise 6

1. Answer the following questions.
   (i) What is meant by the ratio? (ii) Define the proportion.
   (iii) What is meant by the extremes of a proportion?
2. Fill in the blanks.
   (i) The simplest form of a _________ is the same as the lowest form of a fraction.
   (ii) The second and third elements of a proportion are called _________ of a proportion.
   (iii) _________ proportion is a relation in which one quantity increases or decreases in a same proportion by increasing or decreasing the other quantity.
3. Tick (✓) the correct answer.
   (i) A ratio is written by putting:
   (a) : (b) , (c) ; (d) ::
   (ii) $a : b = c : d$, if and only if:
   (a) $a \times b = c \times d$ (b) $a \times c = b \times d$ (c) $b \times c = a \times d$ (d) $c \times d = a \times b$
   (iii) The reduced form of $\frac{1}{4}:\frac{1}{2}$ is:
   (a) 2 : 4 (b) 4 : 2 (c) 2 : 1 (d) 1 : 2
   (iv) 10 : 15 is an equivalent ratio of:
   (a) 15 : 10 (b) 2 : 3 (c) 2 : 5 (d) 3 : 2
   (v) The relation of equality of two ratios is called:
   (a) cross multiplication (b) proportion (c) equivalent ratio

Unit 7

FINANCIAL ARITHMETIC

Student Learning Outcomes

After studying this unit, students will be able to:

- Recognize percentage as a fraction with denominator of 100.
- Convert a percentage to a fraction by expressing it as a fraction with denominator 100 and then simplify.
- Convert a fraction to a percentage by multiplying it with 100%.
- Convert a percentage to a decimal by expressing it as a fraction with denominator 100 and then as a decimal.
- Convert a decimal to percentage by expressing it as a fraction with denominator 100 then as a percentage.
- Solve real life problems involving percentage.
- Define
  - selling price and cost price,
  - profit, loss and discount,
  - profit percentage and loss percentage.
- Solve real life problems involving profit, loss and discount.

7.1 Percentage

Any ratio with a second term of 100 or any fraction with 100 as a denominator is called a percent, e.g. If Zain earns a hundred rupees and gives two rupees out of them to the government as a tax. We can say that he pays 2 percent as a tax which can be represented as 2% as shown below.

$$\text{Zain paid tax} = \text{Rs. } 2 \text{ out of Rs. } 100$$
$$= \frac{2}{100} = 2\% \text{ or 2 percent of the whole}$$

If Zain earns 350 rupees and he has to pay the tax at the same rate (2%) then how much will he pay? We can calculate the amount of the tax by multiplying 2% with 350 rupees. Here result is called the percentage.

7.1.1 Some Basic concepts of Percentage

(i) The fraction with denominator 100 is called percentage. For example; $\frac{35}{100}$ can be written as 35 percent.

(ii) A fraction $\frac{35}{100}$ means 35 parts out of 100 parts. Therefore, percent means per hundred or out of a hundred.

(iii) We use symbol "%" instead of writing the word percent. So, we can write 35 percent as 35%.

(iv) To change a fraction into percentage, simply multiply the fraction by 100%. For example,

$$\frac{4}{25} = \frac{4}{25} \times 1 = \frac{4}{25} \times \frac{100}{100} = \frac{\cancel{400}^{16}}{\cancel{25}} \times \frac{1}{100} = 16\%$$

(v) To change a percentage to a decimal. First, convert it into a fraction, then change it in the decimal. For example,

$$35\% = \frac{35}{100} = 0.35$$

Example 1: Express the following percentages in fractions and decimals.

(i) 4.5% (ii) 420% (iii) 350%

Solution:

(i) 4.5%

Fraction ; $4.5\% = \frac{4.5}{100} = \frac{\cancel{45}^{9}}{\cancel{1000}_{200}} = \frac{9}{200}$

Decimal ; $4.5\% = \frac{4.5}{100} = 0.045$

(ii) 420%

Fraction ; $420\% = \frac{\cancel{420}^{21}}{\cancel{100}_{5}} = \frac{21}{5} = 4\frac{1}{5}$

Decimal ; $420\% = \frac{420}{100} = 4.2$

(iii) 350%

Fraction ; $350\% = \frac{\cancel{350}^{7}}{\cancel{100}_{2}} = \frac{7}{2} = 3\frac{1}{2}$

Decimal ; $350\% = \frac{350}{100} = 3.5$

Example 2: Convert the following into percentages.

(i) $\frac{2}{25}$ (ii) 0.75 (iii) 3.5 (iv) $\frac{3}{5}$

Solution:

(i) $\frac{2}{25}$

Percentage ; $\frac{2}{25} = \frac{2}{\cancel{25}} \times \overset{4}{\cancel{100}}\,\% = 8\%$

(iii) 3.5

Percentage ; $3.5 = \frac{35}{\cancel{10}} \times \overset{10}{\cancel{100}}\,\% = 350\%$

(ii) 0.75

Percentage ; $0.75 = \frac{75}{\cancel{100}} \times \cancel{100}\,\% = 75\%$

(iv) $\frac{3}{5}$

Percentage ; $\frac{3}{5} = \frac{3}{\cancel{5}} \times \overset{20}{\cancel{100}}\,\% = 60\%$

Fraction, Decimal, Ratio and Percentage

We can describe a situation by using fraction, decimal, ratio and percentage. For example, from the figure (a), we can describe 3 shaded parts out of 9 parts as:

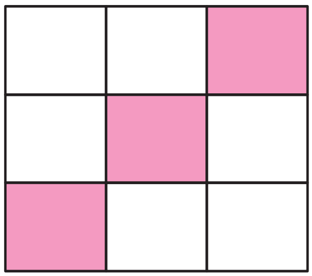


Figure (a)

(i) Fraction: $\frac{\overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}}} = \frac{1}{3}$

(ii) Decimal: $\frac{3}{9} = 0.333$

(iii) Ratio: $3 : 9 = 1 : 3$

(iv) Percentage: $\frac{3}{9} \times 100\% = 33.33\%$

Example 3: Express the following situations in fractions, decimals, ratios and percentages.

(a) 25 students out of 100 students.

(b) 7 workers out of 40 workers.

(c) 16 marks out of 25 marks.

(d) 3 positions out of 80 positions.

Solution:

(a) 25 students out of 100 students.

(i) Fraction : $\frac{\cancel{25}}{\underset{4}{\cancel{100}}} = \frac{1}{4}$

(ii) Decimal : $\frac{25}{100} = 0.25$

(iii) Ratio : $\cancel{25} : \overset{4}{\cancel{100}} = 1:4$

(iv) Percentage : $\frac{25}{\cancel{100}} \times \cancel{100}\,\% = 25\%$

(b) 7 workers out of 40 workers.

(i) Fraction : $\frac{7}{40}$

(ii) Decimal : $\frac{7}{40} = 0.175$

(iii) Ratio : $7 : 40$

(iv) Percentage : $\frac{7}{40} \times 100\% = 17.5\%$

(c) 16 marks out of 25 marks.

(i) Fraction : $\frac{16}{25}$

(ii) Decimal : $\frac{16}{25} = 0.64$

(iii) Ratio : 16 : 25

(iv) Percentage : $\frac{16}{\cancel{25}} \times \overset{4}{\cancel{100}}\% = 64\%$

(d) 3 positions out of 80 positions.

(i) Fraction : $\frac{3}{80}$

(ii) Decimal : $\frac{3}{80} = 0.0375$

(iii) Ratio : 3 : 80

(iv) Percentage : $\frac{3}{80} \times 100\% = 3.75\%$

Using Percentage

In the following, we describe how to calculate the required quantity, if one quantity and its percentage is given.

We shall explain this method for using a percentage by some examples which are given below.

Example 4: Find the following quantities by using percentage.

(i) 15% of 40 (ii) 40% of 10 (iii) 32% of 300
(iv) 125% of 20 (v) 1% of 10,000

Solution:

(i) 15% of 40

$= \frac{15}{100}$ of 40 $\quad \because \quad 15\% = \frac{15}{100}$

$= \frac{\overset{3}{\cancel{15}}}{\underset{20}{\cancel{100}}} \times \overset{2}{\cancel{40}} = 6$

(ii) 40% of 10

$= \frac{40}{100}$ of 10 $\quad \because \quad 40\ \% = \frac{40}{100}$

$= \frac{\overset{4}{\cancel{40}}}{\underset{10}{\cancel{100}}} \times \cancel{10} = 4$

(iii) 32% of 300

$= \frac{32}{100}$ of 300 $\quad \because \quad 32\ \% = \frac{32}{100}$

$= \frac{32}{\cancel{100}} \times \overset{3}{\cancel{300}} = 96$

(iv) 125% of 20

$= \frac{125}{100}$ of 20 $\quad \because \quad 125\% = \frac{125}{100}$

$= \frac{\overset{25}{\cancel{125}}}{\underset{5}{\cancel{100}}} \times \cancel{20} = 25$

(v) 1% of 10,000

$= \frac{1}{100}$ of 10,000 $\quad \because \quad 1\% = \frac{1}{100}$

$= \frac{1}{\cancel{100}} \times \overset{100}{\cancel{10{,}000}} = 100$

EXERCISE 7.1

(1) Express the following in fractions and decimals.

(i) 45%	(ii) 6%	(iii) 56%	(iv) 96%
(v) 18%	(vi) 48%	(vii) 78%	(viii) 89%
(ix) 68%	(x) 15%	(xi) 350%	(xii) 160%

(2) Convert the following into percentages.

(i) $\frac{1}{2}$ (ii) 0.25 (iii) $\frac{7}{2}$ (iv) $\frac{1}{8}$ (v) $\frac{3}{10}$ (vi) $\frac{9}{20}$ (vii) 0.59 (viii) 3.8

(3) Use fraction, decimal, ratio and percentage to express the following situations.

(i) 50 marks out of 100 marks	(ii) 90 metres out of 150 metres
(iii) 48 minutes out of 1 hour	(iv) 8 months out of 1 year.
(v) 6 eggs out of 2 dozen eggs	(vi) 510 marks out of 850 marks
(vii) 700 grams out of 2 kg	(viii) 42 students out of 75 students
(ix) Rs.900 out of Rs.4500	(x) Rs.245 out of Rs.9800
(xi) 1.5 litres out of 90 litres	(xii) 125 ml out of 1 litre

(4) Find the following percentages.

(i) 8% of 50	(ii) 64% of 25	(iii) 75% of 4
(iv) 3.5% of 1000	(v) 50% of 180	(vi) 90% of 190
(vii) 65% of 60	(viii) 18% of 1400	(ix) 18.5% of 2000
(x) 9.5% of 3000		

Word Problem

Example 1: Zara got 80% of 75 marks in a written examination of English. How many marks did she get?

Solution: Total marks = 75 marks

Zara got marks = 80% of 75 marks $= \frac{80}{100} \times 75 = 60$ marks

Here Zara got 60 marks out of 75 marks.

Example 2: Sajid paid $\frac{3}{8}$ of his salary as a house rent. Find percentage of house rent and of remaining amount.

Solution: House rent $= \frac{3}{8}$ of salary

Percentage of house rent $= \frac{3}{8} \times 100\% = 37.5\ \%$

Percentage of remaining amount $= 100\% - 37.5\% = 62.5\ \%$

Example 3: Afzal got 825 marks out of 1100 marks. What is the percentage of Afzal's marks?

Solution: 825 marks out of 1100.

Method 1: By using unitary method

Marks out of 1100 = 825

Marks out of 1 $= \frac{825}{1100}$

Marks out of 100 $= \frac{\overset{75}{\cancel{825}}}{\cancel{1100}} \times \cancel{100}\,\%$

Marks percentage = 75%

Method 2: By using proportion method

Marks	Percentage
1100 ↓	↓ 100%
825	x

Find the proportion of marks and percentage.

$$\frac{1100}{825} = \frac{100\%}{x}$$

$$x = \frac{825}{1100} \times 100\% = 75\%$$

Example 4: The score of Inzmam-ul-Haq was 78 runs in a one-day match, whereas the total score of the team 325 runs. Find the percentage of Inzmam's score.

Solution: 78 runs out of 325 runs.

Method 1: By using unitary method.

Inzmam's score out of 325 runs = 78 runs

Inzmam's score out of 1 run $= \frac{78}{325}$ runs

Inzmam's score out of 100 runs $= \frac{78}{325} \times 100\%$

Inzmam's score percentage $= 24\%$

Method 2: By using proportion method.

Score	Percentage
325	100%
78	x

$$\frac{325}{78} = \frac{100\%}{x}$$

$$x = 100\% \times \frac{78}{325} = 24\%$$

Example 5: Naeem gave 25% of a profit to his partner. If the partner got Rs.4000, what remains with him?

Solution: Method 1: By using unitry method.

We can solve this problem by following two ways.

(i) 25% of the profit = Rs.4000

1% of the profit $= \frac{4000}{25}$

100% of the profit $= \frac{4000}{25} \times 100$

= Rs.16000

Thus, Naeem's share = Rs.16000 – Rs.4000

= Rs.12000

(ii) First find percentage of Naeem's profit.

Percentage of Naeem's profit = 100% – 25%

= 75%

25 % of the profit = Rs.4000

1 % of the profit $= (\frac{4000}{25})$ rupees.

75 % of the profit $= (\frac{4000}{25} \times 75)$ rupees.

= 12000 rupees.

Method 2: By using proportion method.

Here, we also have the same two options for solving this problem.

(i) 25 % of profit = 4000

100% of profit = ?

Profit	Percentage
4000	25%
x	100%

$$\frac{4000}{x} = \frac{25}{100}$$

$$x = \frac{4000 \times 100}{25} = \text{Rs.}16000$$

Thus, Naeem's share= Rs.16000 – Rs.4000

= Rs.12000

(ii) First find percentage of Naeem's share.

Percentage of Naeem's share =

100% – 25% = 75%

Profit	Percentage
4000	25%
x	75%

$$\frac{4000}{x} = \frac{25}{75}$$

$$x = \frac{4000 \times 75}{25} = \text{Rs.}12000$$

EXERCISE 7.2

1. Saeed has Rs.75. He gives 20% of it as alms. What amount remains with him?
2. Komal made a suit of 5.5 metres cloth out of 44 metres. What percentage of the cloth did she use for the suit?
3. 85% of the students in a school of 300 students passed an annual examination. How many of them are fail?
4. The 60% length of a road is 75 km. Find the total length of the road.
5. Sana got 484 marks out of 550 marks. Find the percentage of her marks.
6. In a town, 35% of 15,000 voters did not cast vote in an election. How many people cast vote?
7. In a test match, Shoaib Malik made 134 runs in the first innings and 41 runs in the 2nd innings. Find the percentage of Shoaib's score if total score in both innings of Pakistan was 500 runs.
8. Farooq paid 25% of salary as a house rent and 50% of salary for other expenses. Find the remaining amount if his salary is Rs.8000.
9. Shakeel had Rs.7500. He paid a debt of Rs. 1500. Find the percentage of the remaining amount.
10. Noor spends Rs.1440 out of Rs.2000 and saves the remaining. Find the percentage of his savings.
11. A shoe company found that 4.25% of the production is defective. The company made 28,000 pairs of shoes. How many pairs of shoes were defective?
12. Find the actual amount if 40% of the amount is 60 rupees.
13. Bano spends 70% of her pocket money and saves 30%. Find the amounts she spends and saves, if she gets Rs.1800 as the pocket money.
14. 200 litres pure milk contains 77 litres cream. What is the percentage of cream in pure milk?
15. Every 3 persons are using tobacco out of 5 persons in Pakistan. What is the percentage of tobacco users?

7.2 Profit, Loss and Discount

Cash transaction is the most routine thing of a trader's life. He often purchases things from a wholesaler or a manufacturer after paying any price which is called the cost price and sells them to a customer at any price which is called the sale price. During these cash transactions, what a trader earns or loses is called the profit or loss. We shall explain the profit or loss with the help of following examples.

(i) Anees bought a book for Rs. 50 and sold it for Rs. 75. Is he in profit or loss?
In this example, the cost price of the book is Rs. 50 and the selling price is Rs.75, which is greater than the cost price. Thus profit can be calculated as;

Sale price – Cost price = Profit

We can also calculate the percentage of this profit by the following formula.

$$\text{Profit percentage} = \frac{\text{profit}}{\text{cost price}} \times 100$$

(ii) Sumaira bought a pen for Rs. 18 and sold it to her sister for Rs. 15. Is she in profit or loss?
In this example, the cost price of the pen is Rs. 18 and selling price Rs. 15 which is less than the cost price. It means Sumaira is in loss and her loss can be calculated as;

Cost price – Sale price = Loss

We can also calculate the percentage of this loss by the following formula.

$$\text{Loss percentage} = \frac{\text{loss}}{\text{cost price}} \times 100$$

We calculate the profit or loss percentage at the cost price

Example 1: Find the profit percentage, when the cost price is Rs.490 and selling price is Rs.580.

Solution: Cost price = Rs.490

Selling price = Rs.580

Profit = Rs.580 – Rs.490 = Rs.90

$$\text{Profit \%} = \frac{\text{profit}}{\text{cost price}} \times 100$$

$$\therefore \quad = \frac{90}{490} \times 100 = 18.37\%$$

Example 2: Sher Ali bought 45 kg mangoes for Rs.990. He sold 22 kg at the rate of Rs.15 per kg and 23kg at the rate of Rs.20 per kg. Find if Sher Ali is in profit or loss and also its exact value.

Solution: Cost price of 45 kg mangoes = Rs.990.

Selling price of 22 kg = 22×15 rupees

= Rs.330.

Selling price of 23 kg = 23×20 rupees

= Rs.460

Total selling price = Rs. (330 +460) =Rs.790.

Here, we can see that.

Cost price > Selling price

Cost price – Selling price = Loss.

Rs.990 – Rs.790 = Rs.200

Sher Ali suffered the loss of Rs.200.

Example 3: Nadeem bought 10 kg tomatoes for Rs. 150 and 25 kg tomatoes for Rs.250. He sold all of them at the rate of Rs. 20 per kg. Is he in profit or loss?

Solution: Cost price of 10 kg tomatoes = 150 rupees.
Cost price of 25 kg tomatoes = 250 rupees.
Total cost price of tomatoes = Rs. (250 + 150) = Rs. 400.
Selling price of 35 kg tomatoes = (35×20) rupees = Rs. 700.
In the above, we can see that;

Selling price > Cost price

So, Nadeem is in profit.

Example 4: Azeem bought 90 eggs for Rs.315, 10 eggs were rotten. He sold the eggs at the rate of Rs. 4 per egg. Find if Azeem is in profit or loss and also find its percentage.

Solution: Cost price of 90 eggs = Rs.315.
Remaining eggs = (90 – 10) eggs = 80 eggs
Selling price of 80 eggs = 4×80 = Rs.320.

Selling price – Cost price = Profit
Rs. 320 – Rs. 315 = Rs.5

$$\% \text{ Profit} = \frac{5}{315} \times 100 = 1.59$$

Example 5: A watch is sold for Rs.600 at the loss of 25%. Find the cost price.

Solution: Sale price = Rs.600 Loss % = 25% Cost price = ?

Let the cost price be Rs. 100, then the sale price will be:
Sale price = Cost price – Loss
= Rs. 100 – Rs. 25 = Rs. 75

Cost price	Sale price
100	75
x	600

Resolve, $$\frac{x}{100} = \frac{600}{75}$$

or, $$x = \text{Rs.}\frac{600 \times 100}{75} = \text{Rs. } 800$$

7.2.1 Discount

The difference between the marked price and the sale price is called discount and is given as a percentage of marked price.

Marked price – Sale price = Discount

Example 5. Komal bought a suit on the sale price Rs.450. When the marked price was Rs.525, find the discount percentage.

Solution: Sale price = Rs.450 Is the Marked price = Rs.525
Discount = 525 – 450 = Rs.75 Discount percentage =?

(i) By using unitary method.

Discount out of Rs.525 = Rs.75

Discount out of Re.1 = Rs.$\frac{75}{525}$

Discount out of Rs.100 = $(\frac{75}{525} \times 100)$ %

Discount Percentage = 14.29 %

(ii) By using proportion method.

Marked Price	Discount
525 ↓	↓ 75
100	x

$$\frac{x}{75} = \frac{100}{525}$$

$$x = \frac{75\times100}{525} = \text{Rs.}14.29$$

Discount Percentage = 14.29 %

EXERCISE 7.3

(1) A book seller sold a book for Rs.70 at a gain of 40%. Find the profit.

(2) A shopkeeper sold a toy for Rs.96 at a loss of 20%. Find the loss.

(3) Chand bought a shirt for Rs.250 and sold it for Rs.295. Find the profit percentage.

(4) Waleed bought one dozen pens for Rs.144 and sold each of them for Rs.11. Find the loss percentage.

(5) Saleem bought 90 oranges at the rate of 3 oranges for Rs.10 and sold them at the rate of 2 oranges for Rs.9. Find if Saleem is in profit or loss and also find its percentage?

(6) Shahid bought 80 bananas at the rate of 4 bananas for Rs.5 and sold at the rate of 5 bananas for Rs.8. Find if he is in profit or loss and also find its percentage, when 25% of bananas have been spoiled.

(7) 12 % profit on a computer is Rs.540.

(i) Find the cost price of the computer.

(ii) Find the sale price of the computer.

(8) The cost price of 25 pairs of shoes is Rs.190 each. Find the sale price of each of them, when the retailer has a total gain of Rs.2,875

(9) 25% loss on a mobile set is Rs.475. Find the cost and sale price of the mobile set.

(10) The profit percentage on a bicycle is 40%. Find the cost and sale price of the bicycle, when the shopkeeper got a profit of Rs.500.

(11) The cost price of 18 sweaters is Rs. 425 per sweater and the total gain of the shopkeeper is Rs. 6,750.

(i) Find the sale price of each sweater.

(ii) Find profit percentage.

(12) Sheikh Khalid purchased 80 metres cloth for Rs.2,240. He sold 50 metres cloth at the rate of Rs.30 per metre and 30 metres cloth at the rate of Rs.35 per metre.
   (i) Find if Sheikh Khalid is in profit or loss.
   (ii) Find profit or loss percentage.

(13) Gul Khan purchased 180 chocolates for Rs.2,160. He sold 155 chocolates at the rate of Rs.15 each and 25 chocolates at the rate of Rs.10 each.
   (i) Find if he is in profit or loss.
   (ii) Find profit or loss percentage.

(14) A shirt priced for Rs.150 is sold for Rs.120. Find the percentage discount.

(15) Sarah bought a dinner set for Rs.480 at 20% discount. Find the actual price of the dinner set.

(16) The cost price of a jean is Rs.200 and marked price is 50% more than the cost price. But the shopkeeper sold it at 25% discount.
   (i) Find the marked price.
   (ii) Find the discounted price.

(17) Find the marked price when 9% discount is Rs.81.

(18) Find the marked price of a pair of shoes, when its sale price is Rs.360 and discount percentage is 18%.

Summary

- Any ratio with a second term of 100 or any fraction with 100 as a denominator is called a percentage.
- We use the symbol "%" instead of writing the word percent, i.e., 12 percent can be written as 12 %.
- If selling price is greater than cost price, then it is called Profit, i.e.,
  Profit = sale price – cost price.
- If cost price is greater than sale price, then it is called loss i.e. loss = cost price–sale price.
- We calculate the profit or loss as a percentage of the cost price, i.e.,

$$\text{Profit}\,\% = \frac{\text{profit}}{\text{cost price}} \times 100 \quad , \quad \text{Loss}\,\% = \frac{\text{loss}}{\text{cost price}} \times 100$$

- The difference between the marked price and sale price is called discount, i.e.,
  marked price – sale price = discount

Review Exercise 7

1. Find the percentage of the following.
   (i) Rs. 20 out of Rs 250
   (ii) 30 kg out of 260 kg
   (iii) 250 marks out of 300 marks.
   (iv) 24 min out of 1 hour.
2. Khalid spent Rs.156 out of Rs.1200. What percent of amount did he spend?
3. In a town election, Azeem got 42% of the votes cast and Hamza got the remaining votes. If the total number of votes cast is 40,000. Find the votes obtained by Hamza.

4. Nabeel traveled 75 km by bus and 125 km by train. Find what percent of the total journey did he travel by bus and what percent by train?
5. A shopkeeper bought a pair of shoes for Rs.720 and sold it for Rs.810. Find his profit percent.
6. Komal bought a sewing machine for Rs.5,800 but due to some defects in the machine, she sold it for Rs.5,500. Find her loss percent.
7. A dealer bought 18 toy chairs at Rs.65 per chair. He sold 12 of them at Rs.75 each and the remaining chairs at Rs.60 each. Find his profit or loss %.
8. Fatima bought a doll for Rs.440 after getting a discount of 20 %. Find the marked price of the doll.
9. A mobile is sold for Rs.2160 after giving a discount. If marked price is Rs.2700, find the discount percentage.

Objective Exercise 7

1. Answer the following questions.

(i) Define the percentage. (ii) What should we do to change a fraction into percentage?
(iii) Write the formula for finding the profit. (iv) What is meant by the percent?
(v) What is the formula for finding a discount?

2. Fill in the blanks.

(i) $\frac{1}{100}$ means percent and is denoted by _________ .

(ii) The price that we pay to purchase a thing is called _________ .

(iii) Profit percentage = $\frac{\text{profit}}{\boxed{}}\times 100$.

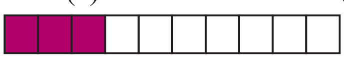
(iv) To change a percentage into a decimal, first, we convert it into a _________ .

3. Tick (✓) the correct answer.

(i) When we change a fraction $\frac{1}{25}$ into percentage, we get:
(a) 1% (b) 4% (c) 25% (d) 0.4%

(ii) By changing 10% into a decimal, we get:
(a) 1 (b) 10 (c) 0.1 (d) 0.01

(iii) 1% of 1000 means:
(a) 1 (b) 10 (c) 100 (d) 1000

(iv) The coloured parts of  are:
(a) 3% (b) 30% (c) 7% (d) 70%

(v) Profit or loss is calculated as the percentage of the:
(a) cost price (b) selling price (c) marked price

Unit 8

INTRODUCTION TO ALGEBRA

Student Learning Outcomes

After studying this unit, students will be able to:

- Explain the term algebra as an extension of arithmetic in which letters replace the numbers.
- Know that chair. He
  - a sentence is a set of words making a complete grammatical structure and conveying full meaning. sentences that are either true or false are known as statements.
  - a statement must be either true or false but not both.
  - a sentence that does not include enough information and it is required to decide whether it is true of false is known as open statement (e.g. $\triangle + 2 = 9$).
  - a number that makes an open statement true is said to satisfy the statement (e.g. $\triangle = 7$ makes the statement $\triangle + 2 = 9$ to modify it to $\triangle + 2 = 9$ true).
  - use English alphabet x in the open statement $\triangle + 2 = 9$ to modify it to $x + 2 = 9$.
- Define variables as letters used to denote numbers in algebra.
- Know that any numeral, variable or combination of numerals and variables connected by one or more of the symbols '+' and '–' is know as an algebraic expression (e.g., $x + 2y$).
- Know that x, $2y$ and 5 are called the terms of the expression $x + 2y + 5$.
- Know that the symbol or number appearing as multiple of a variable used in algebraic term is called its coefficient (e.g. in$2y$, 2 is the coefficient of y).
- Know that the number, appearing in algebraic expression, independent of a variable is called a constant term (e.g. in $x + 2y + 5$, number 5 is a constant term).
- Differentiate between like and unlike terms.
- Know that
  - like terms can be combined to give a single term,
  - addition or subtraction can not be performed with unlike terms.
- Add and subtract given algebraic expressions.
- Simplify algebraic expressions grouped with brackets.
- Evaluate and simplify an algebraic expression when the values of variables involved are given.

8.1 Algebra

Algebra is an important branch of the mathematics that provides us the solution of many complex mathematical problems in an easy way. Especially, when we represent a quantity by a symbol without knowing its numerical value.
The word algebra has been deduced from the Arabic word Al-Jabar. A Muslim mathematician Al-khawarzmi (780-850 AD) wrote a book named Al Jabar-wal-Muqabala in 820 AD, in which, he described the methods of solving difficult and complex mathematical problems. Later on, a translation of this book was published in Europe under the title of "Algebra" thus, Al-Jabar became Algebra.

8.1.1 Relationship between Arithmetic and Algebra

We are well aware of natural numbers 1,2,3,... and use of the basic operations (+, –, ×, ÷) in arithmetic **(The science of numbers)**. In algebra, we use letters $a,b,c,...,z$. in addition of numbers to generalize the arithmetic which helps us to express a quantity without knowing its numerical value. We shall explain it with the help of an example.

$1 + 2 = 3$
$2 + 3 = 5$
$3 + 4 = 7$
....
....
....
$x + y = z$

If $x=1$, $y=2$ then $z=3$
If $x=2$, $y=3$ then $z=5$
If $x=3$, $y=4$ then $z=7$

In above example, $x + y = z$ is giving us a general form which is representing all the given arithmetical statements. Thus, algebra is a general form of the arithmetic.

$x + y = z$ means the sum of the two numbers represented by x and y is equal to the number represented by z.

8.1.2 Sentence

A group of words that makes a complete sense is called a sentence, e.g.

(i) Cows give milk. (ii) Birds fly in the air. (iii) She writes very well.

These are examples of a sentence.

8.1.3 Statement

A statement is a sentence that may be true or false.
For example,

(i) Karachi is in the Punjab. (false)
(ii) Mango is a fruit. (true)
(iii) Thank you. (neither true nor false)

Here, (i) and (ii) are statements but (iii) is not.
Now consider the following statements.

(a) $2 + \square = 4$ (b) $\triangle - 3 = 2$ (c) ○ is a beautiful city.

Out of the above statements, we can't decide which statement is true and which is false, until we get further information of $\square$, $\triangle$ and $\bigcirc$. Such type of statements are called open statements and $\square$, $\triangle$ and $\bigcirc$ are called unknowns or variables. Thus, an open statement is a sentence which has one or more than one unknowns.

Let us examine the following open statement.

$$2 + \square = 4$$

We try to guess the value of $\square$ to make the statement true.

$2 + 1 = 4$ (False) $\quad$ $2 + 2 = 4$ (True)

Here, 2 is the required number which is making the statement true or 2 is satisfying the statement.

Example 1: Replace the unknowns by the numbers to make the statement true.

(i) $6 \times \triangle = 54$ (ii) $15 - \square = 9$ (iii) $8 \div \square = 4$ (iv) $\triangle + 8 = 17$

Solution:

(i) $6 \times \triangle = 54$
$6 \times 9 = 54$
$\therefore \triangle = 9$

(ii) $15 - \square = 9$
$15 - 6 = 9$
$\therefore \square = 6$

(iii) $8 \div \square = 4$
$8 \div 2 = 4$
$\therefore \square = 2$

(iv) $\triangle + 8 = 17$
$9 + 8 = 17$
$\triangle = 9$

8.1.4 Variable and Constant

We have already learnt that in algebra a letter of the alphabet is used to represent a number of values. Here this letter is called a variable, whereas numbers 0,1,2,3,... have definite fixed values are called constants.

EXERCISE 8.1

1. Separate the true, false and open statements.

(i) 5 is a natural number
(ii) $(5 + 4)–2 = (6 + 8) \div 2$
(iii) 9 is a prime number
(iv) $8 \div \square = 4$
(v) $5 \times \triangle = 15 \div \bigcirc$
(vi) -1 is a whole number
(vii) $5 \times 6 = 4 \times 8$
(viii) $0.2 + 0.5 = \square$
(ix) 2 is the only even prime number

2. Replace the unknowns by the numbers to make the statement true.

(i) $x + 2 = 6$ (ii) $p - 1 = 7$ (iii) $m + 15 = 20$
(iv) $6x = 48$ (v) $5 \times x = 75$ (vi) $\frac{2}{3} m = \frac{14}{3}$
(vii) $\frac{1}{2} \div m = 15$ (viii) $2m = 3$ (ix) $x - 0.3 = 0.4$
(x) $x \div 2 = 7$ (xi) $5 + p = 11$ (xiii) $0.4m = 0.8$

8.2 Algebraic Expressions

The expressions in which the numbers or variables or both (numbers and variables) are connected by operational signs are called algebraic expressions.

For example: $5, 4x, a+b, x-y$ etc.

Example 1: Write the following as algebraic expressions.

(i) The sum of a number and 15.

(ii) Multiply the sum of the two numbers by their product.

(iii) Divide the sum of the two numbers by their difference (The 2nd number is less than the 1st)

Solution: (i) Suppose that the number is x.

Algebraic expression: $x + 15$.

(ii) Suppose that the two numbers are x and y.

Product of the two numbers $= xy$

Sum of the two numbers $= x + y$

Algebraic expression: $xy\,(x + y)$

(iii) Suppose that the two numbers are a and b.

Sum of the numbers $= a+b$

Difference of the numbers $= a-b$

Algebraic expressions: $\dfrac{a+b}{a-b}$

8.2.1 Algebraic Terms

The operational signs "+" and "–" separate and connect the parts of an algebraic expression. These parts are called algebraic terms of the algebraic expression.

For example: x, $2y$ and 5 are terms of the expression $x+2y+5$.

Constant Term

In the expression, $x+2y+5$, the term 5 remains unchanged and only values of x and y vary. Here 5 is called a constant term.

Index or Exponential Form

We know that $4\times4=4^2$ and $4\times4\times4=4^3$. These forms are called index or exponential forms and can be read as power of 4.

In algebra, we can write an index or exponential form as:

$$x\times x = x^2 \quad \text{or} \quad a\times a\times a = a^3$$

In x^2, variable x is called base and power 2 is called exponent. We read it as x raise to the power 2.

Coefficient

The multiplying factor of a variable is called its coefficient. For example, in $4x$, 4 is the coefficient of x.

8.2.2 Like or Un-Like Terms

The terms of the same kind only differ by their coefficients are called like terms. Such terms can be reduced to a single term by adding and subtracting. For example,

(i) $a + 2a + 3a = 6a$ (ii) $5xy - xy - 2xy = 2xy$

The terms having different variables or same variables with different exponents are called unlike terms. For example:

(i) x, y, z (ii) ab, bc, ca

Example 2: Write the following in exponential form.

(i) $x \,.\, x \,.\, x \,.\, x \,.\, x$ (ii) $xy \,.\, xy \,.\, xy \,.\, xy$

Solution:

(i) $x \,.\, x \,.\, x \,.\, x \,.\, x$

$(x)^{1+1+1+1+1} = x^5$

(ii) $xy \,.\, xy \,.\, xy \,.\, xy$

$x \,.\, x \,.\, x \,.\, x \,.\, y \,.\, y \,.\, y \,.\, y$

$(x)^{1+1+1+1} \; (y)^{1+1+1+1} = x^4 \cdot y^4$

Example 3: Write the coefficient, base and exponent of each of the following:

(i) $9x^2$ (ii) $6y^4$ (iii) $10m^5$ (iv) $15a^3$

Solution:

(i) $9x^2$
Coefficient $= 9$
Base $= x$
Exponent $= 2$

(ii) $6y^4$
Coefficient $= 6$
Base $= y$
Exponent $= 4$

(iii) $10m^5$
Coefficient $= 10$
Base $= m$
Exponent $= 5$

(iv) $15a^3$
Coefficient $= 15$
Base $= a$
Exponent $= 3$

EXERCISE 8.2

1. Write each of the following word expressions into algebraic expressions:

   (i) x plus y (ii) a minus b
   (iii) m multiplied by n (iv) p divided by q
   (v) The sum of $3x$ and $2y$ (vi) The difference of $5a$ and $4b$ ($4b$ is less than $5a$)
   (vii) The product of x and y
   (viii) The sum of p and q divided by r
   (ix) Half of l multiplied by the difference of n and m. (m is less than n)

2. Write the coefficient, base and exponent of each of the following:

   (i) $5x$ (ii) $16p^2$ (iii) $18l^3$ (iv) $-6k^5$ (v) $\frac{2}{3}q^{-1}$ (vi) $\frac{1}{3}y^{-2}$

3. Write the following in the exponential form:

   (i) $a \,.\, a \,.\, a$ (ii) $x \,.\, x$ (iii) $xy \,.\, xy$
   (iv) $m \,.\, m \,.\, m \,.\, m$ (v) $pq.\, pq.\, pq$ (vi) $abc \,.\, abc$

4. Separate the terms of the following algebraic expressions.

(i) $2a+3b$ (ii) $l-2m+4n$ (iii) $9a^2-12b^2$

(iv) $p^2+2q^2-r^2$ (v) $a+8b-4c$ (vi) $2lm-3mn-4nl$

(vii) $3xy^2+4x^2y+9$ (viii) $\frac{2}{5}xy+\frac{1}{3}yz+\frac{3}{5}xz$ (ix) $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}$

5. Write the algebraic expressions by adding the following terms.

(i) a, b (ii) $x, -y$ (iii) $l, m, -n$

(iv) p, pq, qr (v) xy^2, xz^2, yz^2 (vi) $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$

(vii) $16a^2, -8b^2$ (viii) $\frac{l}{m}, -\frac{m}{n}, -\frac{n}{l}$ (ix) $2ab, 4ac, -3bc$

8.2.3 Addition

In algebra, we follow the same rules for the basic operations (+ , – , ×, ÷) as we use in arithmetic. Let us discuss them one by one.

In arithmetic, we can add only like things. This can be shown by an example. If 2 boys and 3 boys go to the school together then we can say that 5 boys go to school instead of saying 2 boys and 3 boys because we can add them together as given below.

2 boys + 3 boys = (2 + 3) boys = 5 boys.

But when we say 2 boys and 3 girls go to school, then the sentence will remain as it is. This shows that we can't add them together because they are not alike.

We know that algebra is the general form of the arithmetic. Now we can use the same rule for addition in algebra. Consider that we denote boys by a variable x and girls by a variable y, then we can write the above two statements as:

$2x + 3x$

$= (2+3)x = 5x$ … (i)

and $2x+3y$ remains as it is … (ii)

From (i) and (ii), we can examine the following rules, that we use in addition of algebraic expressions.

Rule 1: We can add only like terms and un-like terms will remain as they are.

(i) $x+2x+3x+4x = 10x$ (ii) $x+y+z = x+y+z$

Rule 2: In addition of like terms, we just add their coefficients and write the same variable with the sum.

$$4x + 5x = (4+5)x = 9x$$

We can add algebraic expressions by the following two methods.

Row-wise Addition

(i) Arrange the terms of an expression.

(ii) Add the coefficients of like terms and common variable with the same exponent take by the sum. For example, $x+y$, $y+z$ and $x+z$ are any three expressions. We can add them as

$x+y+y+z+x+z$

$= x+x+y+y+z+z$

$= (1+1)x+(1+1)y+(1+1)z$

$= 2x+2y+2z$

Column-wise Addition

(i) Arrange all the algebraic expressions in the same order.
(ii) Put the like terms in the same column.
(iii) Add them together. For example, $x+y,\ y+z,\ x+z$

$$\begin{array}{rrr} x & +y & \\ & +y & +z \\ x & & +z \\ \hline 2x & +2y & +2z \end{array}$$

Example 1: Add the following algebraic expressions.

(i) $x+y+z,\ 2x+y+z,\ x+2y+z$ (ii) $5l+3m+n\ ,\ l-m+n\ ,\ -3l-m+n$

(iii) $x^2+x^2y+xy^2+y^2\ ,\ 2x^2+xy^2-x^2y+y^2$

Solution: **Method I**

(i) $x+y+z+2x+y+z+x+2y+z$
$=(x+2x+x)+(y+y+2y)+(z+z+z)$
$=(1+2+1)\,x+(1+1+2)\,y+(1+1+1)\,z$
$=4x+4y+3z$

Method II

$$\begin{array}{rrr} x\ + & y\ + & z \\ 2x\ + & y\ + & z \\ x\ + & 2y\ + & z \\ \hline 4x\ + & 4y\ + & 3z \end{array}$$

(ii) $5l+3m+n\ ,\ l-m+n\ ,\ 3l-m+n$

Method I

$(5l+3m+n)+(l-m+n)+(-3l-m+n)$
$=(5l+l-3l)+(3m-m-m)+(n+n+n)$
$=(5+1-3)\,l+(3-1-1)\,m+(1+1+1)\,n$
$=3l+m+3n$

Method II

$$\begin{array}{rrr} 5l\ + & 3m\ + & n \\ l\ - & m\ + & n \\ -3l\ - & m\ + & n \\ \hline 3l\ + & m\ + & 3n \end{array}$$

(iii) $x^2+x^2y+xy^2+y^2\quad ,\quad 2x^2+xy^2-x^2y+y^2)$

Method I

$(x^2+x^2y+xy^2+y^2)+(2x^2+xy^2-x^2y+y^2)$
Arrange them in the same order.
$(x^2+x^2y+xy^2+y^2)+(2x^2-x^2y+xy^2+y^2)$
$=(x^2+2x^2)+(x^2y-x^2y)+(xy^2+xy^2)+(y^2+y^2)$
$=(1+2)\,x^2+(1-1)\,x^2y+(1+1)\,xy^2+(1+1)\,y^2$
$=3x^2+0x^2y+2xy^2+2y^2\quad =3x^2+2xy^2+2y^2$

Method II

$$\begin{array}{rrrr} x^2 & +x^2y & +xy^2 & +y^2 \\ 2x^2 & -x^2y & +xy^2 & +y^2 \\ \hline 3x^2 & & +2xy^2 & +2y^2 \end{array}$$

EXERCISE 8.3

1. Simplify.

(i) $x+x+x+x$ (ii) $2y+3y+4y$ (iii) $6m+3m+m$
(iv) $a+9a+3b$ (v) $3p+q+2p$ (vi) $x+y+x+2y$
(vii) $11a+6a+2a+9b$ (viii) $m+2n+3n+4n$ (ix) $x+y+z+2x+z$
(x) $p+2q+q+r+2p$

2. Answer the following questions.

(i) Ifra had $2p$ chocolates. She bought q more chocolates. How many chocolates she has now?
(ii) Mehak, Naz and Kinza have m, $2n$ and $3l$ books respectively. How many books have they altogether?
(iii) Zain had x candies. He bought more $2x$ candies and y candies. Find the sum of the candies that he has now.

3. Add the following.

(i) ab, bc, bc, bc (ii) $2x^2y, x^2y, xy^2$ (iii) $6m^3, 2m^2, 1, 3m^2$

(iv) $b^2, 3ab, 4ab, 2a^2$ (v) $x^2, -xy, y^2, -xy$ (vi) $p, -2q, -r, -q$

4. Find the sum of the following algebraic expressions.

(i) $a^2+2ab+b^2$, $a^2-2ab+b^2$, a^2-ab-b^2

(ii) $x^3y+2x^2y+y^2$, $x^3y+x^2y+2y^2$, $x^2y-2x^3y-y^2$

(iii) $3mn+2lm+nl$, $3nl+2mn+lm$, $3lm+2nl+mn$

(iv) $2p+3q$, $3q+3r$, $r+3p$

(v) $b+1$, $a+b+2$, 3 , $a+1$

5 Find A+B+C , when

(i)	$A=2a$,	$B=3b$,	$C=4c$
(ii)	$A=x+y$,	$B=x-2y$,	$C=3y-x$
(iii)	$A=s+st$,	$B=g+t$,	$C=s+2g$
(iv)	$A=p+q+r$,	$B=p+q-2r$,	$C=p-2q-r$
(v)	$A=lm+mn$,	$B=mn+nl$,	$C=nl+lm$

8.2.4 Subtraction

Subtraction is an opposite operation of the addition. We use the following rule for subtraction.

Rule: Change the sign of each term of the expression to be subtracted and add them together by using addition rules. For example: $3x+2y$ and $2x-y$ are the two algebraic expressions and we want to subtract $2x-y$ from $3x+2y$, then;

- **Row-wise Subtraction**

$$\begin{aligned}&(3x+2y)-(2x-y)\\ =\ &3x+2y-2x+y\\ =\ &3x-2x+2y+y\\ =\ &(3-2)x+(2+1)y \ = \ x+3y\end{aligned}$$

- **Column-wise Subtraction**

$$\begin{array}{r} 3x+2y \\ \pm\, 2x\mp\ y \\ \hline x+3y \\ \hline \end{array}$$

Example 1: Simplify the following.

Solution: **Method I**

$$\begin{aligned}&(2x^3-3x^2y^2+x^2y)-(2y^3+x^2y^2+x^2y)\\ &=2x^3-3x^2y^2+x^2y-2y^3-x^2y^2-x^2y\\ &=2x^3-3x^2y^2-x^2y^2+x^2y-x^2y-2y^3\\ &=(2)x^3-(3+1)x^2y^2+(1-1)x^2y-(2)y^3\\ &=2x^3-4x^2y^2-2y^3.\end{aligned}$$

Method II

$$\begin{array}{r} 2x^3-3x^2y^2+x^2y \\ \pm\, x^2y^2\pm x^2y\pm 2y^3 \\ \hline 2x^3-4x^2y^2-2y^3 \end{array}$$

EXERCISE 8.4

1. Simplify the following.

(i) $(6x)-(4x)$ (ii) $(13a)-(2a)$

(iii) $(x+1)-(x-1)$ (iv) $(m-n)-(m+n)$

(v) $(2p+q+2r)-(p+q+r)$ (vi) $(2)-(x^2-x^3+2x-1)$

(vii) $(x^3 + x^2y + xy^2 + y^3) - (x^2y + xy^2 + 1)$

(viii) $(3x^2 + 6xy + 9y^2) - (2x^2 - 3xy^2 + xy^2)$

(ix) $(x^3 - x^2y^2 + x^2y) - (y^3 - x^2y^2 + xy^2)$ (x) $3(x^2 - 2xy + y^2) - (2x^2 - xy + y^2)$

2. Subtract $2l - 3m - n$ from $l - 4m - 6n$.
3. Subtract $2a^3 - 3a^2 + 5a + 5$ from $5a^3 + a^2 + 2a - 3$.
4. Subtract $3x^5 - 4x^4 + 8x^3 - 6$ from $8x^5 + 5x^4 - 3x^3 + 4x + 2$.
5. If A = $a + b + c$, B = $a - b + c$, C = $a + b - c$ and D = $-a - b - c$ then find;

   (i) A – B (ii) B – C (iii) A – C (iv) C – D (v) B – D
   (vi) A – D (vii) A + B – C (viii) A – C – D (ix) A – B + D
   (x) (A +B) – (C + D)

6 What must be added to $x^2 + xy + y^2 + 1$ to get $x^3 + 3$?

7. What must be subtracted from $p^5 + p^4 + p^3 + p^2 + p + 1$ to get $p^5 + 1$?

8.2.5 Simplification

We know that brackets are used to indicate the order for performing operations. The four kinds of brackets are:

(i) "—" is called a bar or vinculum.
(ii) "( )" is called a round or curved brackets or parentheses.
(iii) "{ }" is called a curly brackets or braces.
(iv) "[ ]" is called box brackets or square brackets.

In algebra, sometimes, we can't simplify an expression into a single term within the brackets. For example, in $2x - (x + y)$ we can't simplify the expression $(x+y)$. For such a situation,

(i) Expand the brackets. (ii) Simplify the whole expression as given below:

$$2x - (x + y) = 2x - x - y = x - y$$

Hence, $x - y$ is the simplest form of the above mentioned algebraic expression.
–ve sign before the brackets means, change the signs of all the terms within brackets, i.e. $-(a + b) = -a - b$.

Example: Simplify the following.

(i) $[5a - \{3b + (6a - 2a + b)\}]$ (ii) $[2a + \{c - a + (a + \overline{2b + c})\}]$

(iii) $xy - [yz - \{zx + xy + (yz - \overline{zx + xy})\}]$

Solution: (i) $[5a - \{3b + (6a - 2a + b)\}]$

$= [5a - \{3b + (4a + b)\}]$

$= [5a - \{3b + 4a + b\}]$

$= [5a - \{4a + 4b\}]$

$= [5a - 4a - 4b]$

$= [a - 4b] = a - 4b$

(ii) $[2a + \{c - a + (a + \overline{2b + c})\}]$

$= [2a + \{c - a + (a + 2b + c)\}]$

$= [2a + \{c - a + a + 2b + c\}]$

$= [2a + \{2b + 2c\}]$

$= [2a + 2b + 2c]$

$= 2\,[a + b + c]$

(iii) $xy-\left[yz-\left\{zx+xy+\left(yz-\overline{zx+xy}\right)\right\}\right]$

$=xy-\left[yz-\left\{zx+xy+\left(yz-zx-xy\right)\right\}\right]$

$=xy-\left[yz-\left\{\cancel{zx}+\cancel{xy}+yz-\cancel{zx}-\cancel{xy}\right\}\right]$

$=xy-\left[yz-\{yz\}\right] \;=xy-\left[\cancel{yz}-\cancel{yz}\right] \;=xy-0=xy$

EXERCISE 8.5

Simplify the following expressions.

(i) $[a+\{a+(a+\overline{a+a})\}]$

(ii) $[7x-\{4x+(3x-2x)\}]$

(iii) $[5l-\{2m+(6m-3m)\}]$

(iv) $[2y+\{x+x+(x-\overline{2x+x})\}]$

(v) $[x^2+\{2xy+(3y^2-2y^2)\}]$

(vi) $[\{9a^4+\{5a^2+(a^2+1)\}]$

(vii) $[x^2+\{3x^2-(x^2+2x^2)\}]$

(viii) $7l-2\left[3(5l-m)-2(4l+m)\right]$

(ix) $[6a+\{3a+(2a+\overline{a+b})\}]+6c$

(x) $[2x^2-xy-\{xy-(2x^2-x^2-y^2)\}]$

(xi) $8\left[3(4a+5b)-2(6a-5b)\right]$

(xii) $\left[11a-\left\{5b-3(2a+b)\right\}\right]$

(xiii) $[a+c+\{a-c+(a+b+\overline{b-c})\}]$

(xiv) $5x-\left[3y-\left\{4x-\left(5y-\overline{6x-7y}\right)\right\}\right]$

(xv) $2\left(x^2-y^2\right)-3\left[x^2-\left\{y^2-x^2+\left(x^2-\overline{y^2-x^2}\right)\right\}\right]$

8.2.6 Evaluation

The process of finding the absolute or numerical value of an expression by using numbers in place of variables is called evaluation.

The numerical or absolute value of an expression varies according to the given value of variables. For example, if $a=1$ and $b=2$ then the numerical value of $a+b$ is $1+2=3$ and if $a=2$ and $b=3$ then the numerical value of $a+b$ is $2+3=5$.

Example 1: If $a=2$, $b=-3$ and $c=-4$, then evaluate the following.

(i) $ab+bc+ca$ (ii) $\dfrac{bc[b-c]}{a}$

Solution:

(i) $ab+bc+ca$

$=2\times(-3)+(-3)\times(-4)+(-4)\times 2$

$=-6+12-8$

$=-14+12=-2$

(ii) $\dfrac{bc[b-c]}{a}$

$=\dfrac{(-3)\times(-4)[(-3)-(-4)]}{2}$

$=\dfrac{12[-3+4]}{2}=\dfrac{12\times 1}{2}=6$

Example 2: If $a=3$ and $b=4$, then prove that $(a+b)^2=a^2+2ab+b^2$

Solution: L.H.S. $=(a+b)^2=(3+4)^2=(7)^2=49$

R.H.S. $=a^2+2ab+b^2$

$=(3)^2+2\times 3\times 4+(4)^2$

$=9+24+16=49$

So, L.H.S. = R.H.S.

EXERCISE 8.6

1. Evaluate the following when $a = 2$, $b = 1$ and $c = 1$.

(i) $a + b$ (ii) $a - c$ (iii) $b + c$ (iv) $a + b + c$

(v) $a - b$ (vi) $a - b + c$ (vii) $ab + bc$ (viii) $4ab$

(ix) abc (x) $ab - bc + ac$ (xi) $6a - 2b - 2c$ (xii) $a^2 + b^2 + c^2$

(xiii) $\dfrac{a^2 + b^2 - c^2}{2}$ (xiv) $\dfrac{a}{b} + \dfrac{b}{c}$ (xv) $\dfrac{ab}{bc} + \dfrac{ac}{bc}$

2. If $a = 5$ and $b = -3$ then prove that $a + b = 2$.
3. If $a = 1$, $b = 1$ and $c = 9$, then prove that $a - b + c = 9$.
4. If $a = 10$, $b = -10$ and $c = 4$, then prove that $a \times b + 25c = 0$.
5. If $x = 1$ and $y = 1$, then prove that $(x + y)^2 = x^2 + 2xy + y^2$.
6. If $x = 2$ and $y = 1$, then prove that $(x - y)^2 = x^2 - 2xy + y^2$.
7. Evaluate $2 - [2 - \{2 - (2 - \overline{2 - x})\}]$ when $x = 1$.
8. If $a = 1$, $b = 3$ and $c = 1$, then evaluate $b^2 - 4ac$.
9. If $a = 3$, $b = 2$ and $c = 1$, then prove that.

(i) $a + b = b + a$. (ii) $a \times b = b \times a$.

(iii) $(a + b) + c = a + (b + c)$. (iv) $(a \times b) \times c = a \times (b \times c)$.

(v) $a \times (b + c) = a \times b + a \times c$. (vi) $a \times (b - c) = a \times b - a \times c$.

(vii) $a^2 - b^2 = (a + b)(a - b)$.

Summary

- Algebra is a general form of the arithmetic.
- In algebra, a letter is used as a symbol of any number or value which is called a variable.
- A group of words that makes a complete sense is called a sentence.
- The multiplying factor of a variable is called its coefficient.
- The expressions in which the numbers or variables or both (numbers and variables) are connected by operational sings are called algebraic expressions.
- A quantity which has a fixed numerical value is called a constant.
- The terms of same kind only which differ by their coefficients are called like terms.
- In addition of like terms, we just add their coefficients and write the same variable with the sum.
- Brackets are used to indicate the order for performing operations.
- The process of finding the absolute or numerical value of an expression by using numbers in place of variables is called evaluation.

Review Exercise 8

1. Find the sum of :

(i) $3x^2 - x + 7$ and $-2x^2 + 5x - 8$ (ii) $5x^2 - 4x + 2$ and $-3x^2 - 7x + 4$

(iii) $2a - 3b + 4c$ and $5a + 2b - 5c$ (iv) $a - 2b + c$, $5b - 2a$ and $-4a - 3b$

(v) $3l^2+4m-5n^3, 7l^2-8m-6n^3$ and $4l^2-9m-7n^3$ (vi) $p^2+2pq+q^3$, $p^2-2pq+q^2$ and $-p^2-q^2$

2. Subtract the second expression from the first.

(i) $-3a-7b-c$, $3a-8b-6c$ (ii) $19p-q+r$, $8p-3q-4r$
(iii) $2x^3-3x^2+x+5$, $4x^3+5x^2-3x+8$ (iv) $3a-3b+4c-6d$, $4a-6b-c+7d$
(v) $x^2-3xy+7y^2-2$, $-4x^2-6xy-y^2+5$

3. Simplify:

(i) $[3x^2-\{x^2-2y(5x-3y)\}]$ (ii) $x-[2y-\{3x-(2y+3z)\}]$
(iii) $2a-[3a-\{4a-(3b-\overline{2a+3b})\}]$ (iv) $-l-5m-[2l-m-\{3l-2m-(l+2m)\}]$

4. If $x=4$, $y=2$ and $z=5$, then find the value of

(i) $2x-z$ (ii) $5x^2$ (iii) $x+y$ (iv) $x+y-z$ (v) $2xy-yz+y$
(vi) x^2+z^2-2y (vii) $4x^2+2yz-y$ (viii) $4yz-z^2+3x^2$ (ix) $4x^2-3y^2z-8xz$

Objective Exercise 8

1. Answer the following questions.

(i) Define the sentence. (ii) What is meant by an open statement?
(iii) What is called the number that makes an open statement true?
(iv) What is a variable? (v) Define the evaluation.

2. Fill in the blanks.

(i) A __________ can be either true or false but not both.
(ii) Algebra is a __________ form of the arithmetic.
(iii) The multiplying factor of a variable is called its __________.
(iv) The parts of an algebraic expression, separated by the signs + and –, are called __________ .
(v) The terms of the same kind only differ by their coefficients are called __________ terms.

3. Tick (✓) the correct answer.

(i) In $4x^2$, 2 is known as:
(a) base (b) coefficient (c) exponent (d) term

(ii) If, $a = 1$, $b = -1$ and $c = 1$, then $\frac{a^2+b^2+c^2}{3}=?$
(a) 1 (b) 2 (c) $\frac{1}{3}$ (d) $-\frac{1}{3}$

(iii) In x + 2, 2 is known as:
(a) coefficient (b) constant (c) variable (d) exponent

(iv) $x^2+x^2+x+x=?$
(a) x^2 (b) x^2+x^2 (c) $2(x^2+x)$ (d) $(x^2+x)^2$

Unit 9

LINEAR EQUATIONS

Student Learning Outcomes

After studying this unit, students will be able to:

- Define an algebraic equation.
- Differentiate between equation and an expression.
- Define linear equation in one variable.
- Construct linear expression and linear equation in one variable.
- Solve simple linear equations involving fractional and decimal coefficients like $\frac{1}{2}x + 5 = x - \frac{1}{3}$.
- Solve real life problems involving linear equations.

9.1 Algebraic Equations

An open mathematical statement with "=" sign is known as an equation, which means the value of one side is equal to the value of the other side of the statement.
The weighing balance is an excellent example of an equation that we can observe in our daily life, in which:

(i) the two pans of the balance can be considered as two sides of the equation.
(ii) equality sign "=" indicates that the two scale pans are in balance.
(iii) when we subtract, multiply, divide or add a number on both sides of an equation, the equation remains in balance.

Suppose that the weight of one pan is $x+2$ and weight of 2nd pan is 3 if the weights in two pans are equal then the equation will be $x+2=3$

9.2 Linear Equation in one variable

From the above example, we can see that the equation $x+2=3$ contains a single variable with the power of 1. Such types of equations are called the linear equations.

"The equation which contains a single variable with exponent 1 is called the linear equation in one variable", i.e,

$$ax + b = 0$$

9.2.1 Solving an Equation

We can use the idea of a balance to find the value of unknowns. This process or method of finding the values of unknowns is called solving an equation and the value of unknown is called the solution or root of the equation. From the above example,

$$x+2=3 \quad \text{(Subtract 2 from both sides)}$$
$$x+\cancel{2}-\cancel{2}=3-2$$
$$x=1$$

Thus, 1 is the solution or root of the above equation.

Example 2: Find the solution of the following equations and verify the solution.

(i) $\dfrac{x+6}{2}=\dfrac{x+4}{3}$ (ii) $\dfrac{8x+4}{16-4x}=1$

(iii) $\dfrac{1}{2}x+5=x-\dfrac{1}{3}$ (iv) $4x+0.4=5.2$

Solution:

(i) $\dfrac{x+6}{2}=\dfrac{x+4}{3}$

$3(x+6)=2(x+4)$ (Cross multiplication)

$3x+18=2x+8$

$3x-2x=8-18$ (Separate the variables and numbers)

$x=-10$

Verification:

$\dfrac{-10+6}{2}=\dfrac{-10+4}{3}$

$\dfrac{-4}{2}=\dfrac{-6}{3}$

$-2=-2$

L.H.S. = R.H.S.

(ii) $\dfrac{8x+4}{16-4x} = 1$

$8x + 4 = 16 - 4x$ (Cross multiplication)

$8x + 4x = 16 - 4$ (Separate the variables and numbers)

$12x = 12$

$x = \dfrac{12}{12} = 1$

Verification:-

$\dfrac{(8\times1)+4}{16-4(1)}$

$\dfrac{8+4}{16-4} = \dfrac{12}{12} = 1$

L.H.S. = R.H.S.

(iii) $\dfrac{1}{2}x + 5 = x - \dfrac{1}{3}$

Multiply both sides by 6

$6\left(\dfrac{1}{2}x + 5\right) = 6\left(x - \dfrac{1}{3}\right)$

$\overset{3}{\cancel{6}}\left(\dfrac{1}{\cancel{2}}x\right) + 6(5) = 6(x) - \overset{2}{\cancel{6}}\left(\dfrac{1}{\cancel{3}}\right)$

$3x + 30 = 6x - 2$

$6x - 3x = 30 + 2$

$3x = 32$

$x = \dfrac{32}{3}$

Verification:-

$\dfrac{1}{\cancel{2}}\left(\dfrac{\overset{16}{\cancel{32}}}{3}\right) + 5 = \dfrac{32}{3} - \dfrac{1}{3}$

$\dfrac{16}{3} + \dfrac{5}{1} = \dfrac{32}{3} - \dfrac{1}{3}$

$\dfrac{16+15}{3} = \dfrac{32-1}{3}$

$\dfrac{31}{3} = \dfrac{31}{3}$

L.H.S = R.H.S

(iv) $4x + 0.4 = 5.2$

$4x = 5.2 - 0.4$

$4x = 4.8$

$x = \dfrac{\overset{1.2}{\cancel{4.8}}}{\cancel{4}}$

$x = 1.2$

Verification:-

$4(1.2) + 0.4 = 5.2$

$4.8 + 0.4 = 5.2$

$5.2 = 5.2$

L.H.S = R.H.S

EXERCISE 9.1

1. Write an equation for each of the followings.
   (i) The sum of a number and 8 is 14.
   (ii) The difference of a number and 7 is 9.
   (iii) Twice of a number is 16.
   (iv) One-third of a number is 2.
   (v) A number increased by 2 is 4.
   (vi) A number decreased by 4 is 3.
   (vii) Twice of a number increased by 3 is 17.
   (viii) My age and my brother's age is 20 years by adding.
   (ix) Twice of my age increased by 7 years is my mother's age.
   (x) The price of 6 pens is equal to the price of one book.
2. Solve the following equations and verify the solution.
   (i) $2 + 5x = x$
   (ii) $3a - 3 = 0$
   (iii) $x - 3 = 5$
   (iv) $2x + 2 = 14$
   (v) $12x = 36$
   (vi) $\dfrac{x}{6} = 3$
   (vii) $x + 2 = 2x - 1$
   (viii) $\dfrac{2y}{3} = -8$
   (ix) $x + 4 + x - 2 = 0$

(x) $m + 1 + 2m + 5 = 0$ (xi) $2x - 4 = x$ (xii) $\frac{x}{2} + \frac{3x}{2} + \frac{x}{2} + \frac{5x}{2} = 25$

(xiii) $\frac{x}{2} = \frac{7}{2}$ (xiv) $2m - 5 = \frac{1}{2} + \frac{5}{2}$ (xv) $\frac{a}{2} + \frac{a}{2} = 3a + 14$

(xvi) $\frac{6a-4}{2a+2} = 2$ (xvii) $0.9x - 3 = 6$ (xviii) $0.1x + 2.5 = 3$

(xix) $1.5x + 4 = 7$ (xx) $0.25x + 1.5 = 7.5$ (xxi) $\frac{3x - 1.5}{0.9 - 1.5x} = 0$

9.2.2 Word Problem

There are four important steps for solving a problem by using an equation.

(i) What is the required thing?

(ii) Represent the required thing by a variable.

(iii) Write an equation according to the statement.

(iv) Solve the equation and check the solution.

Example 1: Ahsan and Shakeel bought a time clock for Rs. 60. Ahsan paid Rs. 12 more than Shakeel. What amount did Ahsan pay?

Solution: Suppose Shakeel paid $= x$

Then Ahsan paid $= x + 12$

Sum of two amounts $= x + x + 12$

$= 2x + 12$

But actual sum of the two amounts = Rs.60.

Hence,
$$2x + 12 = 60$$
$$2x = 60 - 12$$
$$2x = 48$$
$$x = \frac{48}{2} = 24$$

But x is the amount of Shakeel. It means Shakeel paid Rs. 24. and Ahsan paid Rs. 12 more than Shakeel. So,

Ahsan's amount $= x + 12 = 24 + 12 = 36$

Hence, Ahsan paid Rs. 36.

Example 2:- The age of Kiran's brother is 4 years less than the twice of his sister's age. What will be the age of Kiran when her brother is 22 years old?

Solution:- Suppose Kiran's age $= x$

Then, her brother's age $= 2x - 4$

But her brother's age $= 22$ years.

So,
$$2x - 4 = 22$$
$$2x = 22 + 4$$
$$2x = 26$$
$$x = \frac{\cancel{26}}{\cancel{2}} = 13$$

Hence, Kiran is 13 years old.

EXERCISE 9.2

1. Find the value of m by putting $n = 2$ in each of the followings.

(i) $2m - n = 12$ (ii) $\frac{m}{n} = \frac{9}{2}$ (iii) $\frac{2m}{n} = 9 - 3m$

(iv) $m = 2n + n + 1$ (v) $2m + n - 2 = 3n + 2n$ (vi) $m + n = mn$

2. The price of a toy gun has decreased by Rs.7. Find the original price if the new price is Rs.18
3. The sum of the two numbers is 12. Find the numbers when the one number is twice of the other. (Hint: Suppose one number is x then the other will be $2x$)
4. The product of the two numbers is 72. Find the other number when the one number is 9.
5. The difference of the two numbers is 6. Find the numbers when the one number is $\frac{1}{4}$th of the other.
6. Sabeena, bought a pen and a book for Rs.45. The book was 8 times more expensive than the pen. What are the prices of the book and the pen?
7. Qasim Hussain opened his account book. He observed that the sum of the page numbers of the two pages, in front of him is 93. Find the page numbers.
(Hint: Suppose one page number is x, then the other will be $x + 1$)
8. Imran Farhat and Abdul Razzaq enhanced 69 runs in the score of Pakistan, if the score of Abdul Razzaq is double than the score of Imran Farhat. How many runs Abdul Razzaq requires to complete his half century?

Summary

- A relationship of equality between two algebraic expressions is called an equation.
- When we add, subtract, multiply or divide a number on both sides of an equation, the equation remains in balance.
- The equation which contains a single variable with the greatest exponent of 1 is called the linear equation in one variable.
- The value of unknown in the equation is called the solution or root of the equation. Variable can be transferred from one side to the other side of the equation by changing its sign.
- There are four steps for solving a problem by using an equation.
  (i) What is the required thing?
  (ii) Represent the required thing by a variable.
  (iii) Write an equation according to the statement.
  (iv) Solve the equation and check the solution.

Review Exercise 9

1. Solve the following equations.

(i) $3x + \frac{2}{5} = 2 - x$ (ii) $\frac{x}{4} + \frac{x}{6} = \frac{x}{2} - \frac{3}{4}$ (iii) $\frac{5x-4}{8} - \frac{x-3}{5} = \frac{x+6}{4}$

(iv) $\frac{2}{3}(x-5) - \frac{1}{4}(x-2) = \frac{-3}{2}$ (v) $3(x-4) - 4(2x+3) = 2(x+5)+1$

(vi) $2(x-2) + 3(x-3) = 3(x-5) - 4(x-8)$

2. If a number is doubled and then increased by 7, it becomes 13. Find the number.
3. The length of a rectangle is 6m larger than three times of its breadth. If its perimeter is 148m, find its length and breadth.
4. The sum of four consecutive numbers is 266. Find the numbers
5. The numerator of a fraction is larger than its denominator by 4. If we add 1 to its denominator, the fraction becomes $\frac{3}{2}$. Find the fraction.

Objective Exercise 9

1. Answer the following questions.

(i) Define an equation. (ii) Which equation is called a linear equation?

(iii) What is meant by solving an equation?

(iv) What are four steps for solving a problem by using an equation?

2. Fill in the blanks.

(i) The weighing balance is an excellent example of an _________ .

(ii) A relationship of _________ between two expression is called an equation.

(iii) The value of unknown of an equation is called _________ of the equation.

(iv) The equation which contains a single variable with the greatest exponent of _________ is called the linear equation.

(v) A number or a variable can be transferred from one side to the other side by changing its sign. This operation is called _________ .

3. Tick (✓) the correct answer.

(i) The solution of the equation x – 1 = – 1 is:

(a) 0 (b) 1 (c) 2 (d) –2

(ii) To write an equation, we use the sign:

(a) + (b) – (c) = (d) ∠

(iii) If, $\frac{x-1}{2} = 1$, then $x = ?$

(a) 0 (b) +1 (c) 2 (d) 3

(iv) The statement “my age is twice the age of my brother’s” can be written in the form of equation as:

(a) $x + y = 2$ (b) $x = 2y$ (c) $\frac{x+y}{2}$ (d) $\frac{x}{2} = \frac{y}{2}$

Unit 10

GEOMETRY

Student Learning Outcomes

After studying this unit, students will be able to:

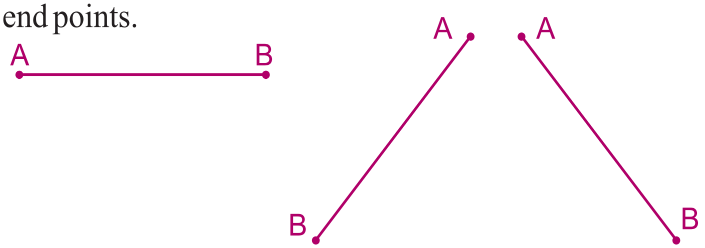
- Add measures of two or more line segments.
- Subtract measure of line segment from a longer one.
- Draw a right bisector of a given line segment using compass.
- Draw a perpendicular to a given line from a point on it using compass.
- Draw a perpendicular to a given line, from a point outside the line, using compass.
- Use compass to:
  - construct an angle equal in measure of a given angle,
  - construct an angle twice in measure of a given angle,
  - bisect a given angle,
  - divide a given angle into four equal angles,
  - construct the following angles: 60°, 30°, 15°, 90°, 45°, $(22\frac{1}{2})^\circ$, 75°, $(67\frac{1}{2})^\circ$, 120°, 150°, 165°, 135°, 105°,
- Construct a triangle when three sides (SSS) are given.
  Caution: Sum of two sides should be greater than the third side.
- Construct a triangle when two sides and their included angle (SAS) are given.
- Construct a triangle when two angles and the included side (ASA) are given.
- Construct a triangle when hypotenuse and one side (RHS) for a right angled triangle are given.

10.1 Introduction

Geometry is an important branch of mathematics which deals with the study of points, lines, surfaces and solids. The word geometry has been deduced from the Greek and Latin words which means, the measurement of the earth or land.

10.1.1 Line Segments

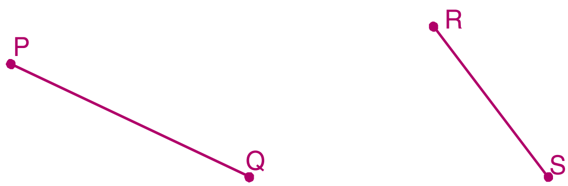
A line segment is one of the basic terms in geometry which help us in the construction of geometrical figures of different shapes and sizes. A line segment is a part of a line which has two distinct end points.



We write the name of a line segment with end points A and B as "line segment AB or $\overline{AB}$". Here we shall learn the method of adding or subtracting the measures of two line segments which is given below.

A line segment of a given length can be drawn by following ways.
- By using a ruler.
- By using a pair of compasses.

10.1.2 Finding the sum of measures of two line segments



In the above figure, $\overline{PQ}$ and $\overline{RS}$ are two line segments. Here we draw a line segment whose length is the sum of the lengths of these line segments.

(i) Draw a ray $\overrightarrow{OE}$. *(By using a rules)*

(ii) Measure the length of line segment $\overline{PQ}$ with the help of a pair of compasses. *(To measure the length of $\overline{PQ}$, place the metal tip of compasses at point P and pencil tip at point Q)*

(iii) Remove the pair of compasses from $\overline{PQ}$ and cut a segment $\overline{OA}$ from the ray $\overrightarrow{OE}$ of the same length. i.e. $m\overline{PQ} = m\overline{OA}$.

(iv) Similarly, measure the length of $\overline{RS}$ and cut another segment $\overline{AB}$ from $\overrightarrow{OE}$. i.e. $m\overline{RS} = m\overline{AB}$ whereas B lies between A and E.

From the above, we can see that:

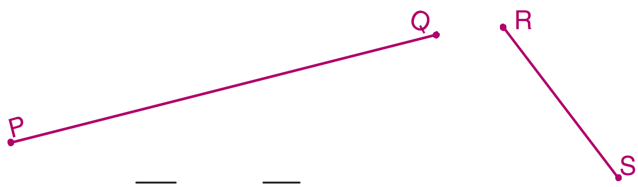
$$m\overline{OB} = m\overline{OA} + m\overline{AB}$$

But we know that, $m\overline{OA} = m\overline{PQ}$ and $m\overline{AB} = m\overline{RS}$. Then,

$$m\overline{OB} = m\overline{PQ} + m\overline{RS}$$

Thus, $\overline{OB}$ is the required line segment.

10.1.3 Finding difference of measures of two line segments



In the above figure, $\overline{PQ}$ and $\overline{RS}$ are two unequal line segments i.e. $m\overline{PQ} > m\overline{RS}$. Now we draw a line segment whose length is the difference of the lengths of two line segments.

(i) Draw a ray $\overrightarrow{OE}$. *(By using a ruler)*

(ii) Measure the length of $\overline{PQ}$ and cut a segment $\overline{OA}$ from the ray $\overrightarrow{OE}$ of the same length. i.e. $m\overline{PQ} = m\overline{OA}$

(iii) Similarly, measure the length of $\overline{RS}$ and cut another segment $\overline{AB}$ from $\overrightarrow{OE}$. i.e. $m\overline{RS} = m\overline{AB}$ but here B lies between O and A.

From the above, we can see that: $m\overline{OB} = m\overline{OA} - m\overline{AB}$

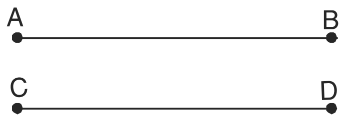
But we know that, $m\overline{OA} = m\overline{PQ}$ and $m\overline{AB} = m\overline{RS}$. Then, $m\overline{OB} = m\overline{PQ} - m\overline{RS}$

Thus, $\overline{OB}$ is the required line segment.

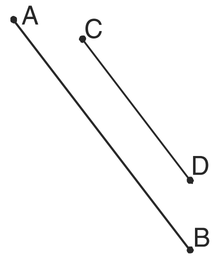
EXERCISE 10.1

1. Draw the line segments to find the sum of measures of the following pairs of line segments.

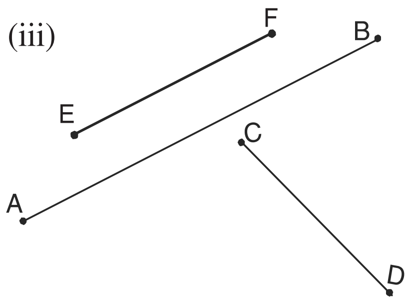
(i)



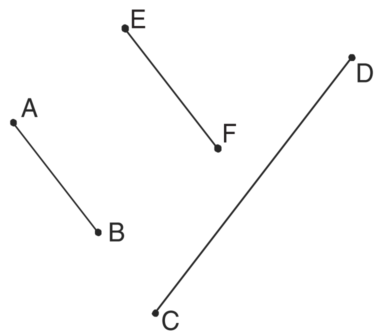
(ii)



(iii)

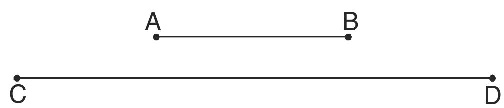


(iv)

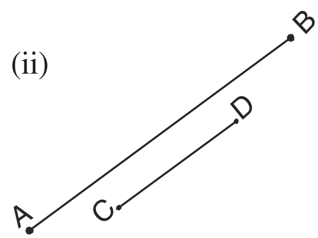


2. Draw the line segments to find the difference of measures of the following pairs of line segments.

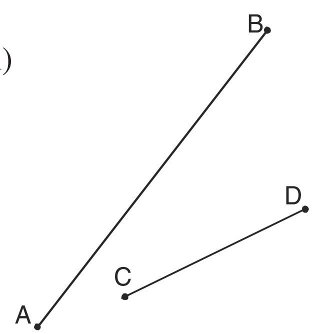
(i)



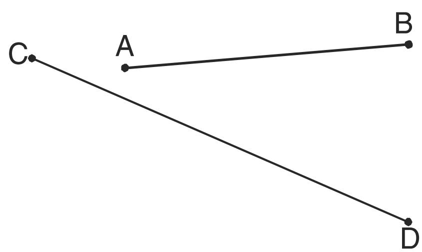
(ii)



(iii)



(iv)



10.1.4 Bisection of a Line Segment

Bi means two and bisection means to divide into two equal sections or parts. Here we shall learn about the construction of a right bisector of a given line by using a pair of compasses and a ruler which is given below.

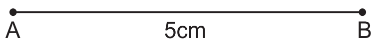
10.1.5 Drawing of a right bisector of a given line segment

A right bisector of a given line segment is a line which is perpendicular to it and passes through its mid-point. The right bisector of a given line segment can be constructed by using a pair of compasses and a ruler as shown in the following example.

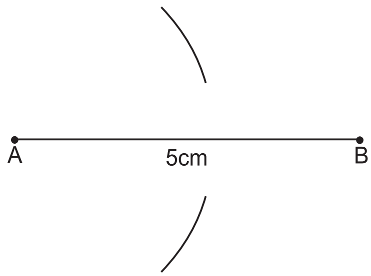
Example1: Draw a right bisector of a 5 cm long line segment.

Solution

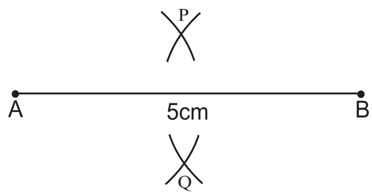
(i) Draw a 5 cm long line segment AB. *(By using a ruler)*



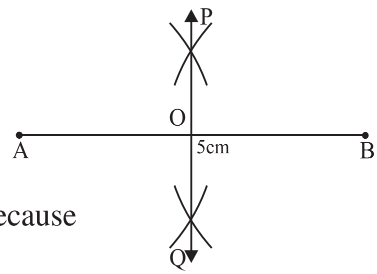
(ii) Consider the point A as centre and draw two arcs of radius more than $\frac{1}{2}\overline{AB}$. i.e. one on each side of $\overline{AB}$. *(By using a pair of compasses)*



(iii) Consider the point B as centre and repeat the process of step (ii). These arcs will cut the previous arcs at any two points P and Q.



(iv) Join the point P and Q by a line (Use a ruler to draw a line that will cut the line segment $\overline{AB}$ at any point O)



Results:

(i) Line segment $\overline{OA}$ and $\overline{OB}$ both have same length.

$\therefore \quad \overline{OA} = \overline{OB}$

(ii) Line $\overleftrightarrow{PQ}$ is perpendicular to the line segment $\overline{AB}$ because

$\angle AOP = \angle BOP = 90^\circ$

$\therefore \quad \overleftrightarrow{PQ} \perp \overline{AB}$

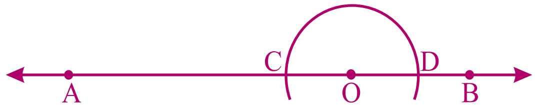
10.1.6 Drawing of a perpendicular to a given line from a point on it

A perpendicular can be drawn to a given line and through a given point on it by using a pair of compasses as shown in the following example.

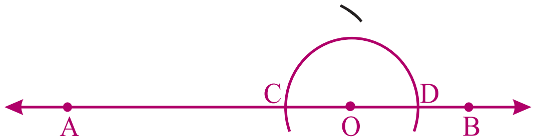
Example 2: Draw a perpendicular on the following line AB and passing through the given point O.

Solution:

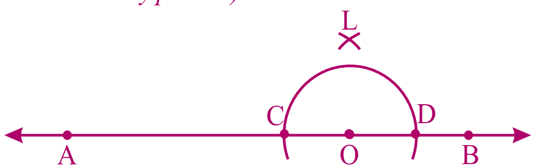
(i) Consider the point O as centre and draw an arc of suitable radius that will cut the line AB at any two points C and D respectively. i.e m$\overline{OC}$ = m$\overline{OD}$



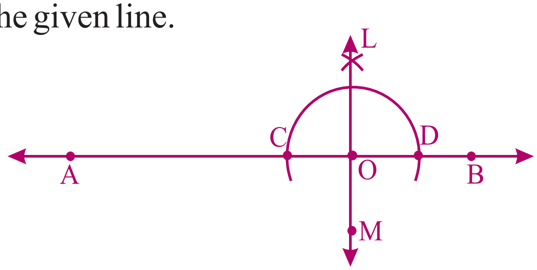
(ii) Consider the point C as centre and draw an arc of radius more than $\overline{OC}$ as given below.



(iii) Now consider the point D as centre and repeat the last process. *(This arc will cut the previous arc at any point L)*



(iv) Draw a line passing through the points O and L that will give a perpendicular to the given line.



The line LM is the required perpendicular.

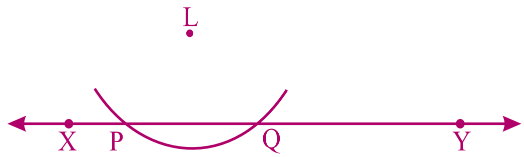
10.1.7 Drawing of a perpendicular to a given line, from a point outside the line

Now we learn a method to draw a perpendicular to a given line form a point outside the line by using a pair of compasses and a ruler.

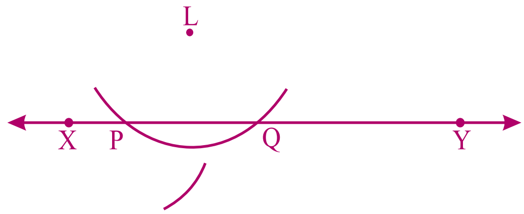
Example 3: Draw a perpendicular from the point L to the given line XY.

Solution:

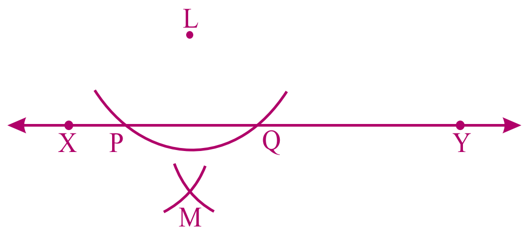
(i) Consider the point L as centre and draw an arc of a suitable radius that will cut the line $\overleftrightarrow{XY}$ at any two points P and Q respectively.



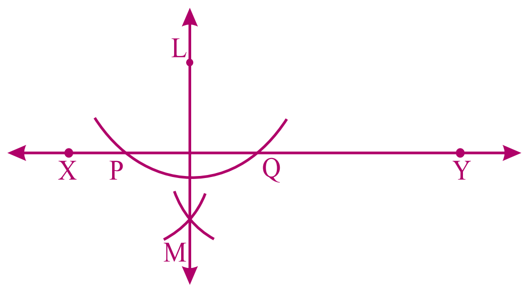
(ii) Consider the point P as centre and draw an arc of a suitable radius as shown in the opposite figure.



(iii) Now consider the point Q as centre and repeat the same process of step (ii). This arc will cut the previous arc at any point M.



(iv) Join the point L and M. *(By using a ruler)*



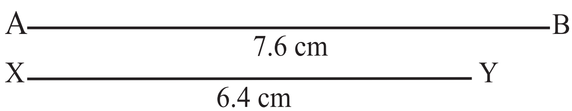
Result: The line LM is the required perpendicular.

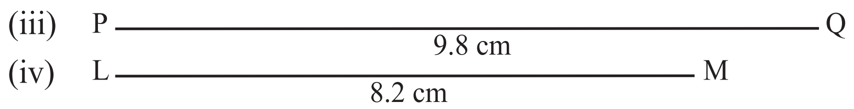
A right bisector always crosses the line at a right angle (90^{O}) that can be checked by using a protractor in above cases.

EXERCISE 10.2

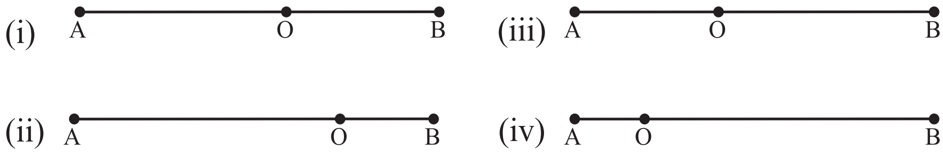
1. Draw the right bisectors of the following line segments by using a pair of compasses.

(i)
(ii)

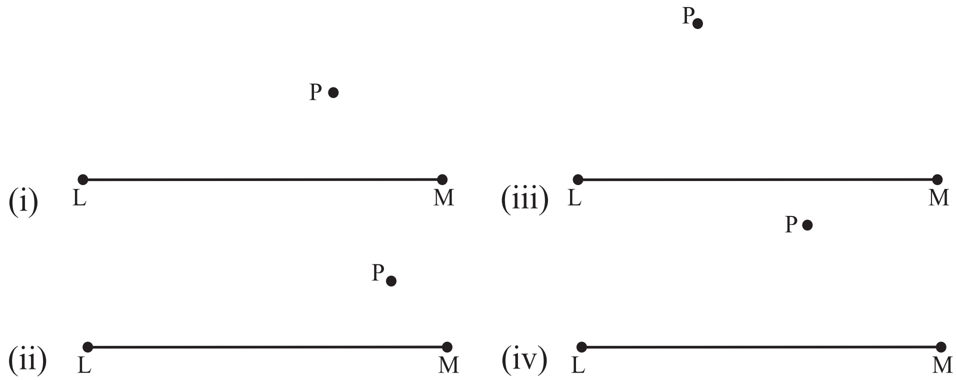




2. Draw perpendiculars from the point O to the line segment AB.



3. Draw perpendiculars from the point P to the line segment LM.



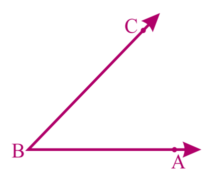
10.2 Construction of Angles

We are familiar with the method of constructing an angle by using the protractor. Now we learn the construction of an angle by using a pair of compasses.

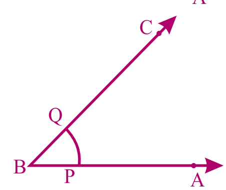
10.2.1 Construction of Congruent Angles

Example 1: Construct an angle of 45° by using a protractor and then construct a congruent angle to it by using a pair of compasses.

Solution:



(i) Construct an angle $\angle$ ABC of 45^{o} as given in the opposite figure. *(By using a protractor)*



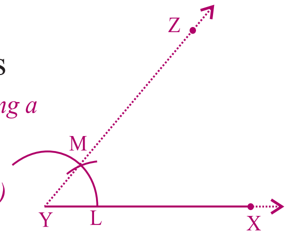
(ii) Consider the point B as centre and draw an arc that will cut the ray BA at point P and BC at the point Q as shown in the given diagram. *(By using a pair of compasses)*

(iii) To draw a congruent angle, first draw a ray YX. *(By using a ruler)*

(iv) Now consider the point Y as centre and draw an arc of same radius as above that will cut the ray YX at any point L. *(By using a pair of compasses)*

(v) Consider the point L as centre and draw an arc of radius PQ. That will cut the previous arc at the point M. *(By using a pair of compasses)*

(vi) Draw a ray YZ passing through the point M. *(By using a ruler)*



$\angle XYZ$ is the required congruent angle.

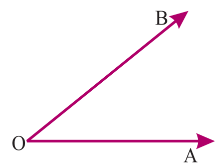
10.2.2 Construction of an Angle Twice in Measurement of a given Angle

By using the given method of construction of congruent angle, we can also construct an angle twice in measurement of a given angle. We learn this method with the help of following example.

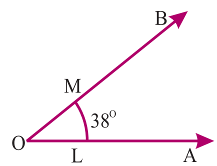
Example 2: Construct the angle of 38° by using a protractor and then construct an angle twice in measurement of it.

Solution:

(i) Construct an angle $\angle AOB$ of 38° as given in the opposite figure. *(By using a protractor)*

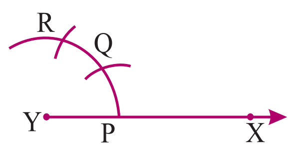


(ii) Consider the point O as centre and draw an arc that will cut the ray OA at point L and OB at the point M as shown in the given diagram. *(By using a pair of compasses)*



(iii) Draw a ray YX. *(By using a ruler)*

(iv) Now consider the point Y as centre of the ray and draw an arc of same radius as drawn in step (iii) that will cut the ray YX at any point P. *(By using a pair of compasses)*

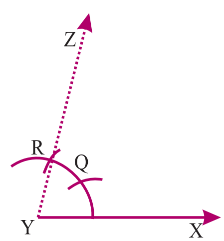


(v) Consider the point P as centre and draw an arc of radius LM. That will cut the previous arc at the point Q *(By using a pair of compasses)*

(vi) Now consider the point Q as centre and draw another arc that will again cut the first arc at point R.

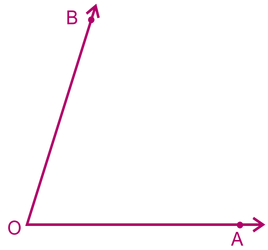
(vii) Draw a ray YZ passing through the point R.

$\angle XYZ$ is the required angle.



10.2.3 Bisection of an angle

We can also divide an angle into the two equal parts that we can observe from the following example.

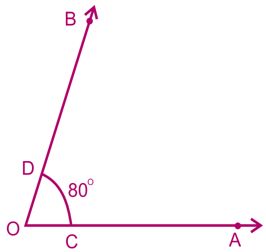


Example 3: Bisect an angle of 80° after construction.

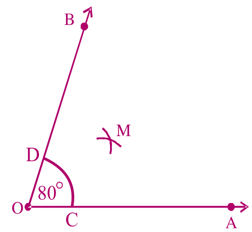
Solution:

(i) Construct an angle $\angle AOB$ of 80° (By using a protractor)

(ii) Draw an arc from the point O. (By using a pair of compasses)



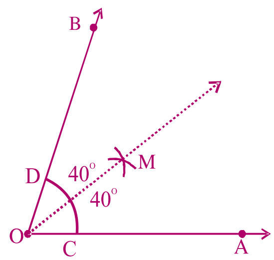
This arc will cut the ray OA at any point C and ray OB at any point D.



(iii) Draw two arcs of same radii by taking the points C and D as centres. (By using a pair of compasses)

These arcs will intersect each other at any point M.

(iv) Join the point O to the point M by a ray OM. (By using a ruler)



The ray OM will cut the angle $\angle AOB$ into the two equal parts.

$$m\angle AOM = m\angle BOM = \frac{80^\circ}{2} = 40^\circ$$

10.2.4 Division of an Angle into Four Equal Angles

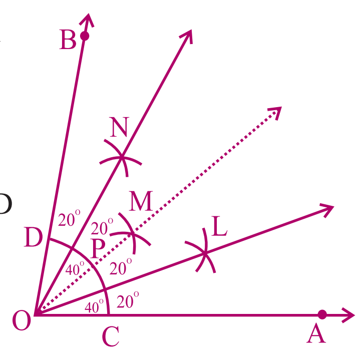
Similarly, we can also divide an angle into four equal angles by further bisecting the already bisected part of an angle as given below.

(i) Draw and bisect the angle according to the above given method.

(ii) Lable the intersecting point of the ray $\overrightarrow{OM}$ as P.

(iii) Draw two arcs of a suitable radii from the points P and C so that they will intersect each other at any point L.

(iv) Again draw two arcs of same radii from the points P and D so that they will intersect at any point N.



The rays $\overrightarrow{OM}$, $\overrightarrow{OL}$ and $\overrightarrow{ON}$ cut the $\angle AOB$ into four equal parts.

$$m\angle AOL = m\angle LOM = m\angle MON = m\angle NOB = \frac{80^\circ}{4} = 20^\circ$$

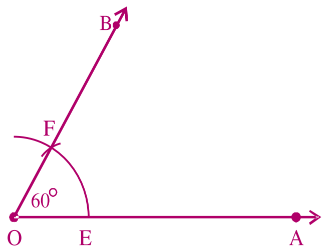
Construction of an angle of 60°

(i) Draw a ray $\overrightarrow{OA}$ (By using a ruler).

(ii) Consider the point O as centre and draw an arc of a suitable radius by using a pair of compasses. This arc will cut the ray $\overrightarrow{OA}$ at point E

(iii) Now consider the point E as centre and draw another arc of same radius which cuts the previous arc at point F. (By using a pair of compasses)

(iv) Draw another ray OB passing through the point F and lable the $m\angle AOB$ of 60°.

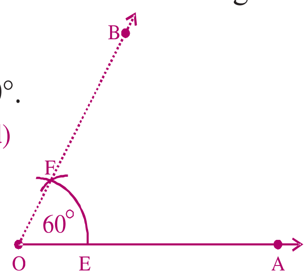


∴ $\angle AOB$ is the required angle of 60°. We can check it by using a protractor.

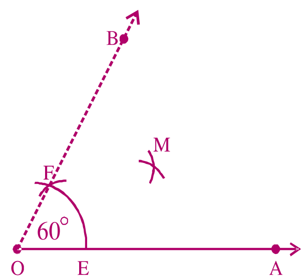
Construction of an angle of 30°

To construct an angle of 30°, we can use the method of bisection of an angle. We have learnt in our previous class that if we divide an angle of 60° into two equal parts, we get the angle of 30°, i.e, $\frac{60^\circ}{2} = 30^\circ$.

(i) Construct an angle of 60°.
(By the above given method)

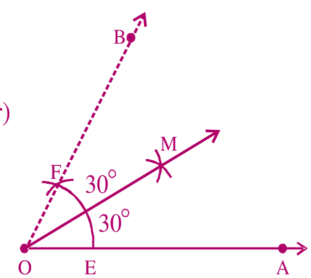


(ii) Draw two arcs of the same radii from the point E and F as centres.
(By using a pair of compasses)



These arcs will intersect each other at any point M.

(iii) Join the point O to the point M by a ray. (By using a ruler)

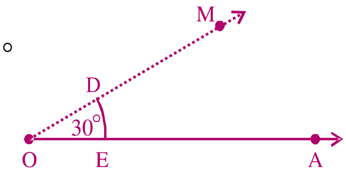


This ray OM will cut the angle $\angle AOB$ into two equal parts.

$$m\angle AOM = m\angle BOM = \frac{60^\circ}{2} = 30^\circ$$

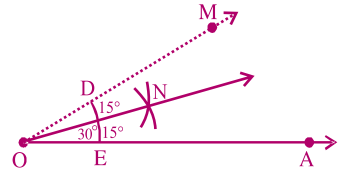
Construction of an angle of 15°

To get the angle of 15°, we bisect the angle of 30°. i.e. $\frac{\overset{15}{\cancel{30}}}{\cancel{2}} = 15°$



(i) Construct the angle of 30°. (By given method)

(ii) Draw two arcs of same radii by taking the points D and E as centres. So that they intersect each other at point N.



(iii) Join the point O to the point N by a ray.

The ray $\overrightarrow{ON}$ will cut the angle into two equal parts.

$$m\angle AON = m\angle MON = \frac{30^\circ}{2} = 15°$$

Construction of an angle of 90°

(i) Draw a ray $\overrightarrow{OA}$ (By using a ruler).

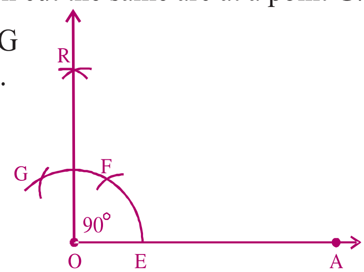
(ii) Consider the point O as centre and draw an arc of a suitable radius by using a pair of compasses. This arc will cut the ray $\overrightarrow{OA}$ at any point E

(iii) Now consider the point E as centre and draw another arc of same radius which cuts the previous arc at point F. (By using a pair of compasses)

(iv) Consider the point F as centre and draw one more arc which cut the same arc at a point G.

(v) Draw two arcs of the same radii by taking the point F and G as centres. These arcs will intersect each other at any point R.

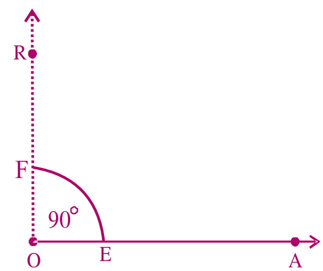
(vi) Join the point O to the point R by a ray.



The angle $\angle AOR$ is the required angle of 90°.

Construction of an angle of 45°

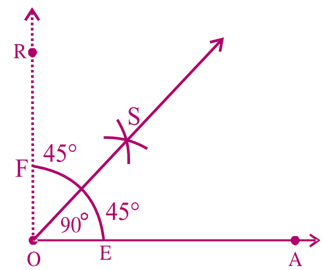
To get the angle of 45°, we bisect the angle of 90°. i.e. $\frac{90^\circ}{2} = 45°$



(i) Construct the angle of 90°. (by given method)

(ii) Draw two arcs of same radii by taking the point E and point F as centres. These arcs will intersect each other at any point S.

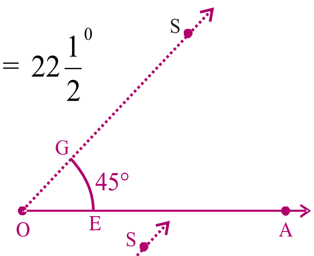
(iii) Join the point O to the point S by a ray.



The ray $\overrightarrow{OS}$ will cut the angle $\angle AOR$ into two equal parts. i.e.

$$m\angle AOS = m\angle SOR = \frac{90^0}{2} = 45°$$

Construction of an angle of $22\frac{1}{2}^0$

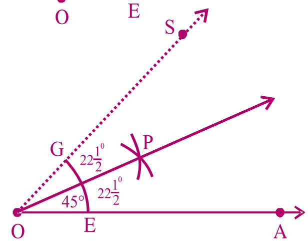


We bisect the angle of 45° to construct the angle of $22\frac{1}{2}$. i.e. $\frac{45^0}{2} = 22\frac{1}{2}^0$

(i) Construct the angle of 45°. (by given method)

(ii) Draw two arcs of same radii by taking the point E and G as centres such that they cut each other at point P.

(iii) Join the point O to the point P by a ray.



The ray $\overrightarrow{OP}$ cut the angle $\angle AOS$ into two equal parts. i.e.

$$m\angle AOP = m\angle POS = \frac{45^0}{2} = 22\frac{1}{2}^0$$

Construction of an angle of 75°

(i) Draw a ray $\overrightarrow{OA}$.

(ii) Take the point O as centre, draw an arc of a suitable radius that will cut the ray $\overrightarrow{OA}$ at any point E.

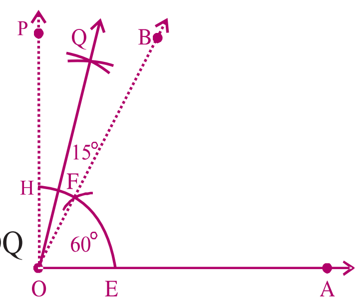
(iii) Take the point E as centre, draw an arc to cut the previous arc at point F that will show an angle of 60°.

(iv) Take the point F as centre, draw an arc to cut the same arc again at point G.

(v) Draw two arcs from the points F and G as centres. That will intersect each other at point P.

(vi) Draw a ray $\overrightarrow{OP}$ that will show an angle of 90°. i.e. $m\angle AOP = 90°$

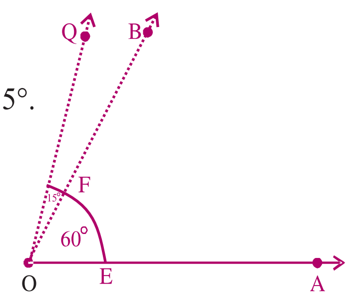
(vii) Draw two arcs by taking the points F and H as centres that intersect each other at point Q.



This is the required angle of 75°. i.e. $m\angle AOQ = \angle AOB + \angle BOQ$

$= 60° + 15° = 75°$

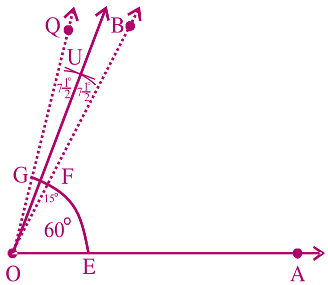
Construction of an angle of $67\frac{1}{2}^0$



(i) Construct an angle of 60° and then its adjacent angle of 15°. (by given method)

(ii) Draw two arcs of same radii by taking points F and G as centres. These arcs will intersect each other at point U.

(iii) Join the point O to the point U by a ray.

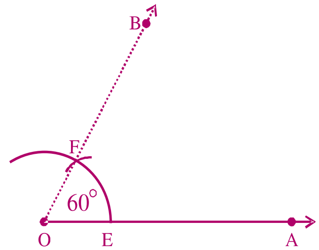


The angle $m\angle AOU$ is the required angle of $67\frac{1}{2}^{0}$ .i.e.
$m\angle AOU = m\angle AOB + m\angle BOU$

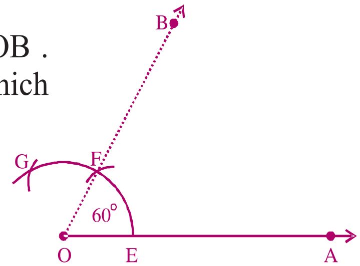
$$= 60° + 7\frac{1}{2}^{0} = 67\frac{1}{2}^{0}$$

Construction of an angle of 120°

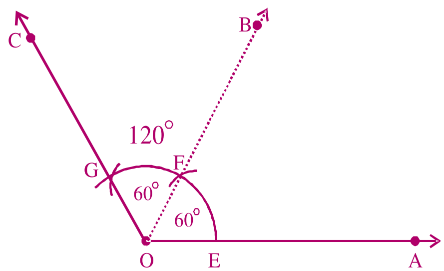
(i) Construct an angle of 60°. (By the given method)



(ii) Now construct another angle of 60° adjacent to $\angle AOB$. Consider the point F as centre and draw one more arc which cuts the previous arc at point G. (By using a pair of compasses)



(iii) Draw another ray $\overrightarrow{OC}$ passing through the point G. you will get the angle $\angle BOC = 60°$.



The two adjacent angles $m\angle AOB = 60°$ and $m\angle BOC = 60°$ give us the angle $m\angle AOC = 120°$.

$$m\angle AOB + m\angle BOC = m\angle AOC$$
$$60° \quad + \quad 60° \quad = 120°$$

Thus, $m\angle AOC$ is the required angle of 120°. We can check it by using a protractor.

Construction of an angle of 150°

(i) Draw a ray OA (By using a ruler).

(ii) Consider the point O as centre and draw an arc of a suitable radius by using a pair of compasses. This arc will cut the ray $\overrightarrow{OA}$ at any point E

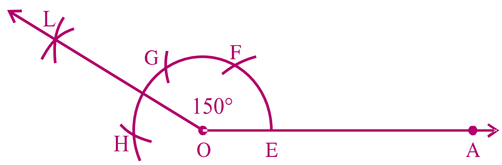
(iii) Now consider the point E as centre and draw another arc of same radius which cuts the pervious arc at point F.

(iv) Now consider the point F as centre and cut the same arc at point G.

(v) Again consider the point G as centre and cut the same arc at point H.

(vi) Draw two arcs from the point G and H as centres that will intersect each other at point L.

(vii) Join the point O to the point L by a ray.



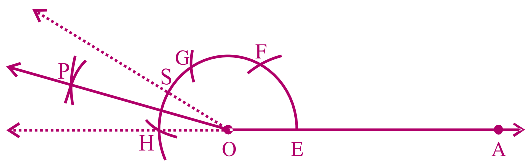
The angle m∠AOL is the required angle of 150°. i.e. m∠AOL = 150°

Construction of an angle of 165°

(i) Construct an angle of 150°. (by given method)

(ii) Draw two arcs from the points S and H that will intersect each other at point P.

(iii) Join the point O to the point P by a ray.



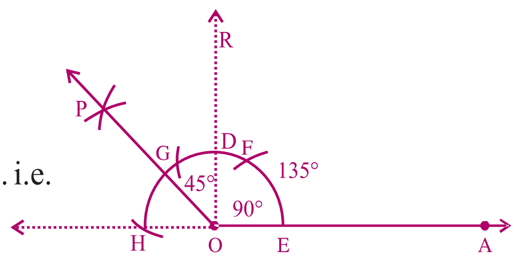
The angle m∠AOP is the required angle of 165°. i.e. m∠AOP = 165°

Construction of an angle of 135°

(i) Construct an angle of 90°. (by given method)

(ii) Take the points D and H as centres to draw two arcs of same radii. These arcs will intersect each other at point P.

(iii) Join the point O to the point P by a ray.



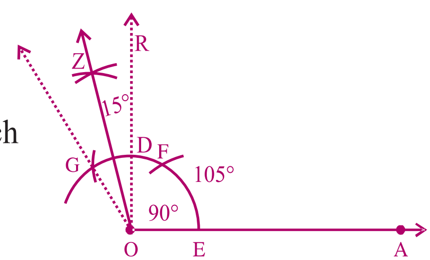
The angle ∠AOP is the required angle of 135°. i.e.
m∠AOP = 90° + 45° = 135°

Construction of an angle of 105°

(i) Construct an angle of 90°. (by given method)

(ii) Take the point D and G as centres to draw two arcs of same radii. These arcs will intersect each other at point Z.

(iii) Join the point O to the point Z by a ray.



The angle ∠AOZ is the required angle of 105°. i.e.
m ∠AOZ = 90° + 15° = 105°

EXERCISE 10.3

1. Construct the congruent angles of the following measurements by using a pair of compasses.

   (i) 35° (ii) 92° (iii) 67° (iv) 56° (v) 118°

2. Draw the twice of the following angles by using a pair of compasses.

   (i) 40° (ii) 75° (iii) 105° (iv) 89° (v) 132°

3. Draw the following angles by using a protractor and bisect them by using a pair of compasses.

   (i) 45° (ii) 120° (iii) 98° (iv) 76° (v) 109°

4. Draw the following angles by using a protractor and divide them into four equal angles.

   (i) 60° (ii) 90° (iii) 180° (iv) 88° (v) 140°

5. Construct the following angles by using a pair of compasses.

   (i) 15° (ii) $(22\frac{1}{2})^\circ$ (iii) $(67\frac{1}{2})^\circ$ (iv) 165° (v) 135°

10.3 Construction of a Triangle

We know that a triangle has six elements, i.e., 3 sides and 3 angles. In construction of a triangle, we do not need all these six elements.

10.3.1 Construction of a triangle when three sides are given

Example: Construct a ΔABC if $m\overline{AB} = 4cm$, $m\overline{BC} = 3cm$ and $m\overline{AC} = 5cm$.

Solution:

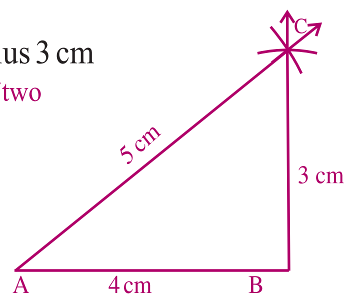
(i) Draw a 4 cm long line segment AB (By using a ruler).

(ii) Consider the point A as centre and draw an arc of radius 5 cm (By using a pair of compasses).

(iii) Now consider the point B as centre and draw an arc of radius 3 cm (This arc will cut the last arc at any point. Name the meeting point of two arcs as C).

(iv) Finally join the point C with the point A and again the point C with the point B.

ΔABC is the required triangle



EXERCISE 10.4

1. Construct the triangle if possible.

(i)	$m\overline{XY} = 5$ cm ,	$m\overline{YZ} = 8$ cm ,	$m\overline{ZX} = 2$ cm
(ii)	$m\overline{AB} = 6$ cm ,	$m\overline{BC} = 4$ cm ,	$m\overline{AC} = 2$ cm
(iii)	$m\overline{BC} = 9$ cm ,	$m\overline{AC} = 12$ cm ,	$m\overline{AB} = 6$ cm
(iv)	$m\overline{LM} = 6.3$ cm ,	$m\overline{MN} = 4.1$ cm ,	$m\overline{LN} = 2.2$ cm
(v)	$m\overline{PQ} = 4.8$ cm ,	$m\overline{QR} = 3.2$ cm ,	$m\overline{RP} = 5.9$ cm

2. Construct the following triangles.

(i)	$m\overline{AB} = 6$ cm ,	$m\overline{BC} = 5$ cm ,	$m\overline{AC} = 4$ cm
(ii)	$m\overline{PQ} = 10$ cm ,	$m\overline{QR} = 7$ cm ,	$m\overline{PR} = 4$ cm
(iii)	$m\overline{DE} = 8$ cm ,	$m\overline{EF} = 9$ cm ,	$m\overline{DF} = 7$ cm
(iv)	$m\overline{XY} = 4.5$ cm ,	$m\overline{YZ} = 5.5$ cm ,	$m\overline{ZX} = 8$ cm
(v)	$m\overline{LM} = 8.8$ cm ,	$m\overline{MN} = 6.6$ cm ,	$m\overline{NL} = 4.4$ cm

10.3.2 Construction of a Triangle when two sides and their included Angle are given

We can construct a triangle when the measurements of two sides and included angle which is formed by two sides are given. For example,

Example: Construct a ΔXYZ if $m\overline{XY} = 5$cm, $m\overline{XZ} = 3$cm and $m\angle YXZ = 75°$.

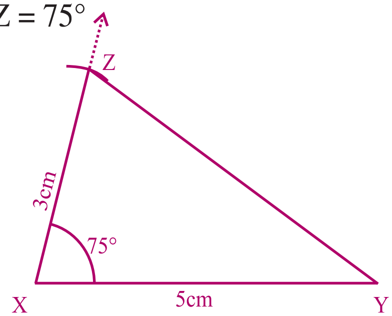
Solution:

(i) Draw a 5 cm long line having end points X and Y .
(By using a ruler)

(ii) Construct an angle of 75° at point X i,e, $m\angle YXZ = 75°$
(Use protractor to draw this angle of 75° that will produce an arm)

(iii) Draw an arc of radius 3cm.
(Use a pair of compasses to draw this arc that will cut the produced arm on the point Z)

(iv) Finally, join the point Z and point Y.
ΔXYZ is the required triangle.



EXERCISE 10.5

1. Construct the following triangles by using a protractor, a pair of compasses and a ruler.

(i)	$m\overline{AB} = 5$ cm ,	$m\overline{BC} = 7$ cm ,	$m\angle ABC = 60°$
(ii)	$m\overline{PQ} = 8.4$ cm ,	$m\overline{PR} = 3.6$ cm ,	$m\angle QPR = 120°$
(iii)	$m\overline{OA} = 8.0$ cm ,	$m\overline{OB} = 6$ cm ,	$m\angle AOB = 90°$
(iv)	$m\overline{LM} = 9$ cm ,	$m\overline{LN} = 6.5$ cm ,	$m\angle MLN = 150°$
(v)	$m\overline{XY} = 5.5$ cm ,	$m\overline{YZ} = 6.6$ cm ,	$m\angle XYZ = 45°$
(vi)	$m\overline{LM} = 6.2$ cm ,	$m\overline{MN} = 4.9$ cm ,	$m\angle LMN = 40°$
(vii)	$m\overline{AB} = 7\ 7$ cm ,	$m\overline{BC} = 6\ 6$ cm ,	$m\angle ABC = 70°$
(viii)	$m\overline{PQ} = 9.2$ cm ,	$m\overline{PR} = 8$ cm, ,	$m\angle QPR = 115°$

2. Draw the following isosceles triangles.

(i) $m\overline{OA} = 5.5$ cm , $m\angle AOB = 30°$ (ii) $m\overline{YX} = 6.3$ cm , $m\angle XYZ = 75°$

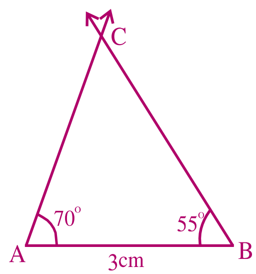
(iii) $m\overline{AB} = 8.3$ cm , $m\angle BAC = 85°$

10.3.3 Construction of a triangle when two angles and included side are given

A triangle can be constructed when we have two angles and their related one side (base of two angles) are given.

Example: Construct the triangle when $\overline{AB} = 3\text{cm}, \angle A = 70°, \angle B = 55°$

Solution:



(i) Draw a 3cm long line segment AB (Use a ruler)

(ii) Construct an angle $\angle A$ of 70° at point A. (Use a protractor)

(iii) Construct an angle $\angle B$ of 55° at point B. (Use a protractor)

(iv) These two angles produce two arms which intersect each other at any point C.

ΔABC is the required triangle.

EXERCISE 10.6

1. Construct the ΔXYZ, when

   (i) $m\overline{XY} = 5$ cm (ii) $m\angle X = 60°$ (iii) $m\angle Y = 30°$.

2. Construct the ΔABC, when

   (i) $m\overline{BC} = 7$ cm (ii) $m\angle B = 45°$ (iii) $m\angle C = 90°$.

3. Construct the ΔPQR, when

   (i) $m\overline{PQ} = 6.8$ cm (ii) $m\angle P = 120°$ (iii) $m\angle Q = 45°$.

4. Construct the ΔABC, when

   (i) $m\overline{AB} = 4.9$ cm (ii) $m\angle A = 90°$ (iii) $m\angle B = 60°$.

5. Construct the ΔLMN when

   (i) $m\overline{LM} = 6$ cm (ii) $m\angle L = 50°$ (iii) $m\angle M = 60°$.

6. Construct the ΔRST, when

   (i) $m\overline{RS} = 5.7$ cm (ii) $m\angle R = 45°$ (iii) $m\angle S = 75°$.

7. Construct the ΔAOB, when

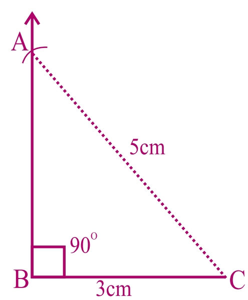
   (i) $m\overline{OA} = 4.5$ cm (ii) $m\angle O = 90°$ (iii) $m\angle A = 30°$.

10.3.4 Construction of a Right-Angled Triangle

We know that a right angled triangle or right triangle is that triangle in which one of its angles is of 90°. This triangle can be constructed, if we have the measurement of hypotenuse and its one side (base or altitude).

Example: Construct a right-angled triangle ΔABC if Hypotenuse = 5 cm and Base = 3 cm.

Solution:



(i) Draw a 3 cm long line segment $\overline{BC}$ (by using a ruler)

(ii) Construct an angle of 90° at the point B that will produce altitude of triangle. (By using a protractor or a pair of compasses)

(iii) Draw an arc of radius 5 cm from the point C. (Use a pair of compasses. This arc will cut the produced altitude at any point A)

(iv) Join the point A and point C.

ΔABC is the required triangle.

EXERCISE 10.7

Construct the right angled triangles for the following.

(i)	Hypotenuse = 8 cm, Base = 4 cm	(ii)	Hypotenuse = 9 cm, Base = 6 cm
(iii)	Hypotenuse = 5 cm, Altitude = 4 cm	(iv)	Hypotenuse = 7.5 cm, Base = 4.5 cm
(v)	Hypotenuse = 6.5 cm, Altitude = 3.5 cm	(vi)	Hypotenuse = 8.2cm, Altitude = 5.8cm

Summary

- The geometry is an important branch of Mathematics which deals with the study of points, lines, surfaces and solids.
- A line segment is a part of a line which has two distinct end points.
- Bisection of a line segment means to divide a line segment into two equal parts.
- A right bisector can be a line, a ray or a line segment which divides another line segment into two equal parts.
- A right bisector always crosses the line at a right angle (90°) that can be checked by using a protractor.
- The two angles of the same measurements are called congruent angles.
- The unit of measuring an angle is the degree which is denoted by "°".
- To construct a triangle with three sides, the sum of the measurements of any two sides is greater than the measurement of 3rd side.
- The sum of interior angles in a triangle is always 180°.
- A right angled triangle can be constructed if we have the measurement of hypotenuse and its one side.

Review Exercise 10

1. Draw two line segments $\overline{AB}$ and $\overline{CD}$ 4.5cm and 3.2cm long respectively. Draw a line segment equal in length to their sum.
2. Draw two line segments $\overline{AB}$ and $\overline{CD}$ 7cm and 2.8cm long respectively. Draw a line segment to find their difference.
3. Draw a line segment $\overline{PQ}$ of length 6cm. Take a point R on it and draw a perpendicular passing through it.
4. Draw a line segment $\overline{LM}$ of length 5cm. Take a point N outside of it and draw a perpendicular on the line passing through the point.

5. Draw the following angles by using a compasses and bisect them.

(i) 60° (ii) 90° (iii) 45°

6. Construct the following triangles.

(i)	$m\overline{AB} = 5cm$,	$m\overline{BO} = 4cm$,	$m\overline{AO} = 3cm$
(ii)	$m\overline{XY} = 6.2cm$,	$m\overline{YZ} = 5.8cm$,	$m\overline{ZX} = 7cm$
(iii)	$m\overline{PQ} = 7cm$,	$m\overline{QR} = 5cm$,	$m\angle Q = 60°$
(iv)	$m\overline{LM} = 4.2cm$,	$m\overline{MN} = 6.4cm$,	$m\angle M = 75°$
(v)	$m\overline{AB} = 7.2cm$	$m\angle A = 65°$	$m\angle B = 35°$

Objective Exercise 10

1. Answer the following questions.
   (i) Write the meaning of the word geometry.
   (ii) What is meant by the right bisector of a line?
   (iii) What are congruent angles?
   (iv) How many elements are required to construct a triangle?
   (v) Define a line segment.

2. Fill in the blanks.
   (i) A __________ is a part of a line which has two distinct end points.
   (ii) The unit of measuring an angle is called __________ .
   (iii) A straight line has an angle of __________ .
   (iv) Only three elements can construct a triangle but one of them must be a __________ .

3. Tick (✓) the correct answer.
   (i) In a line AB, the right bisector passes through its:
   (a) point A (b) point B (c) mid-point (d) none of them
   (ii) Bisection means to divide into parts:
   (a) one (b) two (c) three (d) four
   (iii) A right bisector intersect the line at an angle of:
   (a) 60° (b) 45° (c) 90° (d) 180°
   (iv) The sum of interior angles in a triangle is always:
   (a) 90° (b) 180° (c) 240° (d) 360°

Unit 11

PERIMETER AND AREA

Student Learning Outcomes

After studying this unit, students will be able to:

- Find perimeter and area of a square and a rectangle.
- Find area of path (inside or outside) of a rectangle or square.
- Solve real life problems related to perimeter and area of a square and rectangle.
- Recognize altitude of a geometric figure as the measure of the shortest distance between the base and its top.
- Find area of a parallelogram when altitude and base are given.
- Define trapezium and find its area when altitude and measures of the parallel sides are given.
- Find area of a triangle when measures of the altitude and base are given.

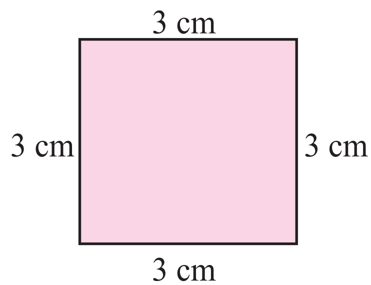
11.1 Perimeter and Area

If someone asks you which thing is bigger in your classroom; a blackboard or a wall clock? Obviously, your answer will be the blackboard because you can observe that the blackboard covers a larger surface on the wall than the wall clock or it has a larger area than the wall clock. Thus, an area can be defined as:

The surface covered by an object in a plane is called area of that object and the measurement of boundary of the surface is called its perimeter.

Here we shall learn the formulae for finding the perimeter and area of a square and a rectangle.

11.1.1 Perimeter and Area of a Square



We know that the given figure is a square with four sides of an equal length. If we add the lengths of these four sides, we get its perimeter as given below.

Perimeter of the square = 3 cm + 3 cm + 3 cm + 3 cm
= 4 × 3 cm = 12 cm

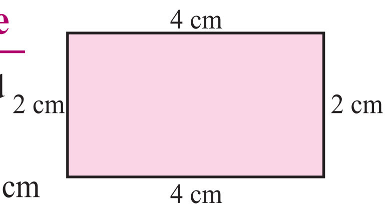
Similarly, the area of given square can also be calculated by finding the product of its length and breadth as:

Area of the square = 3 cm × 3 cm = 9 cm^2

From above, we can conclude the formulae for finding the perimeter and area of a square.

Perimeter of a square = 4 × side   Area of a square = side × side

11.1.2 Perimeter and Area of a Rectangle



The perimeter of the given rectangle can be calculated by adding the measures of its four sides as shown below:

Perimeter of the rectangle = 2 cm + 4 cm + 2 cm + 4 cm
= 4cm + 4cm + 2cm + 2cm
= 8cm + 4cm = 12cm

Similarly, the area of the rectangle can also be calculated by multiplying its length and breadth as:

Area of the rectangle = 2 cm × 4 cm = 8 cm^2

Thus, the formulae for finding the perimeter and area of a rectangle are:
Perimeter of rectangle = 2(length + breadth) , Area of a rectangle = length × breadth
To find the area of a rectangle, the units of length and breadth must be the same.

Example 1: Find the area of a rectangular field whose length is 54 metres and width is 32 metres.

Solution: Length of the field = 54 metres , Width of the field = 32 metres

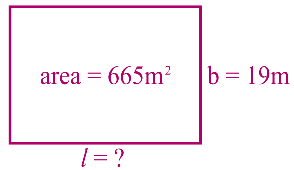
Area of the field = ?

Area of the field = length $\times$ width $= 54\text{m} \times 32\text{m}$

$= 1728\text{m}^2$

Example 2: Determine the length of a rectangular park whose breadth is 19 metre and area is 665 sq. metres.

Solution:



Breadth of the park= 19 metres

Area of the park = 665 sq. metres

Length of the park = ?

Area = length $\times$ breadth

$$\text{or Length} = \frac{\text{area}}{\text{breadth}} = \frac{\cancel{665}^{35}\text{m}^2}{\cancel{19}\text{m}} = 35\text{m}$$

Example 3: A pond is 28.5m long and 16m wide. Find the cost of cementing the floor of the pond at the rate of Rs.110/m^2(Rs.110 per square metre) and cost of fencing at the rate of Rs. 95/m.

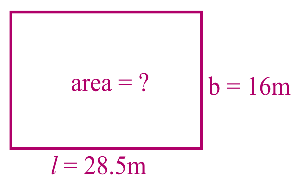
Solution:

Length of the pond = 28.5m, Width of the pond = 16m

Cost of cementing = ? Cost of fencing = ?

Area = length $\times$ breadth

$= 28.5\text{m} \times 16\text{m} = 456\text{m}^2$



Cost of cementing 1m^2 = Rs. 110

Cost of cementing 456m^2 = Rs. (110 $\times$ 456) = Rs. 50,160

Perimeter = 2 (length + breadth)

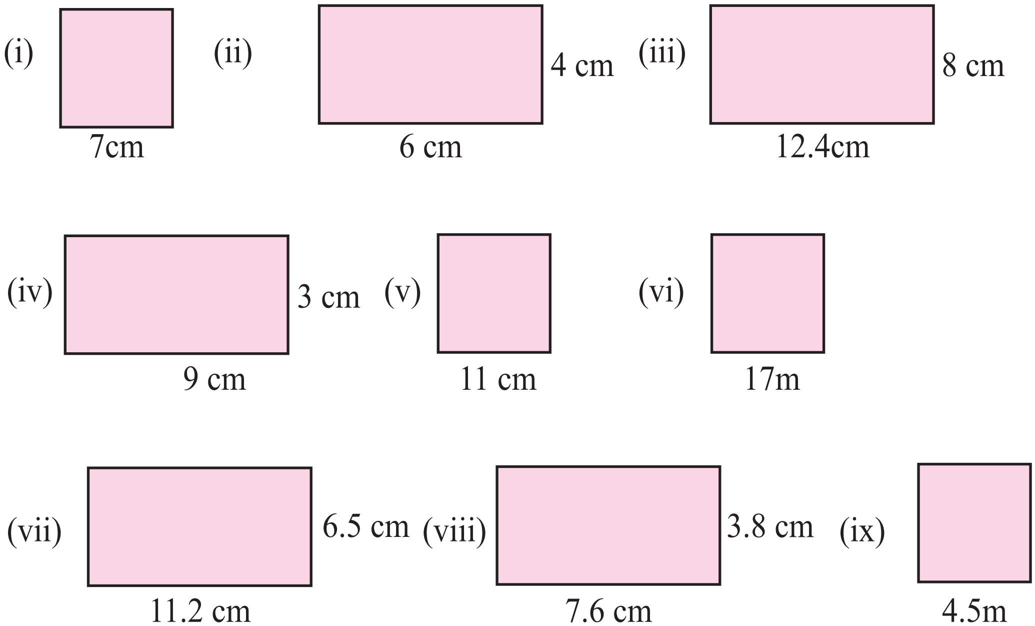
= 2 (28.5m + 16m) = 2 (44.5m) = 89m

Cost of fencing 1m = Rs. 95

Cost of fencing 89m = Rs. (95 $\times$ 89) = Rs. 8455

EXERCISE 11.1

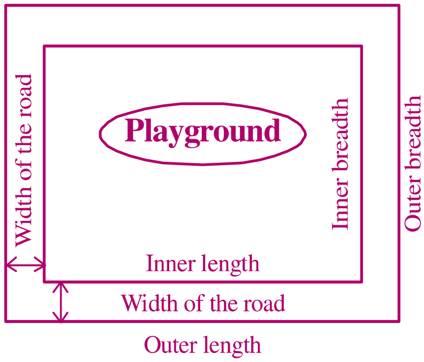
1. Find the area and perimeter of the following squares and rectangles.



2. Find the length of a rectangular park whose breadth is 15m and area is 675m^2.
3. The perimeter of a square garden is 12 km. Find its area.
4. Find the breadth of a swimming pool whose length is 18m and area is 198m^2.
5. Find the area and perimeter of a square shaped garden whose measure of side is 21m.
6. The perimeter of a square shaped room is 36m. Find the cost of tiling the room at the rate of Rs. 182.5 per sq. metre.
7. Find the cost of leveling a playground at the rate of Rs. 150/m^2 whose length is 33m and breadth is 22m. Also find the cost of fencing the playground at the rate of Rs. 100/m.
8. The length of a side of a square shaped field is 48m. Find the cost of ploughing the field at the rate of Rs. 25/m^2 and cost of fencing the field at the rate of Rs. 18/m.
9. A garden is 45m long and 30m wide. Find the cost of repairing the garden at the rate of Rs. 50/m^2 and cost of constructing a wall around it at the rate of Rs. 425/m.

11.1.3 Border Area of a Rectangle

Consider a rectangular shaped playground which has a road all around of it as given in the figure. The area of this road can be calculated by subtracting the area of the small rectangle *(area of portion without road)* from the area of the large rectangle *(area of portion with road)* as shown in the figure.



$\therefore$ Area of a road = Area of a large rectangle − Area of a small rectangle

or we can write it as:

$\therefore$ Area = (Outer length × Outer breadth) – (Inner length × Inner breadth)

From the above figure, we can also observe that if we have length and breadth of one rectangle with width of the road or border surrounding the playground, then we can calculate the length and breadth of the other rectangle. It can be done by adding or subtracting the twice of width of the road.

Length of the large rectangle=Length of the small rectangle+2[Width of the road]

Breadth of the large rectangle=Breadth of the small rectangle+2[Width of the road]

Example: A children park is 35 m wide and 60 m long and has a road of 2.5 m width on its outer side. Find the cost of repairing the road at the rate of Rs.80/m^2.

Solution:

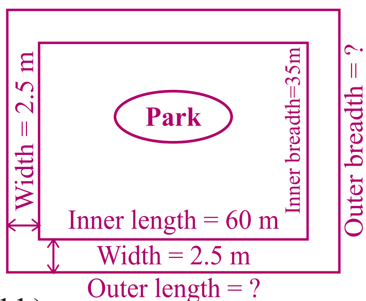
Length of the park = 60 m

Breadth of the park = 35 m

Width of the road = 2.5 m

(i) Area of the road =?

(ii) Cost of repairing =?



(a) Area of children park = (Inner length × Inner breadth)
(Excluding the road)

= 60 m × 35 m

= 2100 m^2

(b) Area of children park = (Outer length × Outer breadth)
(Including the road)

First we find outer length and breadth

Outer length = Inner length + Twice of width of the road

$= 60\text{ m} + 2\times2.5\text{ m}$

$= 60\text{ m} + 5\text{ m} = 65\text{ m}$

Outer breadth = Inner breadth + Twice of width of the road

$= 35\text{ m} + 2\times2.5\text{ m}$

$= 35 + 5\text{ m} = 40\text{ m}$

Area of children park $= 65\text{ m} \times 40\text{ m} = 2600\text{ m}^2$
(including the road)

Now we can calculate the area of the road.

(i) Area of the road = [Area of the large rectangle] – [Area of the small rectangle]

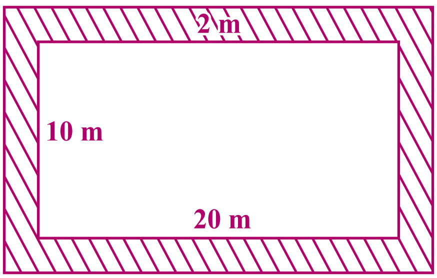
$= 2600\text{m}^2 - 2100\text{m}^2 = 500\text{m}^2$

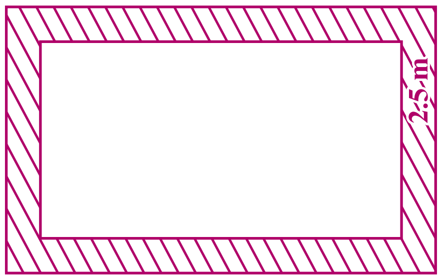
(ii) Cost of repairing of 1m^2 = Rs.80

Cost of repairing of 500 m^2 = Rs.80×500 = Rs.40,000

EXERCISE 11.2

1. Find the area of the border (shaded portion) from the following figures.





2. Find the area of the following borders.

   (i) Inner length = 100 m, Inner breadth = 50 m, Width of the border = 2 m

   (ii) Inner length = 120m, Inner breadth = 70 m, Width of the border = 3 m

   (iii) Outer length = 80 m, Outer breadth = 45 m, Width of the border = 4 m

   (iv) Outer length = 96 m, Outer breadth = 50 m, Width of the border = 3.5 m

3. Find the area of a 4 m wide running track constructed inside of a park, when the length and breadth of the park are 150 m and 80 m respectively.

4. A room is 8 m long and 5 m wide. Find the cost of the flooring tiles at the rate of Rs.40/m² that used in verandah of 1.5 m width which surrounded the room.

5. A 3 m wide road around the garden of 125 m long and 60 m wide is repaired. Find the cost of repairing the road at the rate of Rs.150/m².

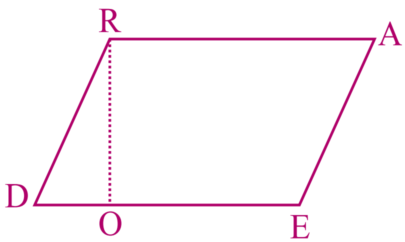
11.1.4 Parallelogram

A plane figure with four straight sides whose opposite sides are parallel and of equal lengths and height is fixed is called a parallelogram.
In a parallelogram, if we draw a perpendicular from any point of its opposite side, then this perpendicular is called height and opposite side is called base. A parallelogram can be constructed if two adjacent sides and their included angles/ diagonals / height are known.

Area of a Parallelogram

To find the area of a parallelogram, we consider a parallelogram DEAR and draw a perpendicular from point R to its opposite side $\overline{DE}$. We get a triangle ΔROD as given in figure (1).

Figure (1)



Now cut this triangle ΔROD and paste it with side $\overline{AE}$ such as given in figure (2). In this way, we get a rectangle ROLA from the parallelogram DEAR.

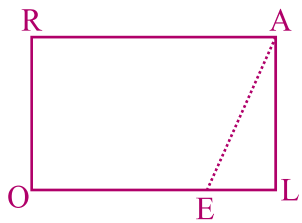


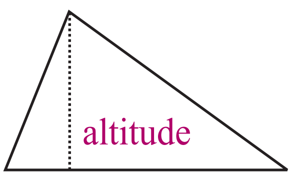
Figure (2)

It means, the area of a rectangle and area of a parallelogram are same and we can use the same formula to find the area of a parallelogram as we use for a rectangle.

Thus, area of a rectangle = Area of a parallelogram
length × breadth = height × base

Altitude or height of a geometric figure

Altitude of a geometric figure is the shortest distance from its top corner to its opposite base as shown below.

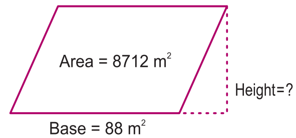


Example: Find the base of a parallelogram when its height is 88 m and the area is 8712 m^2.

Solution:

We know that

Area of a parallelogram = Base × Height



or $$\text{Base} = \frac{\text{Area}}{\text{Height}} = \frac{\overset{99m}{\cancel{8712}m^2}}{\cancel{88}m} = 99m$$

Base = 99m.

EXERCISE 11.3

1. Find the area of a parallelogram shaped pool whose base is 17m and height is 9m.
2. Find the height of a parallelogram shaped hall when its base is 12m and the area of the hall is 216m².
3. Find the area of a parallelogram whose base is 75m and height is 50m.
4. Find the height of a parallelogram whose base is 27m and area is 405m².
5. Find the base of a parallelogram when its height is 16m and area is 560m².
6. Find the cost of levelling a plot of 200m base and 140m height at the rate of Rs. 4.50/m².
7. Find the cost of ploughing a parallelogram shaped field at the rate of Rs. 6/m² whose base is 175m and height is 125m.
8. The height of a parallelogram floor is 25.8m and base is 36.5m. Find the cost of tiles at the rate of Rs. 460/m² that will be used on the floor.

11.1.5 Area of a Trapezium

A trapezium is a quadrilateral with only two parallel sides. To find the area of a trapezium, we consider a trapezium ABCD with two parallel sides AB and CD as shown in the figure (a).

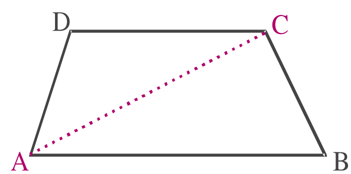


Figure (a)

Now join the point A and point C with a dotted line such that it divides the trapezium ABCD into two triangles ΔACD and ΔABC. Hence we can calculate the area of a given trapezium by finding the area of two triangles as given below.

Area of trapezium ABCD = Area of ΔACD + Area of ΔABC

To calculate the area of a ΔACD, draw a perpendicular line from point A to the extended line $\overline{CD}$ such that both lines meet at a point E as shown in the figure (b).

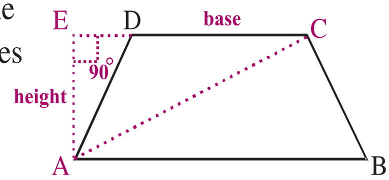


Figure (b)

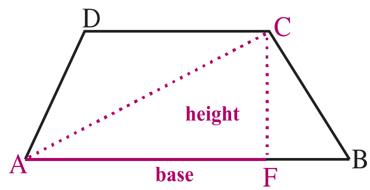
In our previous class we have learnt the formula for finding the area of a triangle. Here we use the same formula.

$$\text{Area of a } \Delta ACD = \frac{1}{2}(\text{base} \times \text{height})$$

$$= \frac{1}{2}(\overline{CD} \times \overline{AE})$$

Similarly, to find the area of a ΔABC, draw a perpendicular line from the point C that meet the line segment $\overline{AB}$ at a point F as given in the figure (c).

Figure (c)



So, area of a $\Delta ABC \quad = \frac{1}{2}(\text{base} \times \text{height}) \quad = \frac{1}{2}(\overline{AB} \times \overline{CF})$

We know that,

Area of a trapezium ABCD = Area of a ΔACD + Area of a ΔABC

$$= \frac{1}{2}(\overline{CD} \times \overline{AE}) \quad + \frac{1}{2}\ (\overline{AB} \times \overline{CF})$$

But from the figure (b) and figure (c) we can examine that,

$$m\overline{AE} = m\overline{CF}$$

So, Area of a trapezium ABCD$= \frac{1}{2}(\overline{CD} \times \overline{AE}) + \frac{1}{2}(\overline{AB} \times \overline{AE})$

$$= \frac{1}{2}\overline{AE}\ (\overline{CD} + \overline{AB})$$

Where, $\quad m\overline{AE}$ = height or perpendicular distance

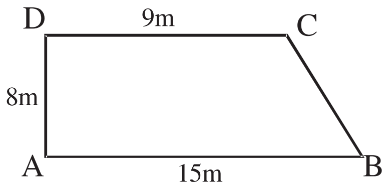
And $m\overline{CD} + m\overline{AB}$ = sum of two parallel sides

Thus, in general we can write the above statement as:

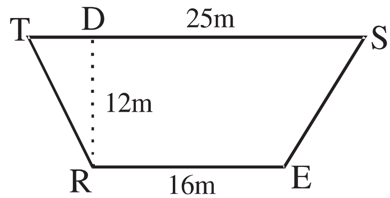
Area of trapezium $= \frac{1}{2}$ [perpendicular distance× sum of lengths of parallel sides]

Example 1: Calculate the area of each of the following trapeziums.

(i)

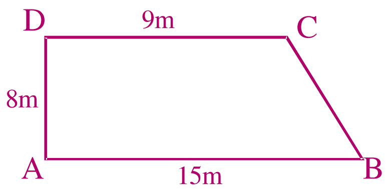


(ii)



Solution:

(i)



In the above figure we can examine that,

Perpendicular distance $= 8m$

Length of 1st parallel side $= 9m$

Length of 2nd parallel side $= 15m$

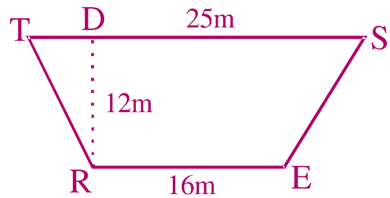
By using a formula,

Area of a trapezium $= \frac{1}{2}$ [perpendicular distance $\times$ sum of lengths of parallel sides]

$$= \frac{1}{\cancel{2}}[\cancel{8m}^{4m} \times (9m + 15m)]$$

$$= [4m \times 24m] = 96m^2$$

(ii)



From the given figure we can see that,

Perpendicular distance $= 12m$

Length of 1st parallel side $= 16m$

Length of 2nd parallel side $= 25m$

Area of a trapezium $= \frac{1}{2}$[perpendicular distance $\times$ sum of lengths of parallel sides]

$$= \frac{1}{\cancel{2}}[\cancel{12m}^{6m} \times (16m + 25m)]$$

$$= [6m \times 41m] = 246m^2$$

Example 2: Find the cost of carpeting of a trapezium shaped floor at the rate of Rs.90/m^2. Where the lengths of parallel sides are 62m and 48m respectively and distance between them is 50m.

Solution: Perpendicular distance $= 50m$
Length of 1st parallel side $= 48m$
Length of 2nd parallel side $= 62m$

Area $= \frac{1}{2}$ [perpendicular distance $\times$ sum of lengths of two parallel sides]

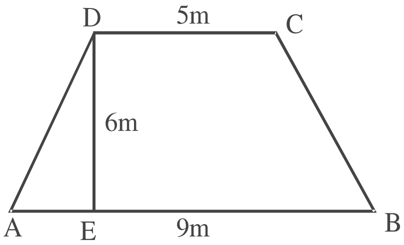
$$= \frac{1}{\cancel{2}}[\cancel{50m}^{25m} \times (48m + 62m)] = 2750m^2$$

Cost of carpeting of $1m^2$ $=$ Rs.90
Cost of carpeting of $2{,}750m^2$ $=$ Rs.(90×2750)
$=$ Rs. 247,500.

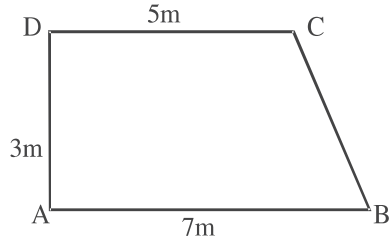
EXERCISE 11.4

1. Calculate the area of each of the following trapezium ABCD:

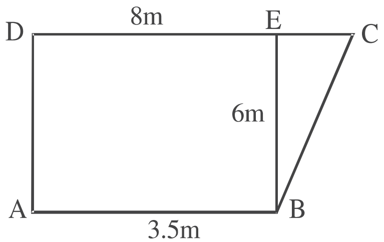
(i)



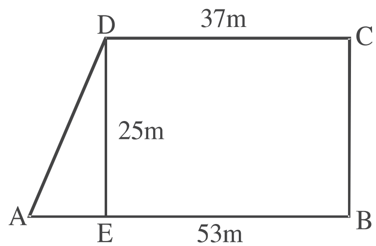
(ii)



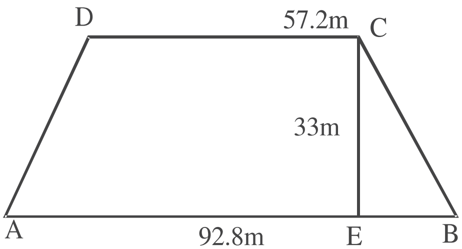
(iii)



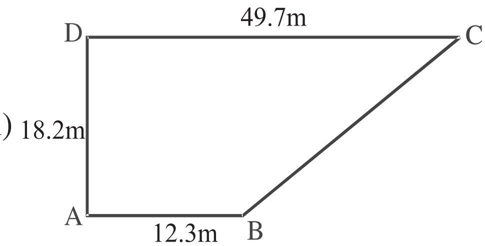
(iv)



(v)



(vi)



2. Find the area of a trapezium whose lengths of its parallel sides are 19m and 24m respectively and distance between them is 14m.

3. A trapezium has 20m and 35m as lengths of two parallel sides and its perpendicular distance is 16m. Calculate its area.

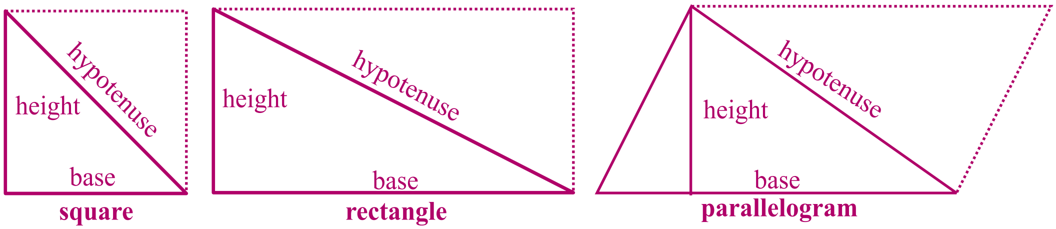
4. The perpendicular distance of a trapezium is 8m and the lengths of parallel sides are 10m and 15m. Calculate the area of the trapezium.

5. A trapezium shaped playground has lengths of its two parallel sides 80m and 120m. Find the cost of its flooring at the rate of Rs.25/m^2, where the distance between two parallel sides is 45m.

6. Find the cost of carpeting a trapezium shaped floor at the rate of Rs.32/m^2, where the lengths of parallel sides of trapezium are 7m and 17m respectively and distance between them is 9m.

11.1.6 Area of a Triangle

When we join two similar triangles from the side of their hypotenuse, they form a square or a rectangle or a parallelogram as shown below.



From above, it can be noticed that if we have the measures of height and base of a triangle, we can calculate its area by using the following formula.

$$\text{Area of the triangle} = \frac{1}{2}(\text{base} \times \text{height})$$

Example 1: Find the area of a triangle whose height is 18cm and base is 24cm.

Solution: We have

Height of the triangle = 18cm

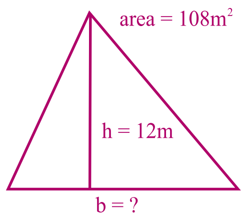
Base of the triangle = 24cm

Area of the triangle = ?

$$\begin{aligned}\text{Area of the triangle} &= \frac{1}{2}(\text{base} \times \text{height})\\ &= \frac{1}{2}(18\text{cm} \times 24\text{cm})\\ &= \frac{1}{2} \times 432\text{cm}^2 = 216\text{cm}^2\end{aligned}$$

Thus, the area of the triangle is 216cm^2.

Example 2: The area of a triangular field is 108m^2 and height is 12m. Find the base of the field.



Solution: We have:

Area of the triangular field = 108m^2

Height of the triangular field = 12m

Base of the triangular field = ?

$$\text{Area} = \frac{1}{2}(\text{base} \times \text{height})$$

$$\text{Base} = \frac{2 \times 108\text{m}^2}{12\text{m}} = 18\text{m}$$

Thus, the base of the triangular field is 18m.

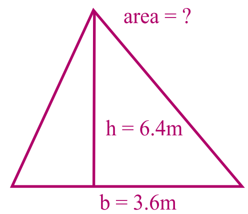
Example 3: A triangular shaped courtyard has the height 6.4m and base 3.6. Find the cost of tiling the courtyard the rate of Rs. 500/m^2.

Solution: We have:

Height of the triangular courtyard = 6.4m

Base of the triangular courtyard = 3.6m

Cost of tiling = ?



$$\text{Area of the triangular courtyard} = \frac{1}{2}(\text{base}\times\text{height})$$
$$= \frac{1}{2}(6.4\text{m} \times 3.6\text{m})$$
$$= \frac{1}{2}(23.04\text{m}^2) = 11.5\text{m}^2$$

Cost of tiling 1m^2 = Rs. 500

Cost of tiling 11.52m^2 = Rs. (500×11.52) = Rs. 5760

EXERCISE 11.5

1. Find the area of the following triangles
   - (i) Base = 8m, Height = 14m
   - (ii) Base = 19m, Height = 16m
   - (iii) Base = 14.4cm, Height = 12.5cm
   - (iv) Base = 6.7m, Height = 10m
   - (v) Base = 5.6m, Height = 6.5m
   - (vi) Base = 20.1 cm Height = 12.8cm
   - (vii) Base = 8.25cm, Height = 6.4cm
   - (viii) Base = 25m, Height = 33m
2. Find the area of a triangular floor whose base is 9m and height is 5.4m.
3. A triangular sandwich has the same height and base. Find the area of the sandwich if its base is 7.4cm.
4. The base of a triangular shaped clock is 28cm and height is 32cm. Find the area of the clock that it will cover on the wall.
5. Find the cost of leveling a triangular playground at the rate of Rs. 25.5 per square metre. The base of the playground is 88m and height is 66m.
6. The base of a triangular shaped field is 246m and height is 125m. How much rice will be produced in this field at the rate of 24 quintal per hectare?
   (Hint: 1 hectare = 10,000m^2)
7. A room is triangular in shape. Its base is 9.4m and height is 8.6m. Find the cost of its wooden floor at the rate of Rs. 250 per square metre.
8. The height of a triangular garden is 54m and base is 92m. Find the number of flowers in the garden if there are 18 flowers on the area of each sq. metre.

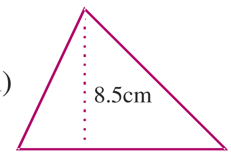
Summary

- The surface covered by an object in a plane is called area of that object and the measurement of boundary of the surface is called its perimeter.
- A square of side 1cm is used as a standard unit for measuring the area of a figure i.e $1cm^2$.
- Perimeter of the square = 4 × length of the side
- Area of the square = length × length
- Perimeter of a rectangle = 2(length + breadth)
- Area of a rectangle = length × breadth
- Border area = [Outer length × Outer breadth] – [Inner length × Inner breadth]
  - ► [Length of the large rectangle] = [Length of the small rectangle] + 2 [Width of the border]
  - ► [Breadth of the large rectangle] = [Breadth of the small rectangle] + 2 [Width of the border]
- A plane figure with four straight sides whose opposite sides are parallel and of equal lengths is called a parallelogram.
- A trapezium is a quadrilateral with only two parallel sides.
- Area of a parallelogram = base × height
- Area of a trapezium = $\frac{1}{2}$ [Perpendicular distance × Sum of lengths of parallel sides]
- Area of a triangle = $\frac{1}{2}$(base × height)

Review Exercise 11

1. Find the perimeter of a square whose area is $676cm^2$.
2. A room is 4.5m long and 4m wide. The floor of the room is to be covered with square marble tiles with length of 0.5m. Find the cost of flooring at the rate of Rs. 500 per tile.
3. Find the cost of repairing of a 2m wide jogging track at the rate of Rs.50 per square metre constructed inside of a park, when the length and the breadth of the park are 100m and 60m respectively.
4. Calculate the cost of flooring of a lm wide verandah at the rate of Rs. 100 per square meter which surroundes a 6m long and 4m wide room.
5. Find the repairing cost of a trapezium shaped park of Kashmir at the rate of 15 rupees per. sq meter whose parallel sides are 65 metre and 115 metre, where the distance between its parallel sides is 38metre.

6. Find the area of given triangles.



Objective Exercise 11

1. Answer the following questions.
   (i) Write the formulae for finding the perimeters of a square and a rectangle.
   (ii) Write the formula for finding the border area of a rectangle.
   (iii) What is a trapezium?
   (iv) Define a parallelogram?

2. Fill in the blanks.
   (i) Area of a __________ = side × side
   (ii) Area of a __________ = height × base
   (iii) Area of a __________ = $\frac{1}{2}$(height × base)
   (iv) The measurement of boundary of a plane figure is called its _________.
   (v) A simple closed figure formed by joining three straight line segments is called ___________.

3. Tick (✓)the correct answer.
   (i) 4 × side is a formula for finding the perimeter of a:
   (a) rectangle (b) square (c) trapezium (d) parallelogram
   (ii) The area of a square of side 2cm is:
   (a) $4cm^2$ (b) $6cm^2$ (c) $8cm^2$ (d) $2cm^2$
   (iii) The perimeter of a rectangle of length 4cm and breadth 2cm is:
   (a) 8cm (b) 6cm (c) 2cm (d) 12cm
   (iv) A quadrilateral with only two parallel sides is called:
   (a) square (b) rectangle (c) parallelogram (d) trapezium
   (v) If, the base of a triangle is 3cm and height is 2cm, then its area will be:
   (a) 2cm (b) 3cm (c) 5cm (d) 6cm

Unit 12

THREE DIMENSIONAL SOLIDS

Student Learning Outcomes

After studying this unit, students will be able to:

- Identify 3D figures (cube, cuboid, sphere, cylinder and cone) with respect to their faces, edges and vertices.
- Define and recognize the units of surface area and volume.
- Find the surface area and volume of a cube and a cuboid.
- Solve real life problems involving volume and surface area.

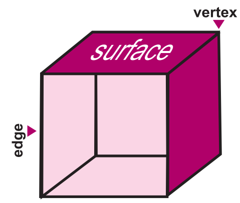
12.1 Introduction

In our daily life we observe many things like chairs, footballs, cupboards, fridges, bricks and dices etc. These all things are called solids.

Cube

A cube is a three dimensional solid formed by six identical square surfaces. The dice, ice cubes, etc are the examples of cubes. It has 6 surfaces, 8 vertices and 12 edges. All of its surfaces are equal in area and all of its edges are equal in length.

surfaces = 6
vertices = 8
edges = 12

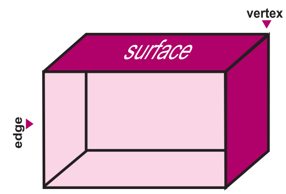


cube

Cuboid

A cuboid is a solid figure formed by six rectangular surfaces. In a cuboid, opposite surfaces and edges are equal and parallel. It has 6 surfaces, 8 vertices and 12 edges. The books, cupboards, match boxes, etc are the examples of cuboids.

surfaces = 6
vertices = 8
edges = 12



cuboid

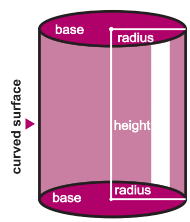
Sphere

A sphere is a solid with completely round surface whose all points on its outer surface are at an equal distance from a fixed point, called the centre of the sphere and the distance from the centre to the points on the sphere is called the radius of the sphere.

sphere

Cylinder

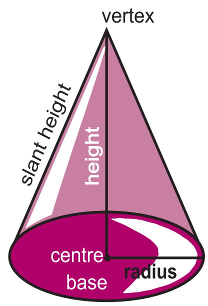
A solid of the shape as shown in the given figure is known as cylinder. The lower or upper circular top of the cylinder is known as its base and its round surface is known as its curved surface. The radius of the circular surface is known as its radius.
The distance between the two parallel circular surfaces is known as its height.



cylinder

Cone

A right cone is a solid figure, formed by a curved surface that slopes up from a round, flat base to a point called the vertex.
The distance between the centre of the base and the vertex is known as its height. The distance of the vertex from any point on the boundary of the circular base is known as its slant height.
The lower circular top is called its base and the radius of the base is also known as the radius of the cone.

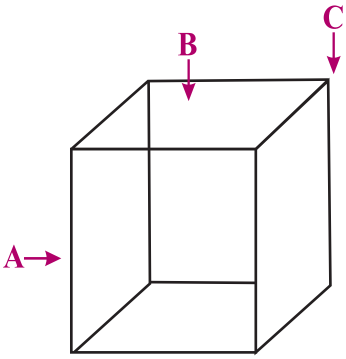


cone

EXERCISE 12.1

1. Write the name of the different parts of given solids.

(i)

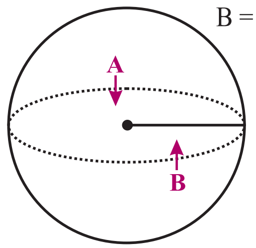


A = ______________

B = ______________

C = ______________

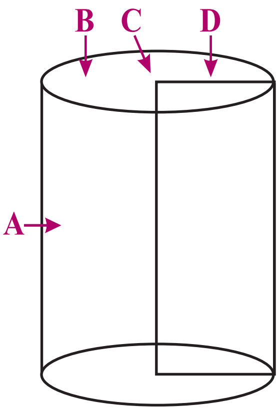
(ii)



A = ______________

B = ______________

(iii)



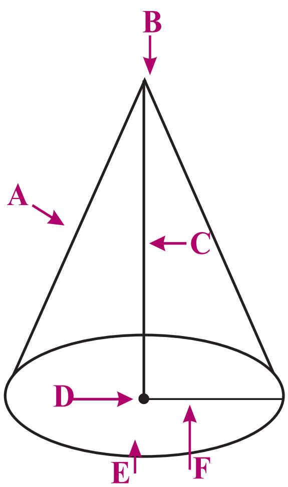
A = ______________

B = ______________

C = ______________

D = ______________

(iv)



A = ______________

B = ______________

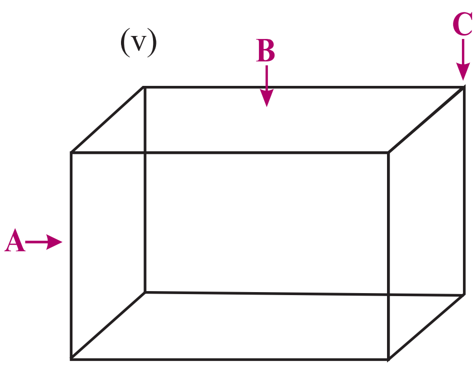
C = ______________

D = ______________

E = ______________

F = ______________

(v)



A = ______________

B = ______________

C = ______________

12.1.1 Volume of three dimensional solids

We can calculate the area of rectangular surface by its length and breadth which is known as the two dimensions of a surface.

But in case of solids such as bricks, tea boxes and so on, we can't calculate the area. These are three dimensional solids and occupy space. Hence, we use the term volume to measure them. So, we can define the volume as;

"The amount of the space which any object occupies in the three dimensions is called its volume".

Units of Volume

Volume is written in cubic unit. For example, the volume of a cube with side 1 cm will be as,

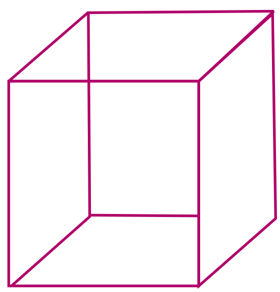
$$\begin{aligned} \text{Volume} &= \text{length} \times \text{breadth} \times \text{height} \\ &= 1\text{ cm} \times 1\text{cm} \times 1\text{ cm} \\ &= 1\text{ cm}^3 \text{ (one cubic centimetre)} \end{aligned}$$

And volume of a cube with side 1 m will be as,

$$\begin{aligned} \text{Volume} &= \text{length} \times \text{breadth} \times \text{height} \\ &= 1\text{ m} \times 1\text{ m} \times 1\text{ m} \\ &= 1\text{ m}^3 \text{ (one cubic metre)} \end{aligned}$$

To find the volume of an object, measurement of all dimensions should be in the same unit.

Volume of Cubes and Cuboids



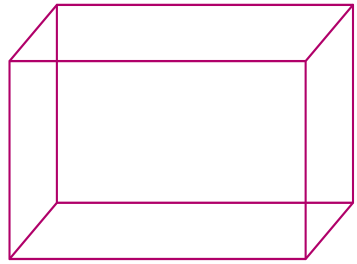
A box-shaped object which has an equal length, breadth and height is called a cube i.e.

$$\text{Length} = \text{Breadth} = \text{Height}$$

$$\begin{aligned} \text{So, the volume of a cube} &= \text{length} \times \text{breadth} \times \text{height} \\ &= \text{length} \times \text{length} \times \text{length} \\ &= (\text{length})^3 \end{aligned}$$

Thus, we can find the volume of a cube by the measurement of a single edge.

Cuboid

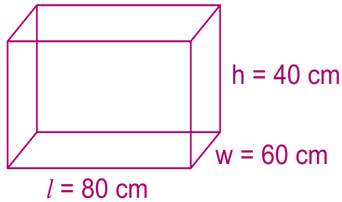


In a cuboid, length, breadth and height are three different measurements. So, we need all the three measurements to find its volume.

$$\text{Volume of a cuboid} = \text{length} \times \text{breadth} \times \text{height}.$$

Example 1: A tin can is 80 cm long, 60 cm wide and 40 cm high. Find the capacity of tin can.

Solution:



Length of the tin can = 80 cm

Width of the tin can = 60 cm

Height of the tin can = 40 cm

Capacity of the tin can = volume =?

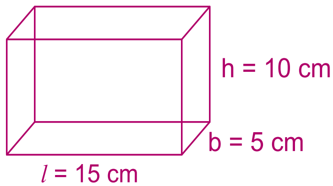
Volume = length × breadth × height

= 80 cm × 60 cm × 40 cm

= 192,000 cm^3

Example 2: How many wooden blocks can fill the space of 1.5 m^3, when the three dimension of each block is 15 cm × 10 cm × 5 cm?

Solution:



Volume of a block = 15cm × 10cm × 5cm

= 750cm^3

$$\text{Total number of blocks} = \frac{\text{total space}}{\text{volume of 1 block}}$$

$$= \frac{15,00,000}{750}$$

= 2,000 blocks

$\because$ 1.5 m^3 = 1.5 × $(100cm)^3$

= 1.5 × 100cm × 100cm × 100cm

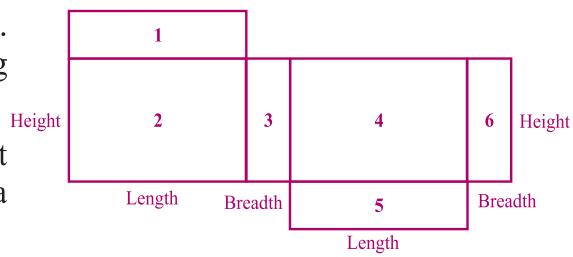
= 1.5 × 1000000 cm^3

= 15,00,000 cm^3

Surface Area of Cuboids

We have already learnt the formulas for finding the areas of different surfaces. We use the same formulas for finding the surface area of a cuboid.

We know that a cuboid is the sum of six flat surfaces that can be shown by unfolding a cuboid as given in the following figure.



From the above figure, we can observe the six rectangular surfaces of a cuboid.

Area of surface 1 = length × breadth

Area of surface 2 = length × height

Area of surface 3 = breadth × height

Area of surface 4 = length × height

Area of surface 5 = length × breadth

Area of surface 6 = breadth × height

By adding all these six surfaces, we can get the surface area of a cuboid.

Surface area of a cuboid = Area of surface 1 + Area of surface 2 + Area of surface 3 + Area of surface 4 + Area of surface 5 + Area of surface 6

Surface area of a cuboid = (length × breadth) + (length × height) + (breadth × height) + (length × height) + (length × breadth) + (breadth × height)

= 2 (length × breadth) + 2 (length × height) + 2 (breadth × height)

= 2[(length × breadth) + (length × height) + (breadth × height)]

Suppose that, length = l, width or breadth = b and height = h
then we can write the above formula as:

$$2\,[(\,l \times b\,) + (\,l \times h\,) + (\,b \times h\,)]$$

Surface Area of Cubes

We know that length, breadth and height of a cube are equal such as

$\therefore$ length = breadth = height

then, surface area of a cube

= 2[(length × length) + (length × length) + (length × length)]

= 2[3 (length)^2]

= 2×3 (length)^2 = 6 length^2

Example 3: Find the surface area of a cube of length 8 cm.

Solution:

Length of a cube = 8 cm

Surface area of a cube = 6(length)^2

= 6× (8 cm)^2

= 6×64 cm^2 = 384 cm^2

Example 4: A box is 3 m long, 2 m high and 1.5 m wide. Find the cost of painting the box at the rate of Rs.3/m^2.

Solution:

Length of the box (l) = 3 m

Height of the box (h) = 2 m

Breadth of the box (b) = 1.5 m

Surface area of the box = $2\,[(\,l \times b\,) + (\,l \times h\,) + (\,b \times h\,)]$

= $2\,[(\,3\text{ m} \times 1.5\text{ m}\,) + (\,3\text{ m} \times 2\text{ m}\,) + (\,1.5\text{ m} \times 2\text{ m}\,)]$

= $2\,[4.5\text{ m}^2 + 6\text{ m}^2 + 3\text{ m}^2\,]$

= 2×13.5 m^2 = 27 m^2

Cost of painting of 1 m^2 = Rs.3

Cost of painting of 27 m^2 = Rs.3×27 = Rs.81

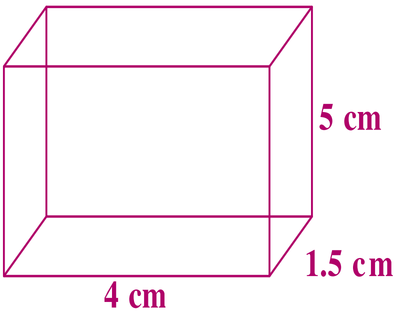
EXERCISE 12.2

1. Find the volume and surface area of the following cubes.

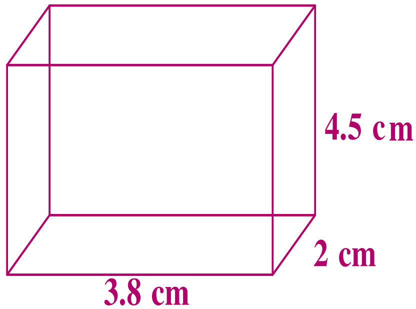
   (i) edge = 4 cm (ii) edge = 7 cm (iii) edge = 2.5 m (iv) edge = 3.2 cm

2. Find the volume and surface area of the following cuboids.

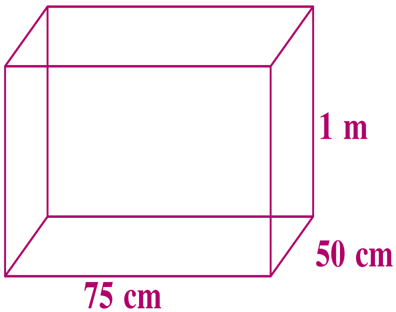
(i)



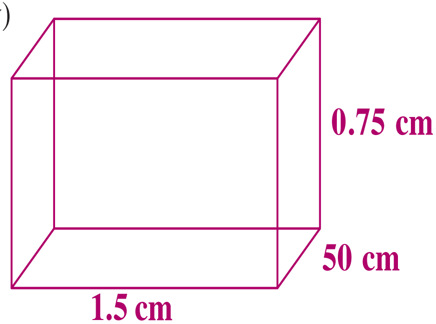
(ii)



(iii)



(iv)



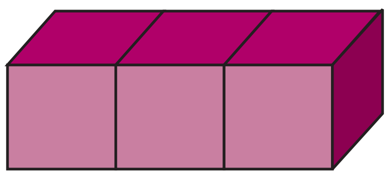
3. A container is 40 m long, 15 m wide and 20 m high. Find the capacity of container.
4. A water tank is 3 m long, 2 m wide and lm high. How many litres of the water can be stored in it?
5. How many cubes of 1cm can be cut from a cuboid of the dimension 10cm × 12 cm × 15 cm?
6. Find the internal surface area of a room whose dimensions are 4 m x 2.5 m x6 m.
7. Find the cost of polishing a cubical wooden box having an edge of 1.5 m at the rate of Rs.40/m^2.
8. A metal cupboard is 1.5 m long, 2 m high and lm wide. Find the cost of painting the cupboard at the rate of Rs.80/m .

Summary

- A solid, whose length, width and height are equal, is called a cube.
- A solid in which at least one out of length width and height is different from the other two is known as a cuboid.
- A sphere is a solid with completely round surface whose all points on its outer surface are at an equal distance from a fixed point.
- The lower or upper circular top of the right circular cylinder is known as its base and its round surface is known as its curved surface.
- A cone is a solid figure, formed by a curved surface that slopes up from a round flat base to a point called the vertex.
- Volume is the measurement of the space occupied by an object.
- To find the volume of an object, measurement of its dimension should be in the same unit.
- Surface area of a cube = 6 length^2
- Volume of a cube = (length)^3
- Volume of a cuboid = length × breadth × height
- Surface area of a cuboid = $2[(l \times b) + (l \times h) + (b \times h)]$

Review Exercise 12

1. The edge of an iron cube is 12cm. Find its volume and cost to colour its total surface at the rate of Rs. 0.5 per sq cm?
2. Three cubes, each of edge 2cm long are placed together as shown in figure. Find the surface area of the cuboid so formed.



3. How many 5cm cubes can be obtained from a cube whose side is 25cm.
4. A match box dimension is 4cm×2.5cm×1.5cm.What will be the volume of a packet containing 6 such match boxes? How many such packets can be placed in a cardboard box whose size is 60cm × 30cm × 24cm?

Objective Exercise 12

1. Answer the following questions.
   - (i) Write down the term used for the distance between two circular surfaces of a cylinder?
   - (ii) Define a cube.
   - (iii) Write the formula for finding the surface area of a cuboid.
   - (iv) Define the volume.

2. Fill in the blanks.
   - (i) A cube has 6 surfaces, 8 vertices and 12 __________ .
   - (ii) A solid in which at least one out of length, width and height is different from the other two is known as __________ .
   - (iii) The solids like tennis ball and football are examples of __________ .
   - (iv) In a cone, the distance of the vertex from any point on the boundary of the circular base is known as its __________.
   - (v) The surface area of a cube = __________ .

3. Tick (✓) the correct answer.
   - (i) A cuboid has the vertices:

     (a) 6 (b) 8 (c) 10 (d) 12
   - (ii) The lower circular surface of a cone is called its:

     (a) height (b) base (c) radius (d) slant height
   - (iii) The surface area of cube of 2cm long side is:

     (a) $6cm^2$ (b) $12cm^2$ (c) $24cm^2$ (d) $48cm^2$
   - (iv) If a cuboid has length = 3cm, breadth = 1cm and height = 2cm then its volume will be:

     (a) $6cm^3$ (b) $12cm^3$ (c) $24cm^3$ (d) $48cm^3$
   - (v) The distance from the centre to the surface of a sphere is called its:

     (a) base (b) height (c) radius (d) vertex

Unit 13

INFORMATION HANDLING

Student Learning Outcomes

After studying this unit, students will be able to:

- Define data and data collection.
- Distinguish between grouped and ungrouped data.
- Draw horizontal and vertical bar graphs.
- Read a pie graph.

13.1 Introduction

Information handling is a branch of the statistics which deals with the collection, analysis, explanation and presentation of a data. The word statistics has been deduced from the Latin word 'statisticum collegium' means council of state) and the Italian word 'statista' means politician.

13.1.1 Data and its Types

A data is a set of information and facts which is represented in the form of figures.
We can collect data by several ways, depending upon the quantity of the data and reason for its collection. For example, Miss Ifra, the manager of a restaurant wants to know about the liking of the food of her restaurant among its customers. For this purpose, she gives a survey form to 100 regular customers and asks them to select only one favourite food of the restaurant. From the survey forms she makes a data table which is given blow:

Food	Chicken tikka	Roasted beef	Fried fish	Mutton karahi	Other food
No. of customers	48	10	20	14	8

From the above table, Miss Ifra concluded that the most favourite food of the restaurant is chicken tikka. Similarly, we can collect and arrange a data according to any set of information to derive results that help us to examine our past and plan our future.

In routine, departments collect information by using a questionnaire. The specimen of a questionnaire is given below.

Tick (✓) your favourite food?

Chicken tikka ☐ Fried fish ☐ Roasted beef ☐

Mutton karahi ☐ Other food ☐

13.1.2 Classification of a Data

Data can be classified into two parts.

- Ungrouped data
- Grouped data

- **Un-grouped data:** The data which provides us information about individuals is called un-grouped data. For example, 11 players of a cricket team enhanced the score of the team in a one day match as given in the following table.

Players	1	2	3	4	5	6	7	8	9	10	11
Score	46	18	23	39	32	27	15	36	9	33	37

- **Grouped data:** The data which provides us the information about groups is called the grouped data. For example, we can represent the data of the above examples in groups as,

Number of players who scored between 1 – 10 = 1
Number of players who scored between 11 – 20 = 2
Number of players who scored between 21 – 30 = 2
Number of players who scored between 31 – 40 = 5
Number of players who scored between 41 – 50 = 1

We can also represent the above data by using a table as given below.

Group	Score	No. of players
1 – 10	9	1
11 – 20	15,18	2
21 – 30	23,27	2
31 – 40	32,33,36,37,39	5
41 – 50	46	1

EXERCISE 13.1

Which table is showing a grouped data?

1.

Student	1	2	3	4	5	6	7	8	9	10	11	12
Marks	581	786	678	725	788	580	690	780	599	509	619	560

2.

Group	Sale of Toys	No. of days
512 – 611	514,610,603,508,607,580, 595,574	8
612 – 711	704,675,650,625,613	5
712 – 811	809,783,766,712	4
812 – 911	877,892,901,910,846,907, 823,825,902	9

3.

Group	Rupees	No. of workers
150 – 199	175,150,195	3
200 – 249	225,245,200,235	4
250 – 299	275	1
300 – 349	315,340	2
350 – 399	365,395,355	3
400 – 449	425,410	2

4.

Player	1	2	3	4	5	6	7	8	9	10	11
Score	62	41	15	59	22	10	8	2	43	7	21

5.

Day	1	2	3	4	5	6	7
Distance (km)	25	43	76	18	90	52	12

6.

Group	Bills	No. of consumers
356 – 455	365,394	2
456 – 555	488,523,549	3
556 – 655	578,594,643	3
656 – 755	732,658,713,698	4

7.

Group	Visitors	No. of customers
6 – 10	7,9,6,7,8,10,9,6	8
11 – 15	12,14,12,11,15,13	6
16 – 20	19,17,19,18,16,20,19,18	8
21 – 25	23,25,22,21,22,24,23	7

13.2 Graph

A graph is a drawing that shows the relationship between numbers and quantities. We use a graph to display a data in a simple, attractive and comprehensible way.

13.2.1 Bar Graph

A graph in which data is represented by a number of rectangular bars is called a bar graph. A bar has a uniform width and an equal distance from the other bars. We use two rays to draw a bar graph which are called axis and marking on axis is called its scale as shown in the figure (i) and (ii).

Fig (i)

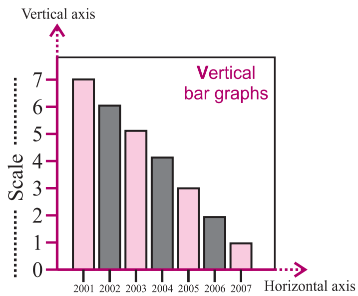
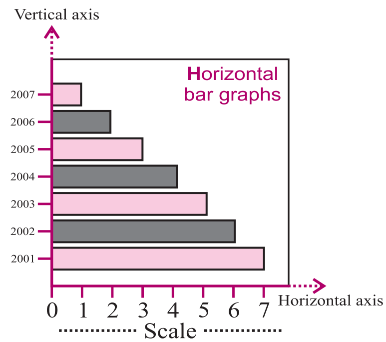


Fig (ii)



A graph with horizontal bars is called horizontal bar graph and a graph with vertical bars is called vertical bar graph.

- **Vertical Bar Graph**

Example1: Nasir got the following marks out of 100 marks

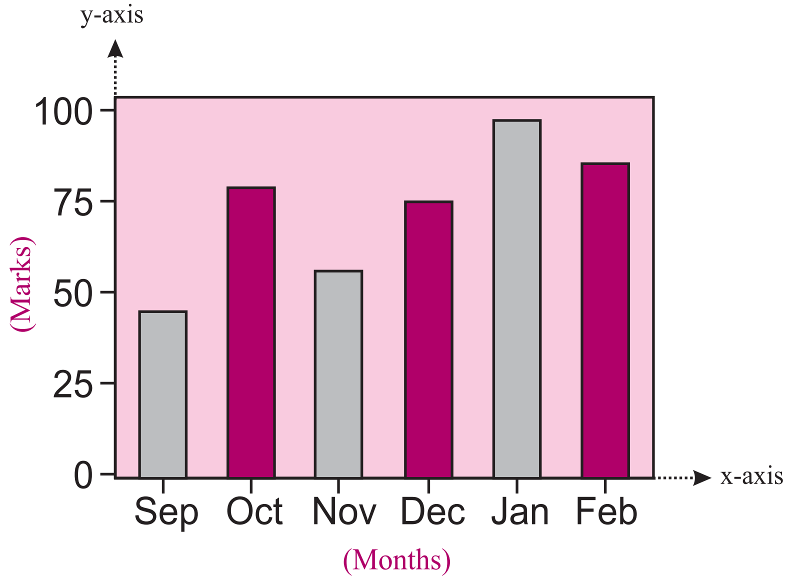
(i) September: 45 marks (ii) October: 80 marks
(iii) November: 60 marks (iv) December: 75 marks
(v) January: 90 marks (vi) February: 85

Solution:

(i) Choose a suitable scale to draw a graph.
Scale: 1 large division represents 25 marks along y-axis.
(ii) Draw x-axis and y-axis with the common point O.
(iii) Indicate marks according to the scale, along y-axis.

1 large division = 25 marks

(iv) Along x-axis, mark each month after a suitable distance.
(v) Draw coloured rectangular bars according to the scale for each month.



Reading the Bar Graph

(i) The bar graph represent in general the marks of Nasir that he got in six tests.
(ii) January is the month in which Nasir got the maximum marks.
(iii) September is the month in which Nasir got the least marks.
(iv) The ratio of the marks of October to the marks of November is: 80:60 or 4:3.
(v) The month of best performance is January and that of worst performance is September.

Horizontal Bar Graph

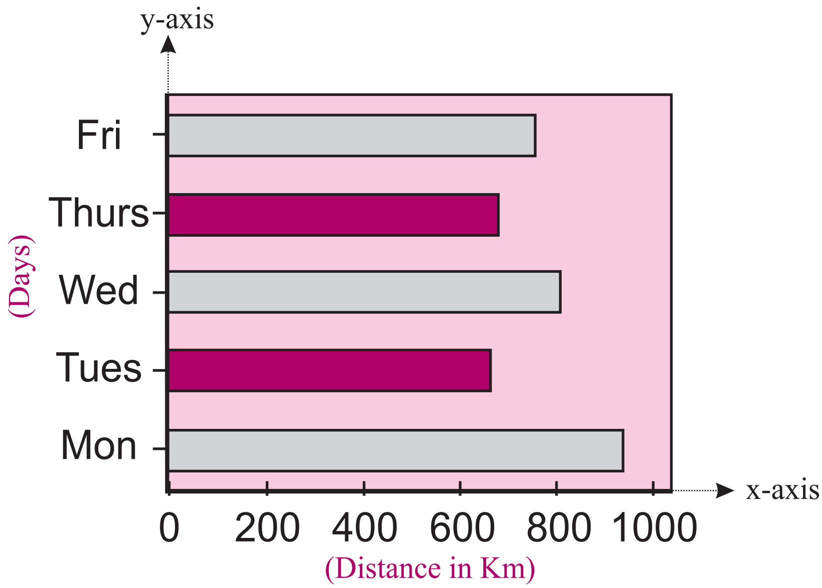
Example 2: Usman traveled to help the poor as given in the following table.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Travel (km)	920	640	800	680	760

By using the above table, draw a horizontal bar graph.

Solution:

(i) Choose a suitable scale to draw a graph.
Scale: 1 large division represents 200 km along x-axis.
(ii) Draw x-axis and y-axis with the common point O.
(iii) Along y-axis mark each day after a suitable distance.
(iv) Draw coloured rectangular bars according to the scale for each day.



Reading the Bar Graph

1. The bar graph is representing in general the distance that Usman traveled in five days.
2. Monday is the day in which Usman traveled the maximum distance.
3. Tuesday is the day in which Usman traveled the least distance.
4. The ratio of distance of Monday to the distance of Tuesday is 920:640 or 23:16.
5. The day of maximum traveling is Monday and that of least traveling is Tuesday.

EXERCISE 13.2

(1) Draw a vertical and horizontal bar graphs by using the following data.

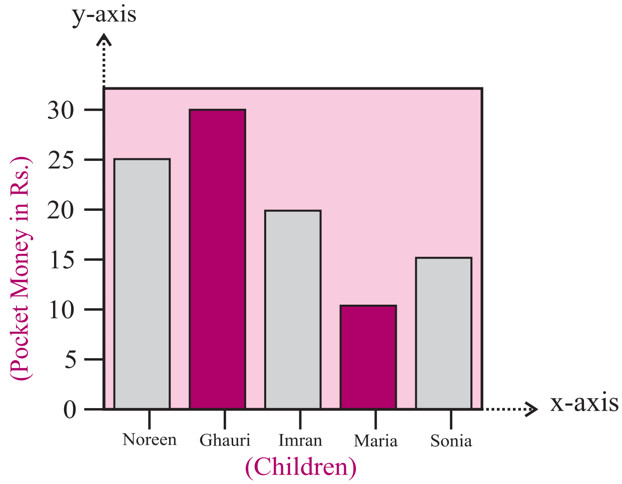
(i)

Month	January	February	March	April	May
Profit(Rs.)	12,000	23,000	18,000	26,000	20,000

(ii)

Time (Hrs)	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}
Temperature (Centigrade)	40	50	35	70	90	65

(2) Read the following vertical bar graph which is showing the daily pocket money of some children and answer the questions.



(i) What is general information we get from the graph?

(ii) Which child is getting maximum pocket money?

(iii) Which child is getting minimum pocket money?

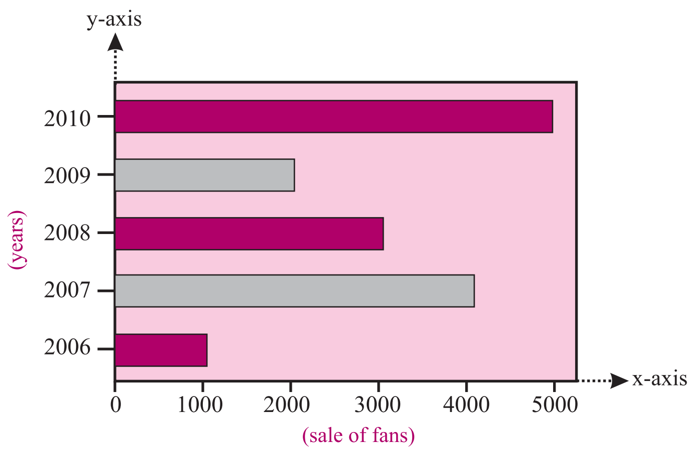
(iv) How many rupees is Noreen getting as a pocket money?

(v) What is the difference between Maria and Imran's pocket money?

(vi) What is the ratio between Ghuari and Imran's pocket money?

(3) A company sold fans during 5 years as given in the following horizontal bar graph. Now read the graph and answer the questions.

(Scale: 1small division represents the sale of 200 fans along x-axis)



(i) What is general information we get from the graph?

(ii) Write the sale of fans in each year.

(iii) What is the sale difference between the years 2006 and 2010?

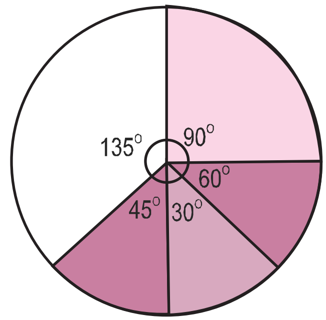
(iv) What is the average sale of 5 years?

(v) Which is the best year of sale?

(vi) What is the ratio between the sale of 2006 and 2010?

13.3 Pie Graph

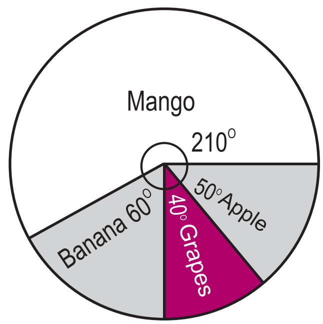
A graph in which data is represented by the sectors of a circle is called a pie graph.



We normally use a pie graph for inter comparison of a data. Although it is little difficult to draw a pie graph than a bar graph and line graph due to its calculations and more use of geometrical instruments like protractor to measure an angle and pair of compasses to draw a circle.

We shall make it clear with the following example.

Example 1: Read the following pie graph which is showing the favourite fruit of 900 boys.



(i) How many boys' favourite fruit is mango?

(ii) How many boys' favourite fruit is banana?

(iii) How many boys' favourite fruit is apple?

(iv) How many boys' favourite fruit is grapes?

(v) Which fruit is the most favourite of the boys?

Solution:

We can find a quantity by the given angle. For this purpose, we use the following formula.

$$\text{Required quantity} = \frac{\text{given angle}}{360} \times \text{total quantity}$$

(i) Mango as a favourite fruit $= \dfrac{\overset{7}{\cancel{210}}}{\underset{12}{\cancel{360}}} \times \overset{75}{\cancel{900}}$ boys $= 525$ boys

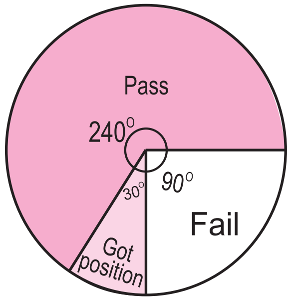
(ii) Banana as a favourite fruit $= \dfrac{\cancel{60}}{\cancel{360}} \times \overset{150}{\cancel{900}}$ boys $= 150$ boys

(iii) Apple as a favourite fruit $= \dfrac{\overset{25}{\cancel{50}}}{\cancel{360}} \times \overset{5}{\cancel{900}}$ boys $= 125$ boys

(iv) Grapes as a favourite fruit $= \dfrac{\cancel{40}}{\underset{9}{\cancel{360}}} \times \overset{100}{\cancel{900}}$ boys $= 100$ boys

EXERCISE 13.3

(1) The following pie graph is showing the result of a class of 120 students. Read the pie graph and answer the questions.



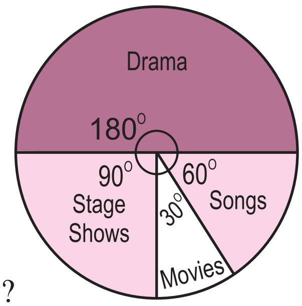
(i) How many students passed the examination?

(ii) How many students got the positions?

(iii) How many students failed?

(2) A media reporter made a chart to show the liking of 1800 girls for different TV programmes.

By using the above pie chart, answer the questions.



(i) How many girls showed the liking for stage shows?

(ii) How many girls showed the liking for dramas?

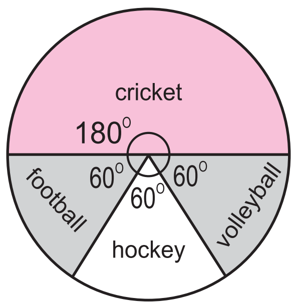
(iii) How many girls showed the liking for movies?

(iv) How many girls showed the liking for songs?

(v) Which programme is the most popular with the girls?

(vi) Which programme is the least popular with the girls?

(3) A teacher made a pie graph to show the likings of 720 students in his school for different games.



(i) In how many sectors the pie graph is divided?

(ii) Which games are indicated by equal sectors?

(iii) How many students like to play Cricket?

(iv) How many students like to play Volleyball?

(v) Which game is played the most?

(vi) How many students like to play Volleyball and Football?

(vii) How many students like to play Cricket and Hockey?

Summary

- A data is a set of information and facts which is represented in the form of figures.
- The data which provides us information about individuals is called ungrouped data.
- The data which provides us information about groups is called the grouped data.
- A graph is a drawing that shows the relationship between numbers and quantities.
- A graph with horizontal bars is called horizontal bar graph and vertical bars is called vertical bar graph.
- A pie graph is used for inter-comparison of a data.

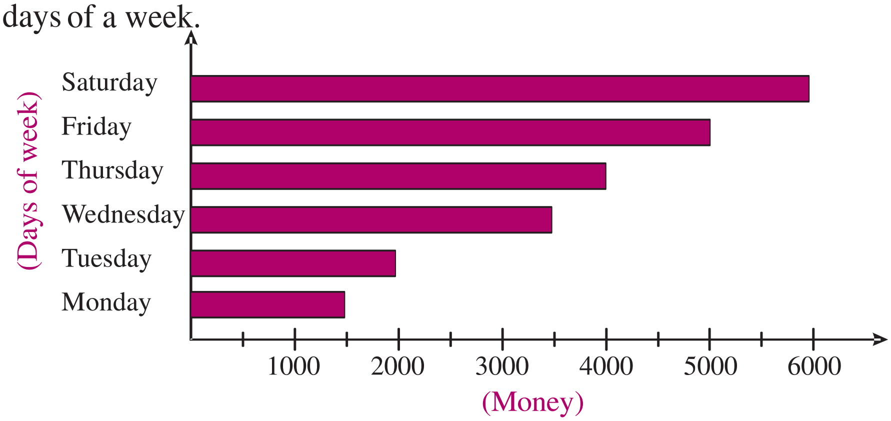
Review Exercise 13

1. The number of students playing different games is as under.

Badminton	= 20 ,	Hockey	= 25
Football	= 20 ,	Carrom	= 10
Cricket	= 30 ,	Volley-ball	= 25

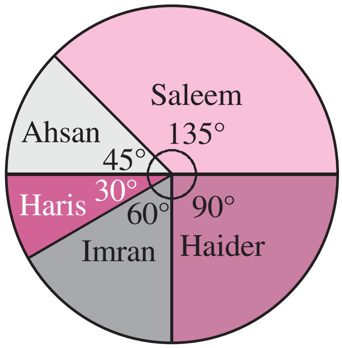
Show this information through a bar graph.

2. Given below is a bar graph that shows the amounts earned by Tariq in six days of a week.



- Read the graph and answer the questions.
  - (i) What amount did Tariq earn in six days?
  - (ii) On which day, did he earn the maximum amount?
  - (iii) On which day, did he earn the minimum amount?
  - (iv) What is general information we are getting from the graph?
  - (v) What is the ratio between the amount of Saturday and Monday?

3. A student made a pie chart to show the result of elections in the college for the seat of union president.



If total 6,480 votes were cast, then answer the following questions by using the above pie chart.

(i) How many votes did the Ahsan get?
(ii) How many votes did the Haider get?
(iii) How many votes did the Imran get?
(iv) How many votes did the Saleem get?
(v) How many votes did the Haris get?
(vi) Who got the seat of union president?

Objective Exercise 13

1. Answer the following questions.

(i) What is data?
(ii) What is meant by a grouped data?
(iii) Name the two sections of a data.
(iv) Define an ungrouped data.
(v) Name the two types of a bar graph.
(vi) Why we use a pie graph?
(vii) What is the sum of angles in a pie graph?

2. Fill in the blanks.

(i) In routine, departments collect information by using a ________.
(ii) A data can be classified into ________ classes.
(iii) A ________ is a drawing that shows the relationship between number and quantities.
(iv) A graph with ________ bars is called horizontal bar graph.
(v) A graph in which data is represented by a number of rectangular bars is called a ________ graph.
(vi) There is no fixed formula for the selection of a ________ while drawing a graph.
(vii) The rays we use to draw a graph are called ________ .

ANSWERS

Exercise 1.1

1. (i), (iii), (v), (vi), (viii) and (ix) are sets.
(ii), (iv) and (vii) are not sets.

2. ii, iv, vii, viii and xi are false statements.
i, iii, v, vi, ix, x and xii are true statements.

3. (i) $\notin$ (ii) $\in$ (iii) $\in$ (iv) $\notin$ (v) $\in$
(vi) $\in$ (vii) $\notin$ (viii) $\notin$ (ix) $\in$ (x) $\in$

4. (i) $0 \in W$ (ii) Lahore $\in$ P (iii) $1 \notin E$
(iv) Sindh $\notin$ B (v) Potato $\in$ V (vi) o $\in$ A
(vii) c $\notin$ C (viii) mango $\notin$ F (ix) $5 \in N$
(x) $4 \notin O$

5. (i) A, B, F, G, J, K and M are sets because their objects are distinct and well-defined.
(ii) C, D and E are not sets, because their objects are not distinct.
(iii) H, I and L are not sets, because their objects are not well defined.

6. (i) Pakistan, India, China, America, Japan.
(ii) Cricket, Hockey, Football.
(iii) 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
(iv) 2, 4, 6, 8, 10, 12, 14, 16
(v) a, e, i, o, u
(vi) September, October, November, December
(vii) Saturday, Sunday, Monday, Tuesday, Wednesday, Thursday, Friday.
(viii) Green, White.
(ix) Ravi, Chanab, Sutlaj, Jehlum, Sindh.
(x) Muharram, Ramzan, Shawal.

Exercise 1.2

1. A = The set of counting or natural numbers less than 7.
B = The set of whole numbers less than 100.
C = The set of four games.
E = The set of all even numbers.
F = The set of four vegetables.
N = The set of all natural or counting numbers.
O = The set of all odd numbers.
W = The set of whole numbers.
X = The set of four family members.

2. A = {Bilal, Babar, Badar}
B = {Iram, Ifra, Iqra}
C = {goat, cow, hen, dog}
D = {sparrow, crow, parrot, pigeon, cuckoo}
E = {Benazir, Nawaz Sharif, Shaukat Aziz}
F = {Lahore, Karachi, Sialkot, Faisalabad, Islamabad}
G = {b, a, n}, I = {a, u}, J = {2007, 2008}
K = {bread, egg, cake}

Exercise 1.3

1. (ii), (iv), (v) and (vi) are empty sets, because such elements do not exist.

2. A, C, D and F are finite sets where B, E and G are infinite sets.

3. (i), (v), (vi) and (vii) are non-equivalent sets.
(ii), (iii) and (iv) are equivalent sets.

4. (i) and (ii) are equal sets (iii) and (iv) are not equal sets.

5. i, iii, v, and vi are false statements.
ii, iii and iv are true statements.

Review Exercise 1

1. (i) {January, February, March, April}
(ii) {U, V, W, X, Y, Z}
(iii) {3, 5, 7, 9, 11}
(iv) {Red, Green, Blue, Yellow}
(v) {Earth, Mars, Venus}

2. (i), (ii) and (iii) are not sets because their objects are not distinct.
(iv) and (v) are not sets because their objects are not well-defined.

3. (i) {2, 4, 6, 8, 10} (ii) {18, 19, 20, 21, 22}
(iii) {O, R, A, N, G, E} (iv) {0, 1, 2, 3, 4}

4. (i) finite set (ii) infinite set
(iii) empty set.

5. (i) and (iii) are equivalent sets.
(ii) and (iv) are non-equivalent sets.

6. (i) B = {t, e, r, h, a}
(ii) B = {$\underline{2}$, $\underline{4}$, 0}
(iii) B = {orange, $\underline{\text{mango}}$, $\underline{\text{apple}}$}

Objective Exercise 1

1. (i) A collection of distinct and well-defined objects is called a set.
(ii) $\in$ means "is the element of the set".
(iii) Descriptive form and tabular form.
(iv) Distinct means the same objects should not appear more than once.
(v) Two sets are equal.

2. (i) non equivalent (ii) well-defined (iii) $\notin$ (iv) empty (v) non equivalent

3. (i) d (ii) c (iii) c (iv) a (v) b (vi) d

Exercise 2.1

1. (i) Successor = 37, predecessor = 35
(ii) Successor = 75, predecessor = 73
(iii) Successor = 200, predecessor = 198
(iv) Successor = 351, predecessor = 349
(v) Successor = 790, predecessor = 788

2. 510, 511, 512 **3.** 1, 2, 3, 4, 5

4. (i) 435 (ii) 989 (iii) 1010 (iv) 5342 (v) 10100 (vi) 13971

5. (i) 10, 11, 12, 13, 14 (ii) 0, 1, 2, 3, 4, 5, 6
(iii) 16, 17, 18,...
(iv) 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98,
(v) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11
(vi) 5, 6, 7, 8, …
(vii) 57, 58, 59, 60, 61, 62, 63, 64
(viii) 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93
(ix) 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34
(x) 48, 49, 50, 51

6. (i)
$\boxed{0}$ 1 2 $\boxed{3}$ 4 $\boxed{5}$ 6 7 8 $\boxed{9}$

(ii)
$\boxed{4}$ 5 6 7 $\boxed{8}$ 9 10 11 $\boxed{12}$ $\boxed{13}$

(iii)
$\boxed{0}$ $\boxed{1}$ $\boxed{2}$ $\boxed{3}$ $\boxed{4}$ 5 6 7 8 9

(iv)
74 75 76 77 78 79 80 81

(v)
5 6 7 8 9 10 11

(vi)
6 7 8 9 10 11 12 13 14 15

7. (i)
|1 point|.........3 points.........|
0 1 2 3 $\boxed{4}$ 5 6 7 8 9

(ii)
|........3 points........|............4 points............|
0 1 2 3 4 5 6 $\boxed{7}$ 8 9 10

(iii)
|..2 points..|...........................9 points.................................... |
0 1 2 3 4 5 6 7 8 9 10 $\boxed{11}$

Exercise 2.2

1. (i) 1218 (ii) 852 (iii) 455 (iv) 486 (v) 5929 (vi) 3192 (vii) 2447 (viii) 1335 (ix) 1556

2. (i)
$$\begin{array}{cccc} & \boxed{7} & 4 & 3 \\ + & 2 & 5 & \boxed{4} \\ \hline & 9 & \boxed{9} & 7 \end{array}$$

(ii)
$$\begin{array}{cccc} & 4 & 9 & 1 \\ + & \boxed{1} & 8 & \boxed{5} \\ \hline & 6 & \boxed{7} & 6 \end{array}$$

(iii)
$$\begin{array}{cccc} & \boxed{1} & 2 & 3 \\ & 2 & \boxed{8} & 4 \\ + & 3 & 4 & \boxed{5} \\ \hline & 7 & 5 & 2 \end{array}$$

(iv)
$$\begin{array}{cccc} & 6 & 9 & 7 \\ - & \boxed{5} & 5 & \boxed{3} \\ \hline & 1 & \boxed{4} & 4 \end{array}$$

(v)
$$\begin{array}{cccc} & 2 & \boxed{3} & 6 \\ - & \boxed{1} & 6 & \boxed{0} \\ \hline & 0 & \boxed{7} & 6 \end{array}$$

(vi)
$$\begin{array}{ccccc} & 6 & 3 & 2 & \boxed{3} \\ - & \boxed{4} & \boxed{3} & \boxed{0} & 9 \\ \hline & 2 & 0 & 1 & 4 \end{array}$$

3. 10 **4.** 199 **5.** 1 **6.** 1

Exercise 2.3

1. (i) $14+\boxed{9}=9+14$
(ii) $\left(1+\boxed{4}\right)+2=\boxed{1}+\left(4+2\right)$
(iii) $\left(\boxed{1}+\boxed{3}\right)+5=1+\left(3+\boxed{5}\right)$
(iv) $4+11=\boxed{11}+\boxed{4}$
(v) $\left(5+7\right)+9=\boxed{5}+\left(\boxed{7}+\boxed{9}\right)$
(vi) $\boxed{6}+7=\boxed{7}+6$
(vii) $11+\boxed{13}=13+\boxed{11}$
(viii) $\left(5+\boxed{10}\right)+\boxed{15}=\boxed{5}+\left(10+15\right)$

Exercise 2.4

1. (i) 5394 (ii) 5369
 (iii) 7777 (iv) 68400
 (v) 31248 (vi) 235458
 (vii) 273948 (viii) 166911
 (ix) 378852
2. (i) 68 (ii) 125 (iii) 223
 (iv) 1122 (v) 118 (vi) 121
 (vii) 56 (viii) 234 (ix) 111
3. 99000 4. 112 5. 984

Exercise 2.5

1. (i) $\boxed{4}\times 2=\boxed{2}\times 4$ (ii) $7\times 9=\boxed{9}\times\boxed{7}$
 (iii) $3\times(9-6)=3\times 9-3\times\boxed{6}$
 (iv) $5\times\boxed{6}=6\times\boxed{5}$
 (v) $2\times(1+2)=\boxed{2}\times 1+2\times 2$
 (vi) $7+(1+6)=(7+1)+\boxed{6}$
 (vii) $3\times(2\times 5)=(\boxed{3}\times\boxed{2})\times\boxed{5}$
 (viii) $1\times(\boxed{11}-\boxed{12})=\boxed{1}\times 11-\boxed{1}\times 12$
 (ix) $\boxed{2}\times(11\times 9)=(2\times 11)\times\boxed{9}$
 (x) $9\times(\boxed{5}+4)=\boxed{9}\times 5+\boxed{9}\times\boxed{4}$

Review Exercise 2

1. 0, 1, 2, 3
4. (i) 1000 (ii) 3579 (iii) 7709
 (iv) 211 (v) 5504 (vi) 9000
 (vii) 110889 (viii) 5535
 (ix) 881892 (x) 118
 (xi) 121 (xii) 123

Objective Exercise 2

1. (i) A number is a concept that help to answer the question of how many?
 (ii) The symbols which are used to represent the numbers.
 (iii) The sum of 0 and a whole number is always the whole number itself.
 (iv) 1
 (v) By changing the order of two numbers, the result does not change.
2. (i) 0 (ii) natural (iii) even
 (iv) whole (v) multiplicative
3. (i) d (ii) b (iii) c
 (iv) d (v) d

Exercise 3.1

1. (i) 1, 3, 7, 21
 (ii) 1, 2, 3, 4, 6, 9, 12, 18, 36
 (iii) 1, 2, 3, 4, 6, 8, 12, 16, 24, 48
 (iv) 1, 3, 9, 11, 33, 99
2. (i) 3, 6, 9, 12, 15 (ii) 5, 10, 15, 20, 25
 (iii) 9, 18, 27, 36, 45
 (iv) 12, 24, 36, 48, 60
3. i, v and viii are odd numbers.
 ii, iv, vi and vii are even numbers.
4. (i) 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47
 (ii) 29, 31, 37, 41, 43, 47, 53, 59
 (iii) 37, 41, 43, 47 (iv) 79, 83, 89
5. The multiples of 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95
 The multiples of 10 = 10, 20, 30, 40, 50, 60, 70, 80, 90
6. The multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48
 The multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48
7. 4, 6, 8, 9, 10, 12, 14, 15, 16, 18
8. 44, 45, 46, 48, 49 9. 2, 3, 5, 7, 11, 13

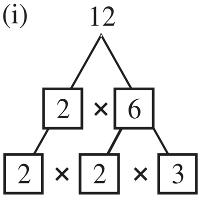
Exercise 3.2

1. i, ii, iv, viii, ix and xii are odd numbers.
 iii, v, vi, vii, x and xi are even numbers.
2. i, vi, vii, x, xii are divisible by 3, ii, iv, v, xi are divisible by 4.
 iii, vii, viii, ix, x are divisible by 5.
3. i, iii, iv, vi, ix, x, xi, xii are divisible by 8
 ii, v, vii, viii, ix, x xi are divisible by 9.
4. iii, v, vi, viii are divisible by 11.
5. i, ii, iv, vi, viii, ix, xi are divisible by 12
 iv, v, vii, x, xii are divisible by 15.
6. i, ii, iv, vi are divisible by 25.

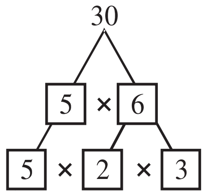
Exercise 3.3

1. (i) 13^3 (ii) 7^4 (iii) 29^2
 (iv) 5^6 (v) 11^4 (vi) 3^6
 (vii) $2^3\times 3^2\times 5^2$ (viii) $7^2\times 11\times 23^2$

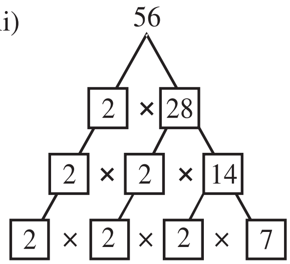
2. (i)



(ii)

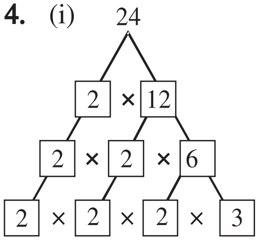


(iii)

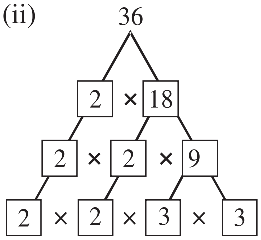


3. (i) $2^2 \times 5$ (ii) $2^2 \times 3^2$ (iii) 2×7^2
(iv) $3^2 \times 5^2$ (v) $2^3 \times 3^3$ (vi) $3^2 \times 7^2$
(vii) 2^8 (viii) $2^3 \times 7^2$
(ix) $2 \times 3 \times 5^3 \times 7$ (x) $2 \times 3 \times 5 \times 7 \times 11$
(xi) $2 \times 3 \times 7^3$ (xii) $2^5 \times 3 \times 13$

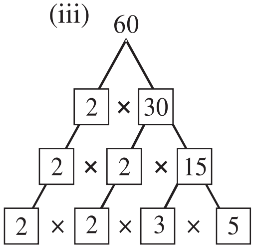
4. (i)



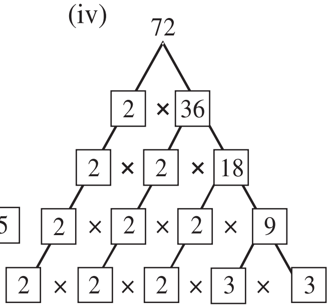
(ii)



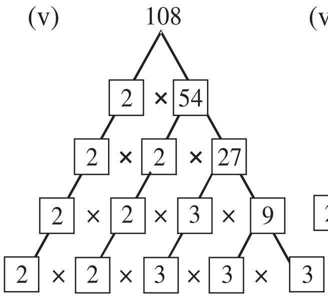
(iii)



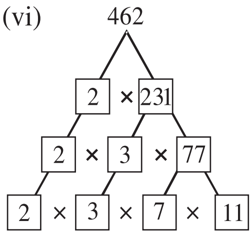
(iv)



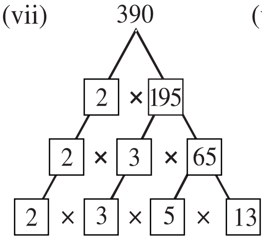
(v)



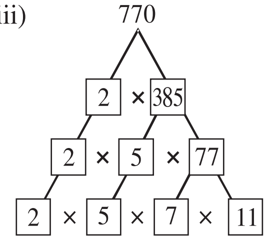
(vi)



(vii)



(viii)



Exercise 3.4

1. (i) 1, 2 (ii) 1,2,4 (iii) 1,5 (iv) 1,2,3,6
(v) 1,2,5,10 (vi) 1,2,4

2. (i) 12 (ii) 5 (iii) 7 (iv) 3
(v) 13 (vi) 4 (vii) 2 (viii) 22 (ix) 5

3. (i) 6 (ii) 11 (iii) 18 (iv) 24
(v) 22 (vi) 12 (vii) 2 (viii) 11 (ix) 7

4. (i) 8 (ii) 7 (iii) 23 (iv) 2 (v) 27
(vi) 4 (vii) 2 (viii) 31 (ix) 133

Exercise 3.5

1. (i) 4 (ii) 30 (iii) 12 (iv) 56 (v) 18
(vi) 24 (vii) 14 (viii) 30 (ix) 18 (x) 18
(xi) 24 (xii) 66

2. (i) 72 (ii) 80 (iii) 180 (iv) 308 (v) 160
(vi) 540 (vii) 225 (viii) 252 (ix) 72 (x) 1575
(xi) 315 (xii) 150

3. (i) 162 (ii) 630 (iii) 7700 (iv) 1260 (v) 8400
(vi) 3600

4. 5775 **5.** 60 **6.** 203 **7.** 208

Exercise 3.6

1. 36 **2.** 75 **3.** 9m **4.** 36m
5. (i) 36cm^2 (ii) 12 **6.** 23940cm
7. 170 litres **8.** 52 students **9.** 2 : 04 p.m
10. 25cm by 25cm

Review Exercise 3

1. (i) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38
(ii) 5, 10, 15, 20, 25, 30, 35 (iii) 7, 14, 21, 28, 35
(iv) 9, 18, 27, 36

2. Even numbers = 2, 4, 6, 8, 10, 12, 14, 16, 18
Odd numbers = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19
Prime numbers = 2, 3, 5, 7, 11, 13, 17, 19
Composite numbers = 4, 6, 8, 9, 10, 12, 14, 15, 16, 18

3. i, iii, iv, vi, vii and viii are divisible by 2.
i, ii, v, vi, vii and viii are divisible by 3.
i, ii, iii and v are divisible by 5.

4. (i) $2^2 \times 3^2 \times 5^2$ (ii) $2^4 \times 3^4$ (iii) $2^4 \times 3^2 \times 7^2$
(iv) $2^2 \times 3^4 \times 11^2$

5. (i) 24 (ii) 35 (iii) 11
6. (i) 11 (ii) 7 (iii) 5
7. (i) 600 (ii) 702 (iii) 5250
8. (i) 4212 (ii) 11550 (iii) 4368
9. 277488 **10.** 69

Objective Exercise 3

1. (i) A number that divides the given number exactly.
(ii) A number having exactly two factors; 1 and the number itself.
(iii) 1 (iv) A number is divisible by 3, if the sum of its digits is divisible by 3.
(v) The process of writing a number into prime factors.
(vi) 1^{st} number × 2^{nd} number = HCF × LCM
2. (i) co-prime (ii) composite (iii) 2
(iv) 2 (v) Factorization
3. (i) b (ii) b (iii) b (iv) d (v) a

Exercise 4.1

2. (i) > (ii) < (iii) <
(iv) < (v) > (vi) <
3. –101 **4.** –199
5. (i) 3, 4, 5 (ii) –1, 0, 1, 2
(iii) –5, –4, –3, –2 (iv) –2, –1, 0, 1, 2, 3
6. (i) 1,2 (ii) –2, –1 (iii) –1,–2
7. 1,0,–1 **8.** –1,0,1,2
9. (i) 3 (ii) 8 (iii) 5 (iv) 9 (v) 6 (vi) 2
10. 0
11. (i) Asc = –4, –2, 0, 1 Des = 1, 0, –2, –4
(ii) Asc = –4, –3, 0, 1 Des = 1, 0, –3, –4
(iii) Asc = –3, –2, 2, 3 Des = 3, 2, –2, –3

Exercise 4.2

1. (i) – 4 (ii) – 6 (iii) + 4
(iv) – 1 (v) – 6 (vi) – 9
2. (i) + 7 (ii) + 16 (iii) – 10
(iv) – 16 (v) + 8 (vi) – 1
(vii) – 4 (viii) – 6 (ix) – 3
(x) – 24 (xi) + 35 (xii) – 46
3. (i) – 3 (ii) + 10 (iii) – 15
(iv) + 2 (v) – 11 (vi) + 19
(vii) – 50 (viii) – 100
4. (i) + 9 (ii) – 7 (iii) + 7
(iv) – 4 (v) + 100 (vi) – 23

Exercise 4.3

1. (i) + 3 (ii) + 3 (iii) – 4
(iv) + 2 (v) + 19 (vi) – 25
(vii) + 8 (viii) – 21 (ix) + 90
(x) – 32 (xi) + 235 (xii) – 30
2. (i) + 9 (ii) + 4 (iii) + 2
(iv) + 5 (v) + 6 (vi) – 16
3. (i) + 2 (ii) – 13 (iii) + 22
(iv) – 11 (v) + 3 (vi) + 25
4. + 222 **5.** – 33

Exercise 4.4

1. (i) – 18 (ii) – 9 (iii) – 16
(iv) + 11 (v) – 8 (vi) + 3
(vii) + (viii) –
2. (i) + 12 (ii) + 12 (iii) – 25
(iv) – 56 (v) + 36 (vi) – 24
(vii) + 50 (viii) – 77 (ix) + 72
(x) + 72 (xi) – 150 (xii) – 49
(xiii) + 36 (xiv) + 65 (xv) – 880
3. (i) + 1 (ii) – 24 (iii) – 72
(iv) – 108 (v) + 3200 (vi) + 4500

Exercise 4.5

1. (i) + 6 (ii) +4 (iii) + 13
(iv) + 9 (v) – 9 (vi) –2
(vii) – 20 (viii) + 9 (ix) – 5
(x) + 3
2. (i) 4 (ii) 8 (iii) – 20
(iv) – 5 (v) – 72 (vi) 13
(vii) 2 (viii) – 4 (ix) 3
3. (i) 14 (ii) – 13 (iii) 4
(iv) – 7 (v) – 9 (vi) – 50

Review Exercise 4

2. (i) 6 (ii) 28 (iii) 43
3. (i) +1, 0, –1, –6, –10
(ii) +4, +3, +2, –2, –3, –4
5. (i) +61 (ii) – 20 (iii) –28
(iv) +36
6. (i) –13 (ii) +15 (iii) –11
(iv) –12

Objective Exercise 4

1. (i) The whole number together with the negative numbers is called integers.
(ii) 9 (iii) subtraction (iv) –1, 0
2. (i) positive (ii) 0 (iii) negative
(iv) directed numbers

3. (i) a (ii) c (iii) d (iv) b (v) a

Exercise 5.1

1. $\frac{2}{3}$ **2.** $2\frac{2}{3}$ **3.** $2\frac{8}{15}$ **4.** $\frac{2}{7}$ **5.** $15\frac{5}{6}$

6. $1\frac{8}{15}$ **7.** $7\frac{1}{2}$ **8.** $1\frac{3}{4}$ **9.** 5.125 **10.** 4.25

11. 8.055 **12.** 2.55 **13.** 11.232 **14.** 1.8852

15. 5.678

Exercise 5.2

1. 9 litre **2.** 2 metre **3.** $2\frac{5}{12}$ feet

4. Rs.3300 **5.** $4\frac{1}{2}$ hours **6.** 196 papers

7. Rs.29 **8.** Rs.977.5

9. Rs.1018.59 **10.** Rs.478

Review Exercise 5

1. $\frac{5}{24}$ **2.** 5 **3.** $\frac{8}{9}$ **4.** 1.375

5. 10.5 **6.** 107.163 **7.** $2\frac{1}{5}$ cm **8.** $\frac{11}{12}$ m

Objective Exercise 5

1. (i) (a) – (b) ( )
(c) { } (d) [ ]

(ii) BODMAS rule means to perform four operations as;

First	Brackets off
Second	Division
Third	Multiplication
Fourth	Addition
Last	Subtraction

(iii) (a) What do you know?
(b) What do you want to know?
(c) What is the proper operations?

(iv) box brackets

2. (i) BODMAS rule (ii) basic (iii) { }
(iv) paranthesis (v) viniculum

3. (i) b (ii) d (iii) d (iv) b (v) a

Exercise 6.1

1. (i) 3 : 4 (ii) 2 : 7 (iii) 9 : 11
(iv) 1 : 13 (v) 5 : 6 (vi) 8 : 13
(vii) 14 : 23 (viii) 10:99 (ix) a : b
(x) x : y

2. (i) $\frac{2}{3}$ (ii) $\frac{7}{4}$ (iii) $\frac{19}{20}$ (iv) $\frac{99}{100}$
(v) $\frac{1}{10}$ (vi) $\frac{4.1}{5.2}$ (vii) $\frac{a}{b}$ (viii) $\frac{x}{y}$

3. (i) 1 : 3 (ii) 5 : 8 (iii) 3 : 2
(iv) 6 : 1 (v) 7 : 1 (vi) 15 : 2
(vii) 1 : 3 (viii) 1 : 20 (ix) 6 : 4 : 3
(x) 3 : 4 : 5 (xi) 1 : 2 : 3 (xii) 100 : 10 : 1

4. (i) 2 : 5 (ii) 5 : 2 (iii) 1 : 5
(iv) 73 : 48 (v) 1 : 7 : 15

5. (i) 1:10 (ii) 1:2 (iii) 4:5
(iv) 3:2 (v) 100:1 (vi) 1:6

Exercise 6.2

1. (i) P = 8 (ii) P = 1.5 (iii) P = 24

2. (i) x = 14 (ii) x = 9 (iii) x = 1
(iv) x = 3

3. 72 **4.** 12 **5.** 6 **6.** 9 machines

7. 96 litres **8.** 6 days

9. 28 days **10.** Rs.2250 **11.** 36 minutes

12. 72 more persons

Review Exercise 6

1. (i) 7 : 10 (ii) 5 : 17 (iii) 2 : 5
(iv) 2 : 3

2. (i) 1 : 5 (ii) 1 : 6 (iii) 5 : 6

3. (i) 7 : 15 (ii) 6 : 7 (iii) 2 : 15

4. Usman = Rs. 425, Waleed = Rs. 255,
Total = Rs. 850

5. 280m **6.** 17 litres **7.** 75 days **8.** 585 goats

Objective Exercise 6

1. (i) The numerical compassion between two quantities of the same kind is called ratio.

(ii) A proportion shows the relation of equality of two ratios.

(iii) 1st and 4th elements in a proportion.

2. (i) ratio (ii) means (iii) direct

3. (i) a (ii) c (iii) d
(iv) b (v) b

Exercise 7.1

1. (i) $\frac{9}{20}$, 0.45 (ii) $\frac{3}{50}$, 0.06
(iii) $\frac{14}{25}$, 0.56 (iv) $\frac{24}{25}$, 0.96
(v) $\frac{9}{50}$, 0.18 (vi) $\frac{12}{25}$, 0.48
(vii) $\frac{39}{50}$, 0.78 (viii) $\frac{89}{100}$, 0.89
(ix) $\frac{17}{25}$, 0.68 (x) $\frac{3}{20}$, 0.15

(xi) $3\frac{1}{2}$, 3.5 (xii) $1\frac{3}{5}$, 1.6

2. (i) 50% (ii) 25% (iii) 350%
(iv) 12.5% (v) 30% (vi) 45%
(vii) 59% (viii) 380%

3.

	Fraction	Decimal	Ratio	Percentage
(i)	$\frac{1}{2}$	0.5	1 : 2	50%
(ii)	$\frac{3}{5}$	0.6	3 : 5	60%
(iii)	$\frac{4}{5}$	0.8	4 : 5	80%
(iv)	$\frac{2}{3}$	0.67	2 : 3	66.67%
(v)	$\frac{1}{4}$	0.25	1 : 4	25%
(vi)	$\frac{3}{5}$	0.6	3 : 5	60%
(vii)	$\frac{7}{20}$	0.35	7 : 20	35%
(viii)	$\frac{14}{25}$	0.56	14 : 25	56%
(ix)	$\frac{1}{5}$	0.2	1 : 5	20%
(x)	$\frac{1}{40}$	0.025	1 : 40	2.5%
(xi)	$\frac{1}{60}$	0.017	1 : 60	1.67%
(xii)	$\frac{1}{8}$	0.125	1 : 8	12.5%

4. (i) 4 (ii) 16 (iii) 3
(iv) 35 (v) 90 (vi) 171
(vii) 39 (viii) 252 (ix) 370
(x) 285

Exercise 7.2

1. Rs.60 2. 12.5% 3. 45 students 4. 125 km
5. 88% 6. 9,750 voters 7. 35% 8. Rs.2000
9. 80% 10. 28% 11. 1190 shoes 12. Rs.150
13. Spend = Rs.1260/, Save: Rs.540
14. 38.5% 15. 60%

Exercise 7.3

1. Rs.20 2. Rs. 24 3. 18% 4. 8.33%
5. Profit % = 35% 6. Loss %, 4%
7. (i) Rs.4500 (ii) Rs.5040 8. Rs.305
9. (i) Cost price = Rs.1900
(ii) Selling price = Rs.1425
10. (i) Cost price = Rs.1250
(ii) Selling price = Rs.1750
11. (i) Selling price = Rs.800
(ii) % profit = Rs.88.24%
12. (i) Profit (ii) 13.84% 13. (i) Profit
(ii) 19.2%

14. 20% 15. Rs.600 16. (i) Rs.300 (ii) Rs.225
17. Rs.900 18. Rs.439

Review Exercise 7

1. 8% (ii) 11.5% (iii) 83.33% (iv) 40%
2. 13% 3. 23200 votes 4. 37.5% by bus, 62.5 % by train
5. 12.5% 6. 5.17% 7. 7.7% 8. Rs. 550 9. 20%

Objective Exercise 7

1. (i) The fraction with denominator 100 is called percentage.
(ii) Multiply the fraction with 100%.
(iii) Profit/loss = sale price – cost price
(iv) Out of 100.
(v) Discount = Marked price – sale price.
2. (i) % (ii) cost price (iii) cost price
(iv) fraction
3. (i) b (ii) c (iii) b (iv) b (v) a

Exercise 8.1

1. i, ii, ix, are true statements. iii, vi, vii are false statements. iv, v, viii are open statements.
2. (i) x = 4 (ii) P = 8 (iii) m = 5
(iv) x = 8 (v) x =15 (vi) m = 7
(vii) $m = \frac{1}{30}$ (viii) $m = \frac{3}{2}$ (ix) x = 0.7
(x) x = 14 (xi) p = 6 (xii) m = 2

Exercise 8.2

1. (i) x + y (ii) a – b (iii) mn
(iv) $\frac{p}{q}$ (v) 3x + 2y (vi) 5a – 4b
(vii) xy (viii) $\frac{p+q}{r}$ (ix) $\frac{l}{2}$ (n – m)
2. (i) Co–efficient = 5
Base = x
Exponent = 1

(ii) Co–efficient = 16
Base = p
Exponent = 2

(iii) Co–efficient = 18
Base = l
Exponent = 3

(iv) Co–efficient = –6
Base = k
Exponent = 5

(v) Co–efficient = $\frac{2}{3}$
Base = q
Exponent = –1

(vi) Co–efficient = $\frac{1}{3}$
Base = y
Exponent = –2

3. (i) a^3 (ii) x^2 (iii) x^2y^2
(iv) m^4 (v) p^3q^3 (vi) $a^2b^2c^2$

4. (i) 2a, 3b (ii) l, –2m, 4n
(iii) $9a^2, -12b^2$ (iv) $p^2, 2q^2, -r^2$
(v) a, 8b, –4c (vi) $2lm, -3mn, -4nl$
(vii) $3xy^2, 4x^2y, 9$ (viii) $\frac{2}{5}xy, \frac{1}{3}yz, \frac{3}{5}xz$
(ix) $\frac{a}{b}, \frac{b}{c}, \frac{c}{a}$

5. (i) a + b (ii) x – y (iii) $l + m - n$
(iv) p + pq + qr (v) $xy^2 + xz^2 + yz^2$
(vi) $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ (vii) $16a^2 - 8b^2$
(viii) $\frac{l}{m}-\frac{m}{n}-\frac{n}{l}$ (ix) 2ab + 4ac – 3bc

Exercise 8.3

1. (i) 4x (ii) 9y (iii) 10m
(iv) 10a + 3b (v) 5p + q
(vi) 2x + 3y (vii) 19a + 9b
(viii) m + 9n (ix) 3x + y + 2z
(x) 3p + 3q + r

2. (i) (2p + q) chocolates
(ii) $(m + 2n + 3l)$ books
(iii) (3x + y) candies

3. (i) b (a + 3c) (ii) xy (3x + y)
(iii) $6m^3 + 5m^2 + 1$ (iv) $2a^2 + 7ab + b^2$
(v) $x^2 - 2xy + y^2$ (vi) p – 3q – r

4. (i) $3a^2 - ab + b^2$ (ii) $2y(2x^2 + y)$
(iii) $6(lm+mn+nl)$ (iv) 5p + 6q + 4r
(v) 2a + 2b + 7

5. (i) 2a + 3b + 4c (ii) x + 2y
(iii) 2s + st + t + 3g (iv) 3p – 2r
(v) $2(lm + mn + nl)$

Exercise 8.4

1. (i) 2x (ii) 11a (iii) 2 (iv) –2n
(v) p + r (vi) x^3-x^2-2x+3 (vii) $x^3 + y^3 - 1$
(viii) $x^2 + 6xy + 3xy^2 - xy^2 + 9y^2$
(ix) $x^3+x^2y-xy^2-y^3$ (x) $x^2 - 5xy + 2y^2$

2. $-l - m - 5n$ **3.** $3a^3 + 4a^2 - 3a - 8$

4. $5x^5 + 9x^4 - 11x^3 + 4x + 8$

5. (i) 2b (ii) 2(c–b) (iii) 2c
(iv) 2(a + b) (v) 2(a + c)
(vi) 2(a+b+c) (vii) a – b + 3c
(viii) a + b + 3c (ix) –a + b – c
(x) 2(a + 2c)

6. $x^3 - x^2 - xy - y^2 + 2$ **7.** $p^4 + p^3 + p^2 + p$

Exercise 8.5

1. (i) 5a (ii) 2x (iii) $5(l - m)$
(iv) 2y (v) $x^2 + 2xy + y^2$
(vi) $9a^4 + 6a^2 + 1$ (vii) x^2
(viii) $-7l + 10m$ (ix) 12a + b + 6c
(x) $3x^2 - 2xy - y^2$ (xi) 200b
(xii) 17a – 2b (xiii) 3a + 2b – c
(xiv) 15(x – y) (xv) $2(x^2 - y^2)$

Exercise 8.6

1. (i) 3 (ii) 1 (iii) 2 (iv) 4
(v) 1 (vi) 2 (vii) 3 (viii) 8
(ix) 2 (x) 3 (xi) 8 (xii) 6
(xiii) 2 (xiv) 3 (xv) 4

7. 1 **8.** 5

Review Exercise 8

1. (i) $x^2 + 4x - 1$ (ii) $2x^2 - 11x + 6$
(iii) 7a – b – c (iv) –5a + c
(v) $14l^2 - 13m - 18n^3$ (vi) $p^2 + q^3$

2. (i) $-6a + b + 5c$ (ii) $11p + 2q + 5r$
(iii) $-2x^3 - 8x^2 + 4x - 3$
(iv) $-a + 3b + 5c - 13d$
(v) $5x^2 + 3xy + 8y^2 - 7$
3. (i) $2x^2 + 10xy - 6y^2$ (ii) $4x - 4y - 3z$
(iii) $5a$ (iv) $-l - 8m$
4. (i) 3 (ii) 80 (iii) 6 (iv) 1 (v) 8 (vi) 37
(vii) 82 (viii) 63 (ix) -156

Objective Exercise 8

1. (i) A group of words that makes a complete sense is called a sentence.
(ii) An open statement is a sentence which has one or more than one unknowns.
(iii) solution
(iv) A variable is a symbol, usually a letter, that stands for a number.
(v) An evaluation is a process of finding the absolute value of an expression.
2. (i) statement (ii) general (iii) coefficient
(iv) terms (v) like
3. (i) c (ii) a (iii) b (iv) c

Exercise 9.1

1. (i) $x + 8 = 14$ (ii) $x - 7 = 9$
(iii) $2x = 16$ (iv) $\frac{x}{3} = 2$
(v) $x + 2 = 4$ (vi) $x - 4 = 3$
(vii) $2x + 3 = 17$ (viii) $x+y=20$
(ix) $2x + 7 = y$ (x) $6x = y$
2. (i) $x = -\frac{1}{2}$ (ii) $a = 1$
(iii) $x = 8$ (iv) $x = 6$
(v) $x = 3$ (vi) $x = 18$
(vii) $x = 3$ (viii) $y = -12$
(ix) $x = -1$ (x) $m = -2$
(xi) $x = 4$ (xii) $x = 5$
(xiii) $x = 7$ (xiv) $m = 4$
(xv) $a = -7$ (xvi) $a = 4$
(xvii) $x = 10$ (xviii) $x = 5$
(xix) $x = 2$ (xx) $x = 24$
(xxi) $x = 0.5$

Exercise 9.2

1. (i) 7 (ii) 9 (iii) $\frac{9}{4}$
(iv) 7 (v) 5 (vi) 2
2. Rs.25 3. 4 and 8 4. 8 5. 8 and 2
6. book = Rs. 40,Pen = Rs. 5
7. Page # 46 and 47 8. 4 runs

Review Exercise 9

1. (i) $x = \frac{2}{5}$ (ii) $x = 9$ (iii) $x = 8$
(iv) $x = \frac{16}{5}$ (v) $x = -5$ (vi) $x = 5$
2. 3 3. length = 57m, breadth = 17m
4. 65, 66, 67, 68 5. $\frac{9}{5}$

Objective Exercise 9

1. (i) An equation is an open mathematical statement with "=" sign.
(ii) The equation which contains a single variable with the greatest exponent of 1 is called a linear equation.
(iii) The process of finding the values of unknowns is called solving an equation.
(a) What is the required thing?
(b) Represent the required thing by a variable.
(c) Write an equation according to the statement.
(d) Solve the equation and check the solution.
2. (i) equation (ii) equality (iii) solution
(iv) 1 (v) transposition
3. (i) a (ii) c (iii) d (iv) b

Objective Exercise 10

1. (i) The measurement of the earth.
(ii) A right bisector of a line is a line which is perpendicular to it and passes through its mid-point.
(iii) Two angles of the same measurement are called congruent angle.
(iv) Only three elements
(v) A line segment is a part of a line which has two distinct end points.
2. (i) line segment (ii) degree (iii) 180^o
(iv) side
3. (i) c (ii) b (iii) c (iv) b

Exercise 11.1

1. (i) area = 49 cm^2, perimeter = 28 cm
(ii) area = 24 cm^2, perimeter = 20 cm
(iii) area = 99.2 cm^2, perimeter = 40.8 cm
(iv) area = 27 cm^2, perimeter = 24 cm
(v) area = 121 cm^2, perimeter = 44 cm
(vi) area = 289 m^2, perimeter = 68 m

(vii) area = 72.8 cm^2, perimeter = 35.4 cm
(viii) area = 28.88 cm^2, perimeter = 22.8 cm
(ix) Area = 20.25 m^2, perimeter = 18 m

2. 45m **3.** 9km^2 **4.** 11m
5. area = 441m^2 perimeter = 84m
6. Rs. 14,782.50
7. cost of leveling = Rs. 1,08,900, cost of fencing = Rs. 11000
8. cost of ploughing = Rs. 57,600, cost of fencing = Rs. 3,456
9. cost of repairing = Rs. 67,500, cost of constructing wall = Rs. 63, 750

Exercise 11.2

1. (i) 136 m^2 (ii) 175 m^2
2. (i) 616 m^2 (ii) 1176 m^2
(iii) 936 m^2 (iv) 973 m^2
3. 1776 m^2 **4.** Rs.1920 **5.** Rs.171,900

Exercise 11.3

1. Area = 153m^2,
2. 18 m **3.** 3750 m^2 **4.** 15 m **5.** 35 m
6. (i) Cost of leveling = Rs.126,000
7. Rs. 1,31,250 **8.** Rs. 4,33,182

Exercise 11.4

1. (i) 42m^2 (ii) 18m^2 (iii) 34.5m^2
(iv) 1125m^2 (v) 2475m^2 (vi) 564.2m^2
2. 301m^2 **3.** 440m^2 **4.** 100m^2
5. Rs.112,500 **6.** Rs.3456

Exercise 11.5

1. (i) 56m^2 (ii) 152m^2 (iii) 90cm^2
(iv) 33.5m^2 (v) 18.2m^2 (vi) 128.64cm^2
(vii) 26.4cm^2 (viii) 412.5m^2
2. 24.3m^2 **3.** 27.38cm^2 **4.** 448cm^2 **5.** Rs. 74,052
6. 36.9 quintal **7.** Rs. 10,105 8. 44712 flowers

Review Exercise 11

1. 104cm **2.** Rs. 36000 **3.** Rs. 31200 **4.** Rs. 2400
5. Rs.51300 **6.** (i) 52.7 cm^2 (ii) 6.075 cm^2

Objective Exercise 11

1. (i) Square = 4(length)
Rectangle = 2 (length + breadth)

(ii) $\text{Border Area} = \left(\text{outer length} \times \text{outer breadth}\right) - \left(\text{inner length} \times \text{inner breadth}\right)$

(iii) A quadrilateral with only two parallel sides.
(iv) A plane figure with four straight sides whose opposite sides are parallel and of equal lengths is called a parallelogram.

2. (i) square (ii) parallelogram (iii) triangle
(iv) perimeter (v) triangle
3. (i) b (ii) a (iii) d (iv) d (v) b

Exercise 12.2

1. (i) 64cm^3, 96cm^2 (ii) 343 cm^3, 294cm^2
(iii) 15.625cm^3, 37.5cm^2
(iv) 32.768cm^3, 61.44cm^2
2. (i) 30cm^3, 67cm^2 (ii) 34.2cm^3, 67.4cm^2
(iii) 375000 cm^3, 32500cm^2
(iv) 56.25cm^3, 227.25cm^2
3. 12000m^3 **4.** 6000 litres **5.** 1800 cubes
6. 98m^2 **7.** Rs.540 **8.** Rs.1040

Review Exercise 12

1. Volume, 1728 cm^3, Rs. 432 **2.** 56cm^2
3. 125 cubes **4.** volume of the packet = 90cm^3
No. of packets = 480

Objective Exercise 12

1. (i) height (ii) A solid with an equal length, width and height.
(iii) $2\left[(l\times b)+(l\times h)+(b\times h)\right]$
(iv) Measurement of the space occupied by an object.
2. (i) edges (ii) cuboid (iii) sphere
(iv) slant height (v) 6 (length)^2
3. (i) b (ii) b (iii) c (iv) a (v) c

Exercise 13.1

2, 3, 6 and 7 are showing a grouped data.

Exercise 13.2

2. (i) The bar graph is showing in general the daily pocket money of some children.
(ii) Ghauri (iii) Maria (iv) Rs. 25
(v) Rs. 10 (vi) 3 : 2
3. (i) The bar graph is representing the sale of fans of a company in five years.

(ii) 2006 = 1000 fans 2009 = 2000 fans
2007 = 4000 fans 2010 = 5000 fans
2008 = 3000 fans

(iii) 4000 fans (iv) 3000 fans (v) 2010 (vi) 1 : 5

Exercise 13.3

1. (i) 80 students (ii) 10 students
(iii) 30 students

2. (i) 450 girls (ii) 900 girls
(iii) 150 girls (iv) 300 girls
(v) Drama (vi) Movies

3. (i) 4 sectors
(ii) Football, hockey and volleyball
(iii) 360 students (iv) 120 students
(v) cricket (vi) 240 students
(vii) 480 students

Review Exercise 13

2. (i) Rs. 22000 (ii) Saturday (iii) Monday
(iv) The graph is representing in general the amount that Tariq earned in six days.
(v) 4 : 1

3. (i) 810 votes (ii) 1620 votes
(iii) 1080 votes (iv) 2430 votes
(v) 540 votes (vi) Saleem

Objective Exercise 13

1. (i) A data is a set of information and facts.
(ii) A grouped data provides information about groups.
(iii) Ungrouped and grouped data.
(iv) The data which provides us the information about individuals.
(v) Horizontal and vertical bar graph.
(vi) A pie graph is used for inter comparison of a data.
(vii) 360^0

2. (i) questionnaire (ii) two (iii) graph
(iv) horizontal (v) bar (vi) scale
(vii) axis

NOTES FOR THE TEACHERS

We live in a mathematical world. When we decide on a purchase, make a picnic plan or pay the utility bills, we rely on mathematical understanding.

Students have different abilities, needs and interests. Effective teaching of Mathematics requires understanding what students know and need to learn. Then there is a challenge to support them to learn it. This curriculum is mathematically rich, provides students with opportunities to learn important mathematical concepts and procedures in accordance with understanding and solving their problems through Maths. Here are some suggestions for teaching this book in your classroom.

Unit 1: Sets

When your students begin to learn about sets, it is essential that they understand exactly what is a collection. Give the students more examples and tell them only those collections whose objects are distinct and well-defined, called sets. A set can be classified with the help of its number of elements.

- If there are countable number of elements in a set, then it is a finite set.
- If there are uncountable number of elements in a set, then it is an infinite set.
- If there is no element in a set, then it is an empty or null set that can be represented by the symbol { } or ϕ.

Write any two equivalent sets on the blackboard and ask the students to establish one-to-one correspondence between them. Now explain to the students that these are equivalent sets and we use symbol $\leftrightarrow$ to write them. In the same way, you can explain about equal and non-equivalent sets. Enrich your teaching of sets by giving examples of subsets and supersets.

Unit 2: Whole Numbers

The introduction of whole numbers provides us an opportunity to understand the meaning of numbers and their role in our daily life. Compare bigger whole numbers like million, billions and trillions with lacs, crores and arebs in the classroom. This will create an interest about bigger number. Give some examples about laws of addition and laws of multiplication related to the real life.

Unit 3: Factors and Multiples

Here we have given some simple examples to understand the relation between a factor and a multiple but you may wish to develop additional worksheets. Give your students more examples and plenty of practice to learn the tests of divisibility. It may be noted that the divisibility test also provides us an easy method to separate even and odd numbers. You can help the students to present the prime factorization in terms of a factor tree. Make sure you have thoroughly discussed that how H.C.F and L.C.M can help us to solve a real- life problem.

Unit 4: Integers

When introducing integers, make sure every child knows about whole numbers very well. Draw a line on the floor of the classroom and mark integers on it to show that each positive integer corresponds to the negative integer and the absolute value of an integer is always positive because it tells us only the distance from the starting point.

Unit 5: Simplification

Before starting the simplification, make sure your students know the methods to perform the basic operations involving fractions and decimals. Tell the students simplification rule is also named as BODMAS rule that helps us to remember the order in which operations are done. You can use your blackboard to present simple all four-operation sum. Ask the students 'Which part of the sum should be solved first or what is the next operation to tackle?'

Unit 6: Ratio and Proportion

When introducing the concept of ratio and proportion, be sure to establish a link between fraction and ratio. Your students need to understand that ratio is another form of a fraction. The first element denotes the numerator and the second element denotes the denominator.

Here your working on multiples and factors will also help the students in writing the reduced form of a ratio and equivalent ratios. The section on direct and inverse proportion includes a discussion on how one quantity changes by changing the other quantity. Give your students plenty of practice to identify the relation between two quantities.

Ask your students to draw some tables which represent the relation of direct and inverse proportion.

Unit 7: Financial Arithmetic

Your success in teaching percentage depends largely on the extent to which your students understand the concept of fraction because percentage is also a special kind of fraction with 100 as denominator. If the students have the basic concepts of fractions and they need to if they are to understand percentage, half the battle is won.

Discuss the ways in which percentage relates to everyday problems. Offer extra credit to your students who bring in examples of 'real problems they encountered.' This will encourage them to think about percentage in their off time.

Unit 8: Introduction to Algebra

Algebra can be one of the most difficult subject to teach because it may seem less visual. But if taught correctly, it provides the basics for critical thinking and logic skills.

Here your teaching focuses on important mathematical concepts your students must know to be successful in algebra. As students improve in their knowledge of it, start challenging them to answer thought provoking questions, not the "what is the answer?" type. Keep the question simple. It is surprising how useful a very basic question can be.

Make sure that students do their homework. It is the best way to keep track of each student's progress and ensure that all are getting the practice needed to grasp algebra.

Unit 9: Linear Equations

This section focuses on the ways to help students to grasp the meaning, significance and practical application of a linear equation. Before starting, make sure your students understand the concept of an equation very well. Give them examples related to the weighing balance. It is essential that students remember and internalize this basic idea as they progress with the subject.

Also tell the students that transposition is an important method we often use to find the value of an unknown in an equation.

Students learn best when examples are presented one by one with increasing complexity. The first example problem is that your work should be extremely simple so that every single student in the classroom will completely follow it. This builds their confidence; then increase the difficulty of the problems that you solve on the blackboard.

Unit 10: Geometry

Before starting the section geometry, it is important to check that each student mastered its basic concepts like point, line, ray, line segments, etc. Plenty of practical work should accompany in the review of these concepts, so that students become accurate in use of geometrical instruments.

Tell the students that the diagrams are essential parts of the explanations and exercises, and theyy need to read them along with text. While teaching this section you may need to draw diagrams on the blackboard from time to time. This will help the students to understand what they are learning. At the end, students must get enough opportunity and practice in drawing these diagrams on their notebooks.

Unit 11: Perimeter and Area

Ask the students to think what they already know about and perimeter. Use the following tips to teach this section.

- Explain in the language students can understand easily. Relate the topic to reallife concepts.
- Take a ruler and measure the length and width of few things in the classroom such as a book, top of the table or desk, floor of the classroom, etc. Help the students to calculate the area and perimeter of these things.
- Tell students that area is measured in square units, i.e. cm^2, m^2, $feet^2$ and so on.
  While solving the problems, draw diagrams that help the students to visualize the situation.

Unit 12: Three dimensional Solids

To introduce the concept of three dimensional solids, draw some shapes on the blackboard such as cube, cuboid, sphere, cylinder and cone. Ask students to think about where they have seen these shapes throughout the school and at home. Also teach them vocabulary such as vertex, radius, base, slant height, etc by using the diagrams to describe the shapes and their basic properties.

Explain by drawing a cube or cuboid on the blackboard what each side of the figure is and the shapes that make up the figure, so that the students will be able to understand the makeup of the formula. Then demonstrate each formula and ask if there are any questions.

Unit 13: Information Handling

Explain the students that information handling actually is the part of the statistics. It is the study of such methods as are used in the collection of data, its analysis and presentation.

Encourage the students to collect data themselves and with their friends but the emphais should be on presenting it in a variety of forms such as bar graph or pie graph.

Ask the students to draw graphs of their own on notebooks. To be successful they need to understand what the graph is saying and relate that to the information they have been given. This requires high-level thinking, especially if you ask them to justify what they have done. It is probably helpful to ask them different questions as:

- How do you know that?
- Which bar is easiest to identify and why?
- Which sector has the biggest angle?

Offer them plenty of opportunities to think without insisting on quick answer. After they have had this chance, find out their ideas. These questions would also help the students to realize the significance of the labels on axis.

GLOSSARY

Absolute value: The absolute value of a number is its distance, in either direction, from zero on a number line.

Addition Property of Equality: If the same number is added to each side of an equation, the two sides remain equal.

Additive inverses: Two numbers whose sum equals zero are called additive inverses.

Algebraic expression: A mathematical phrase involving a variable or variables, numbers, and operations.

Angle bisector: An angle bisector is a ray that divides an angle into two congruent angles.

Angle: An angle is made up of two rays with a common endpoint.

Area: The area of a figure is the amount of space that it encloses.

Associative property of Addition: Changing the grouping of the numbers does not change the sum.

Associative property of Multiplication: Changing the grouping of the factors does not change the product.

Axis: Either of two lines drawn perpendicular to each other in a graph.

Bar graph: A graph that uses bars to show data.

Base: When a number is written in exponential form, the number that is used as a factor is the base.

Bisection: To divide into two equal parts.

Capacity: Capacity is a measure of the amount of space an object or a liquid occupies.

Common multiples: Multiples that are shared by two or more numbers are common multiples.

Commutative Property of Addition: Changing the order of the numbers does not change the sum.

Compass: A compass is a geometric tool used to draw circles and arcs.

Composite number: A number that has more than two factors is called a composite number.

Common factor: A factor that is the same for two or more numbers.

Cone: A cone is a three-dimensional figure with one circular base and one vertex.

Congruent: Having the same size and shape.

Cross products: The cross products of the proportion $\frac{a}{b}=\frac{c}{d}$ are a × d and b × c.

Cubic Unit: A unit measuring volume, consisting of a cube with edges one unit long.

Cylinder: A three-dimensional figure that has two circular bases which are parallel and congruent.

Data: Information that is gathered.

Denominator: The number below the fraction bar in a fraction; the total number of equal parts in all.

Degree (^{o}): A unit of measure for angles.

Discount: The amount a price is reduced is called the discount.

Distributive Property: If a, b, and c are any numbers, then.

$$a \times (b + c) = a \times b + a \times c$$
$$a \times (b - c) = a \times b - a \times c$$

Divisibility rules: Rules that are used to find if a number is divisible by numbers such as 2, 3, 4, 5, 6, 9, or 10.

Divisible: One number is divisible by another number if the remainder is zero.

Division Property of Equality: If both sides of an equation are divided by the same nonzero number, the two sides remain equal.

Discount: The amount by which the regular price of an item is reduced.

Edge: A line segment where two faces meet in a solid figure.

Equation: A mathematical sentence that contains an equal sign, = , is an equation.

Equivalent fractions: Fractions that have the same simplest form are equivalent fractions.

Even number: An even number is a whole number that is divisible by 2.

Exponent: An exponent tells you how many times a number, or bas, is used as a factor.

Factor tree: A factor tree is used to find a number's prime factors.

Factor: One number is a factor of another if it divides that number with no remainder.

Geometry: An important branch of mathematics that deals with the study of points, lines, surfaces and solids.

Height: The height, or altitude, of a parallelogram or triangle is the length of a perpendicular segment from a vertex to the line containing the base.

Highest common factor: The highest common factor of two or more numbers is the greatest number that is a factor of all the numbers.

Identity Property of Addition: The sum of zero and any number a is a.

Identity Property of Multiplication: The product of 1 and any number a is a.

Improper fraction: A fraction whose numerator is greater than or equal to its denominator is called an improper fraction.

Inequality: An inequality is a statement comparing expressions that are not equal.

Integers: Integers are the set of whole numbers and their opposites.

Inverse Operations: Operations that "undo" each other, such as addition and subtraction, or multiplication and division (except multiplication by 0).

Least common multiple: The least number that a common multiple of two or more numbers is the least common multiple.

Line segment: A segment is part of a line. It consists of two points and all the points on the line that are between the two points.

Line: A straight path of points that goes on forever in two directions.

Mixed number: A mixed number shows the sum of a whole number and a fraction.

Midpoint: The point that divides the segment into two segments of equal length.

Monomial: A polynomial that has only one term is a monomial.

Multiple: A multiple of a number is the product of that number and any nonzero whole number.

Multiplication Property of Equality: If each side of an equation is multiplied by the same number, the two sides remain equal.

Multiplicative inverse: The reciprocal of a number is also called its multiplicative inverse.

Negative integers: Integers that are less than zero.

Null set: A set having no element.

Numerator: The number above the fraction bar in a fraction; the number of objects or equal parts being considered.

Odd number: An odd number is a nonzero whole number that is not divisible by 2.

Opposite numbers: Two number that are the same distance from zero on a number line, but in different directions, are opposites.

Order of operations: The order in which operations are done in calculations. Work inside parentheses is done first. The multiplication ad division are done in order from left to right and finally addition and subtraction are done in order from left to right.

Parallelogram: A parallelogram is a quadrilateral with both pairs of opposite sides parallel.

Percent (%): A percent is a ratio that compares a number to 100.

Perimeter: The perimeter of a figure is the distance around it.

Perpendicular bisector: A perpendicular bisector is a line perpendicular to a line segment that passes through the midpoint of the line segment.

Perpendicular lines: Lines that intersect to form right angles are perpendicular.

Pie graph: A pie graph shows the parts of a whole. The total must be 100% or 1.

Place Value: The place value system allows us to represent any number using one or more of 10 digits. The value of a digit depends on its place within the number.

Point: An exact location in space.

Prime factorization: Writing a composite number as the product of its prime factors is called prime factorization.

Proportion: A proportion is an equation that states two ratios are equal. The cross products of a proportion are always equal.

Range: The range of a set of data is the difference between the greatest and the least values in the set.

Ratio: A ratio is a comparison of two numbers.

Ray: A ray is part of a line. It consists of one endpoint and all the points of the line on one side of the endpoint.

Reciprocal: Two numbers are reciprocals if their product is 1. Dividing by a number is the same as multiplying by the reciprocal of that number.

Right angle: A right angle is an angle with a measure of 90°.

Simplest form of a fraction: A fraction is in simplest form when the only common factor of the numerator and denominator is 1.

Simplest form: A fraction for which the greatest common factor of the numerator and denominator is 1; also, lowest terms.

Singleton Set: A set having a single element.

Solid figure (also solid): A figure that has three dimensions and volume.

Solution of an equation: The solution of an equation is the value of the variable that makes the equation true.

Surface area (SA): The sum of the areas of each face of a solid.

Three-dimensional figure: Figures, such as buildings, that do not lie in a plane are three-dimensional. The dimensions are length, width, and height.

Tree diagram: A tree diagram displays all the possible outcomes of an event.

Triangle: A triangle is a polygon with three sides.

Variable: A variable is a symbol, usually a letter, that stands for a number.

Volume: The volume of a three-dimensional figure is the number of cubic units needed to fill the space inside the figure.

***x*-axis:** The x-axis is the horizontal number line that, together with the y-axis, forms the coordinate plane.