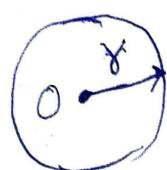


MENSURATION

CHAPTER-14 [Areas Related to Circles]

Circle - Circle is the locus of a point which moves in such a way that its distance from fixed point 'O' remains constant.



$r \rightarrow$ radius
 $O \rightarrow$ fixed point 'O' called center

Ex: Bangle, coin, wheel, Moon, sun, sunflower papad.

Circumference - The boundary (perimeter) of a circle is called its circumference.

- ⑧ → It is the distance covered by travelling once around a circle.

$$\boxed{\text{Circumference} = 2\pi r}$$

$r \rightarrow$ radius
 $\pi \rightarrow$ constant

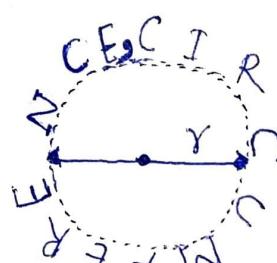
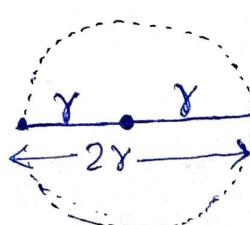
$\pi = 3.14$	$\pi = \frac{22}{7}$
--------------	----------------------

Special Case -

- * Circumference of a circle
 $= 2\pi r$ or πd

$r \rightarrow$ radius $d \rightarrow$ diameter of the circle

$$\boxed{d = 2r}$$



Soln-

diameter of a wheel = 84 cm.

\Rightarrow radius of wheel = $\frac{84}{2} = 42 \text{ cm}$

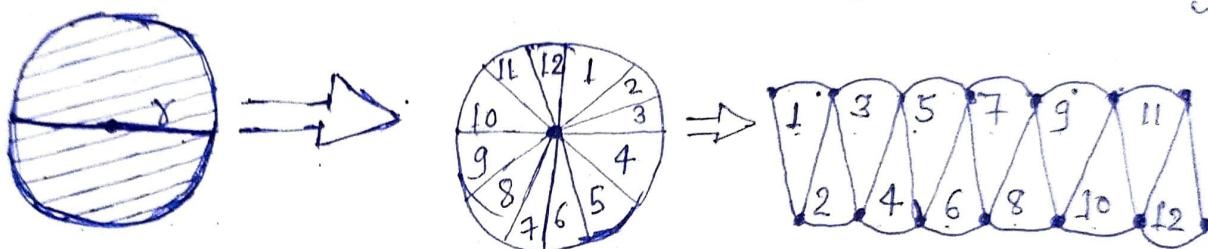
distance covered in 1 revolution = $2 \times \frac{\pi r}{7} \times 42$.

$$792 \text{ m} = 79200 \text{ cm}$$

$$= 264 \text{ cm}$$

No of complete revolution to cover 792m = $\frac{79200}{264}$
= 300 Ans.

Lec: 02 \rightarrow Area of the Circle $\leftarrow [\pi r^2]$



Area of the Circle = Area of the rectangle
= length \times breadth
= $\pi r \times r$

$$\boxed{\text{Area of the Circle} = \pi r^2}$$

* $\boxed{\text{Area of the semi-circle} = \frac{1}{2} \pi r^2}$

* Length of the Arc -



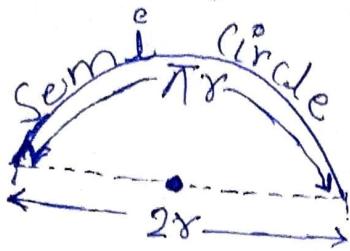
or

$$\boxed{\text{Arc length} = r\theta}$$

but here:-

θ : must be in radian

* Perimeter of a semicircle = $\pi r + 2r = r(\pi + 2)$



* distance moved by a wheel in 1 rotation = circumference of the wheel

* Number of Rotation in 1 minute = $\frac{\text{distance moved in 1 minute}}{\text{circumference}}$

Examples — find the circumference of the wheel cycle whose diameter is 21 cm.

Soln. — diameter = 21 cm

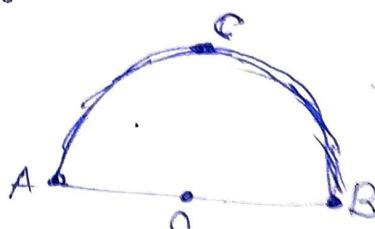
$$(\pi) \text{radius} = \frac{21 \text{ cm.}}{2} \quad (\because d=2r; r=\frac{d}{2}) \\ = 10.5 \text{ cm}$$

$$\text{Circumference} = 2\pi r = 2 \times 3.14 \times 10.5 = 66 \text{ cm} \quad \underline{\text{Ans.}}$$

Ex-2. If the diameter of a semicircle plot is 14 m.
Then find its perimeter.

Soln. — $d = 14 \text{ m} \Rightarrow 2r = 14$
 $\Rightarrow r = 7$

$$\text{Perimeter} = \text{Semicircumference} + \text{diameter (AB)} \\ (\text{ACB}) \\ = 22 + 14 = 36 \text{ m} \quad \underline{\text{Ans.}}$$



Ex-3. A wheel has diameter 84 cm. Find how many complete revolutions must it make to cover 792 meters.

Ex:- find area of circular park whose circumference is 22m:

Soln:- Let radius of circle = r m.

$$\Rightarrow \text{Circumference} = 2\pi r = 22.$$

$$\Rightarrow r = \frac{22}{2\pi} = \frac{11}{\pi}$$

$$\Rightarrow r = \frac{11}{\frac{22}{7}} = \frac{7}{2}.$$

$$\begin{aligned}\text{Area of circle} &= \pi r^2 = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 = \frac{22}{7} \times \frac{49}{4} \\ &= \frac{22 \times 7}{4} = \frac{77}{2}\end{aligned}$$

$$\underline{\text{Area of circle} = 38.5 \text{ m}^2.}$$

Ex:- If the perimeter and area of a circle are numerically equal, then find the radius of the circle?

Soln:- Let radius of circle = r

$$\text{Then, Perimeter of circle} = 2\pi r$$

$$\text{Area of circle} = \pi r^2$$

Accn to question -

$$\text{Area of circle} = \text{Perimeter of circle}$$

$$\Rightarrow \pi r^2 = 2\pi r$$

$$\Rightarrow \boxed{r = 2} \quad \underline{\text{Ans.}}$$

Ex:- difference b/w circumference and diameter of circular path plot is 105m. find the area of the circular plot.

Soln:- Circumference = $2\pi r$

Dia Diameter = $2r$

Accn to question-

$$2\pi r - 2r = 105$$

$$\Rightarrow 2r(\frac{22}{7} - 1) = 105$$

$$\Rightarrow 2r \left(\frac{22-7}{7} \right) = 105$$

$$\Rightarrow 2r \times 15 = \frac{105 \times 7}{7} \Rightarrow r = \frac{49}{2}$$

$$\text{Area of circular plot} = \pi r^2 = \frac{22}{7} \times \left(\frac{49}{2} \right)^2$$

$$= \frac{22}{7} \times \frac{49}{2} \times \frac{49}{2}$$

* Area of circular plot = $\frac{77 \times 49}{2} = 1886.5 \text{ m}^2$

Ex:- The radius of two circles are 8cm and 6cm respectively. equal. find the radius of the circle having its area equals to the sum of the areas of the two circles.

Soln:- Area of first circle = $\pi 6^2 = 36\pi$

Area of second circle = $\pi 8^2 = 64\pi$

$$\begin{aligned}\text{Sum of areas of two circles} &= 64\pi + 36\pi \\ &= 100\pi\end{aligned}$$

$$\text{Area of new circle} = 100\pi$$

$$\Rightarrow \pi R^2 = 100\pi$$

$$\Rightarrow R = 10 \text{ cm}$$

Ex:- An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 45cm. Find area b/w two consecutive ribs of the umbrella.

Soln-

$$\text{Area of rib} = \frac{1}{8} \times \text{Area of umbrella}.$$

$$\text{radius} = r = 45 \text{ cm.}$$

$$\text{Area of Umb.} \Rightarrow \pi r^2$$

$$= \pi \times (45)^2$$

$$= \frac{22}{7} \times 45 \times 45 = \frac{22}{7} \times 2025$$

$$\text{Area of rib} = \frac{1}{8} \times \text{area of umbrella} = \frac{1}{8} \times \frac{22}{7} \times 2025.$$

$$= \frac{22275}{28} \text{ cm}^2$$

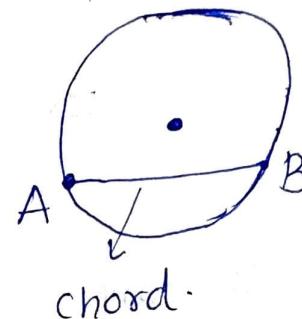
$$\text{Area b/w two consecutive rib} = \frac{22275}{28} \text{ cm}^2 \quad \underline{\text{Ans.}}$$

Lec: 03: Area Related to the Circle

(I) Chord - A line segment joining any two points on a circle is called a chord of the circle.

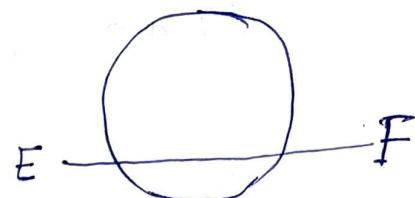
- In this figure AB is chord

fb-studyonlineclasses



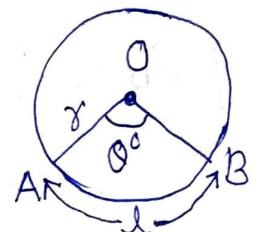
(II) Secant - A line which intersects a circle at two points on a circle is called a secant of the circle.

- In this figure EF is secant



(III) Arc:- Arc is part of the circle

- In this figure AB is an arc and it is written as \overarc{AB}



$$\text{Length of the Arc } AB = \frac{\theta}{360} \times 2\pi r$$

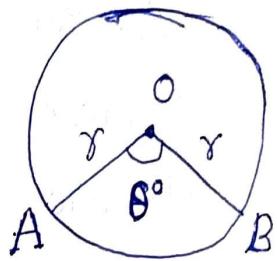
$$l = \frac{\pi r \theta}{180}$$

Sector of a Circle :-

The region bounded by two radii of a circle and the arc intercepted by them is called a sector of the circle.

(I) Perimeter of a sector:-

$$\text{Perimeter} = 2r + \frac{\pi r \theta}{180^\circ}$$



(II) Area of a sector:-

$$\text{Area} = \frac{\theta^\circ}{360^\circ} \times \pi r^2 = \frac{\pi r^2 \theta^\circ}{360^\circ}$$

(III) When Length of the arc (l) is given, Then :-

$$\text{Area} = \frac{1}{2} lr$$

$$\left(\text{since } l = \frac{\pi r \theta}{180^\circ} \right)$$

[fb-study online classes
YouTube- Rankers study point]



Example :-

Ex:-1 A sector is cut from a circle of diameter 21cm. If the angle of the sector is 150° , find its area.

Soln:-

$$\text{Diameter} = 21\text{cm} \Rightarrow \text{radius} = \frac{21}{2}\text{cm}$$

$$\text{Angle of sector} = 150^\circ$$

$$\text{Area of the sector} = A = \frac{\theta}{360} \times \pi r^2 = \frac{150}{360} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^2$$

$$= \frac{5}{12} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$$

$$= 144.38 \text{ cm}^2$$

Ex:- A sector is cut from a circle of radius 42 cm. The angle of sector is 120° . Find the length of its arc and area.

Soln:- Length of arc of sector angle θ and radius

$$l = \frac{\theta}{180} \times \pi r$$

$$\Rightarrow l = \frac{120}{180} \times \frac{22}{7} \times 42 = 88 \text{ cm}$$

$$\Rightarrow \text{Area of the sector} = A = \frac{\theta}{360} \times \pi r^2$$

$$\begin{aligned} &= \frac{120}{180} \times \frac{22}{7} \times (42)^2 \\ &= \frac{1}{3} \times 22 \times 6 \times 42 \\ A &= 1848 \text{ cm}^2 \end{aligned}$$

Ex:- The length of minutes hand of a clock is 14 cm. Find the area swept by the minutes hand in 5 minutes. [use $\pi = \frac{22}{7}$]

Sol No:-

Length of minutes hand = 14 cm

As, the minutes hand rotate in 5 minutes.

Area swept by the minute hand =

The Area of a sector of angle 30° , in a circle of radius 14 cm

$$\text{The required area} = \frac{30^\circ}{360^\circ} \times \pi r^2 = \left[\frac{30^\circ}{360^\circ} \times \frac{22}{7} \times (14)^2 \right] \text{cm}^2$$

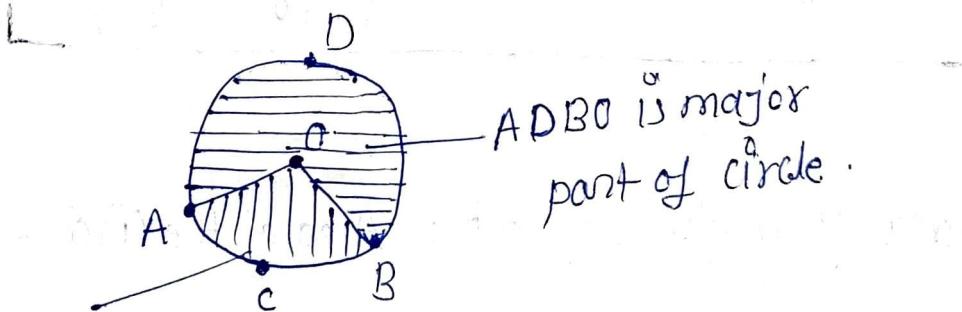
$$= \frac{154}{3} \text{cm}^2 = 51.33 \text{cm}^2 \quad \underline{\text{Ans.}}$$

lec:04

Area of a segment :-

Segment of a circle -

A segment of a circle is defined as the part of circle bounded by a chord and the arc-



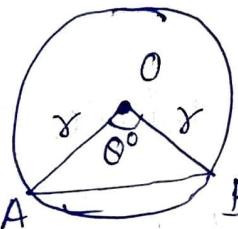
AOBG is
minor segment
of circle

⇒ Rankers study point / YouTube.

* Area of major segment and minor segment-

Area of minor segment =

Area of sector OAB - Area of triangle OAB



$$= \text{Area of sector OAB} - \frac{1}{2} \times \text{Base} \times \text{height}$$

$$= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} \times OA \times OB$$

$$= \frac{\theta}{360} \pi r^2 - \frac{1}{2} \times r \times r \sin \alpha$$

$$= \frac{\theta}{360} \pi r^2 - \frac{1}{2} r^2 \sin \alpha \quad \Rightarrow \text{Rankers study point/YouTube}$$

$$= \left(\frac{\pi \theta}{360} - \frac{\sin \alpha}{2} \right) r^2 \quad \text{study online classes/face book.}$$

$$\therefore \boxed{\text{Area of minor segment} = \left(\frac{\pi \theta}{360} - \frac{\sin \alpha}{2} \right) r^2}$$

Area of major segment = Area of the circle - Area of minor segment

$$= \pi r^2 - \left(\frac{\pi \theta}{360} - \frac{\sin \alpha}{2} \right) r^2$$

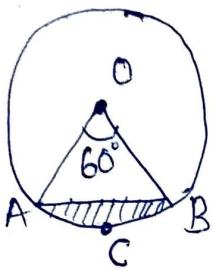
$$= \left(\pi - \frac{\pi \theta}{360} + \frac{\sin \alpha}{2} \right) r^2$$

Ex:- In a circle of radius 21 cm, an arc subtends an angle of 60° at the center -

- (i) find the length of arc
- (ii) Area of the minor segment
- (iii) Area of the major segment.

Soln:-

$$(I) \text{ Length of arc} = \frac{2\pi r \theta}{360^\circ} \text{ cm.}$$



$$= \left(2 \times \frac{22}{7} \times 21 \times \frac{60^\circ}{360^\circ} \right) \text{ cm.}$$

$$= 22 \text{ cm.}$$

(III)

$$\text{Area of minor segment} =$$

$$\left(\text{Area of sector } \frac{\theta}{360^\circ} \text{ of circle} \right) - \left(\text{area of the } \triangle OAB \right)$$

$$= \left(231 - \frac{1}{2} r^2 \sin \theta \right) \text{ cm}^2 = \left[231 - \left(\frac{1}{2} \times 21 \times 21 \times \sin 60^\circ \right) \right]$$

$$= 231 - \frac{441\sqrt{3}}{4} \text{ cm}^2$$

$$= 1155 \text{ cm}^2 - 40.05 \text{ cm}^2$$

(IV) Area of the major segment \Rightarrow

$$= \text{Area of the circle} - \text{Area of the minor segment}$$

$$= (\pi r^2 - 231) \text{ cm}^2 = \left(\frac{22}{7} \times 21 \times 21 - 231 \right) \text{ cm}^2$$

$$= 1155 \text{ cm}^2$$

Ex:- A chord AB of a circle of radius 15 cm subtends an angle of 60° at the center of the circle. Find the area of major and minor segment. [take $\pi = 3.14$, $\sqrt{3} = 1.73$]

$$\text{Soln: } A = r^2 \left[\frac{\pi \theta}{360^\circ} - \frac{1}{2} \sin \theta \right]$$

Here, $r = 15 \text{ cm}$, and $\theta = 60^\circ$

$$= (15)^2 \left[\frac{3.14 \times 60^\circ}{360^\circ} - \frac{1}{2} \sin 60^\circ \right]$$

$$= 225 \left[\frac{3.14}{6} - \frac{\sqrt{3}}{4} \right] \text{ cm}^2$$

$$= 225 [0.5233 - 0.4325] \text{ cm}^2$$

$$= 225 \times 0.0908 \text{ cm}^2$$

$$= 20.43 \text{ cm}^2$$

Area of major segment = Area of circle - Area of minor segment

$$= [3.14 \times (15)^2 - 20.43] \text{ cm}^2$$

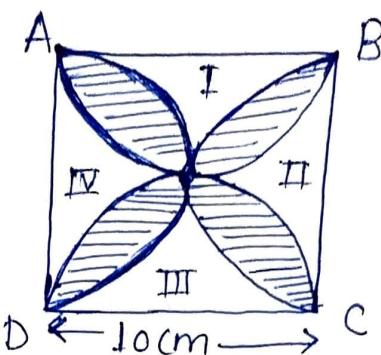
$$= [706.5 - 20.43] \text{ cm}^2$$

$$= 686.07 \text{ cm}^2$$

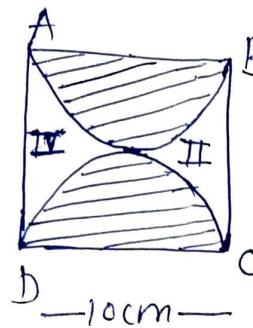
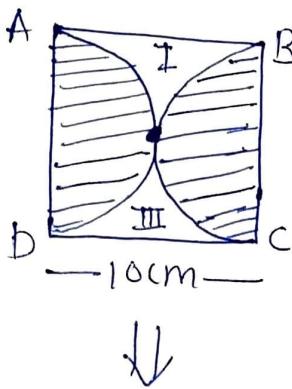
Ans:

[Rankers study point / YouTube]
fb- study online classes]

Ex:- find the area of the shaded design in adjoining fig. where ABCD is a square of side 10cm and semicircle are drawn with each sides of the square as diameter. ($\pi=3.14$)



Soln:-



\Downarrow

Area of Region

I + III

Area of region.

(IV) + (II)

= area of ABCD -

$$= \text{area of } ABCD - (\text{sem } AD + BC) \quad (\text{sem } AB + CD)$$

$$= 100 - \left(\frac{\pi(5)^2}{2} + \frac{\pi(5)^2}{2} \right) = 100 - \frac{50\pi}{2}$$

$$= 100 - \frac{25\pi}{2} - \frac{25\pi}{2} = 100 - 25\pi$$

$$= 100 - \frac{50\pi}{2} = 100 - 25\pi$$

Rankers study point / YouTube.
fb - Study online classes.

Area of shaded region = Area of ABCD - (Area of I + II + III + IV)

$$\begin{aligned} &= 100 - (3.14 \times 25) \\ &= 50(\pi - 2) \\ &= 50(3.14 - 2) \\ &= 57 \text{ Ans.} \end{aligned}$$

$$= 100 - (100 - 25\pi + 100 - 25\pi)$$

$$= 100 - (200 - 50\pi)$$

$$= 50\pi - 100 = 100(\pi - 1)$$