

Chapter # 5

Exercise# 5.1

Q1 $9s^2t + 15s^2t^3 - 3s^2t^2$

Solution

$$9s^2t + 15s^2t^3 - 3s^2t^2$$

Take common $3s^2t$

$$= 3s^2t(3 + 5t^2 - t)$$

Q2 $10a^2b^3c^4 - 15a^3b^2c^2 + 30a^4b^3c^2$

Solution

$$10a^2b^3c^4 - 15a^3b^2c^2 + 30a^4b^3c^2$$

Take common $5a^2b^2c^2$

$$= 5a^2b^2c^2(2bc^2 - 3a + 6a^2b)$$

Q3 $ax - a - x + 1$

Solution

$$ax - a - x + 1$$

Taking common

$$= a(x - 1) - 1(x - 1)$$

Taking common

$$= (x - 1)(a - 1)$$

Q4 $x^2 - 2y^3 - 2xy^2 + xy$

Solution

$$x^2 - 2y^3 - 2xy^2 + xy$$

Arrange it

$$= x^2 + xy - 2xy^2 - 2y^3$$

Taking common

$$= x(x + y) - 2y^2(x + y)$$

Taking common

$$= (x + y)(x - 2y^2)$$

Q5 $4x^2 + 4 + \frac{1}{x^2}$

Solution

$$4x^2 + 4 + \frac{1}{x^2}$$

$$= (2x)^2 + 2(2x)\frac{1}{x} + \left(\frac{1}{x}\right)^2$$

As we know that

$$= a^2 + 2ab + b^2 = (a + b)^2$$

$$= \left(2x + \frac{1}{x}\right)^2$$

Q6 $4(x + y)^2 - 20(x + y)z + 25z^2$

Solution

$$4(x + y)^2 - 20(x + y)z + 25z^2$$

$$= [2(x + y)]^2 - 2[2(x + y)](5z) + (5z)^2$$

As we know that $a^2 - 2ab + b^2 = (a - b)^2$

$$= [2(x + y) - 5z]^2$$

Q7

$$\frac{x^4}{y^4} - \frac{y^4}{x^4}$$

Solution

$$\frac{x^4}{y^4} - \frac{y^4}{x^4}$$

$$= \frac{(x^2)^2}{(y^2)^2} - \frac{(y^2)^2}{(x^2)^2}$$

$$= \left(\frac{x^2}{y^2}\right)^2 - \left(\frac{y^2}{x^2}\right)^2$$

Using Formula $a^2 - b^2 = (a + b)(a - b)$

$$= \left(\frac{x^2}{y^2} + \frac{y^2}{x^2}\right) \left(\frac{x^2}{y^2} - \frac{y^2}{x^2}\right)$$

$$= \left(\frac{x^2}{y^2} + \frac{y^2}{x^2}\right) \left[\left(\frac{x}{y}\right)^2 - \left(\frac{y}{x}\right)^2\right]$$

Using Formula $a^2 - b^2 = (a + b)(a - b)$

$$= \left(\frac{x^2}{y^2} + \frac{y^2}{x^2}\right) \left(\frac{x}{y} + \frac{y}{x}\right) \left(\frac{x}{y} - \frac{y}{x}\right)$$

Q8 $2x^2 - 288$

Solution

$$2x^2 - 288$$

Taking common

$$= 2(x^2 - 144)$$

$$= 2[(x)^2 - (12)^2]$$

Using Formula $a^2 - b^2 = (a + b)(a - b)$

$$= 2(x + 12)(x - 12)$$

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Q9 $1 - u^2 + 2uv - v^2$

Solution

$$1 - (u^2 - 2uv + v^2)$$

$$\text{Using Formula } a^2 - 2ab + b^2 = (a - b)^2$$

$$= (1)^2 - (u - v)^2$$

$$\text{Using Formula } a^2 - b^2 = (a + b)(a - b)$$

$$= [1 + (u - v)][1 - (u - v)]$$

$$= (1 + u - v)(1 - u + v)$$

Q10 $25a^2b^2 - 20abc + 4c^2 - 16d^2$

Solution

$$25a^2b^2 - 20abc + 4c^2 - 16d^2$$

$$\text{Using Formula } a^2 - 2ab + b^2 = (a - b)^2$$

$$= (5ab)^2 - 2(5ab)(2c) + (2c)^2 - (4d)^2$$

$$= (5ab - 2c)^2 - (4d)^2$$

$$\text{Using Formula } a^2 - b^2 = (a + b)(a - b)$$

$$= (5ab - 2c + 4d)(5ab - 2c - 4d)$$

Exercise# 5.2

Q1 $x^4 + 64$

Solution

$$x^4 + 64$$

$$= (x^2)^2 + (8)^2$$

$$\text{Add and Subtract } 2(x^2)(8)$$

$$= (x^2)^2 + (8)^2 + 2(x^2)(8) - 2(x^2)(8)$$

$$\text{Using Formula } a^2 + b^2 + 2ab = (a + b)^2$$

$$= (x^2 + 8)^2 - 16x^2$$

$$= (x^2 + 8)^2 - (4x)^2$$

$$\text{As } a^2 - b^2 = (a + b)(a - b)$$

$$= (x^2 + 8 + 4x)(x^2 + 8 - 4x)$$

$$= (x^2 + 4x + 8)(x^2 - 4x + 8)$$

Q2 $4x^4 + 81$

Solution

$$4x^4 + 81$$

$$= (2x^2)^2 + (9)^2$$

$$\text{Add and Subtract } 2(2x^2)(9)$$

$$= (2x^2)^2 + (9)^2 + 2(2x^2)(9) - 2(2x^2)(9)$$

$$\text{Using Formula } a^2 + b^2 + 2ab = (a + b)^2$$

$$= (2x^2 + 9)^2 - 36x^2$$

$$= (2x^2 + 9)^2 - (6x)^2$$

$$\text{As } a^2 - b^2 = (a + b)(a - b)$$

$$= (2x^2 + 9 + 6x)(2x^2 + 9 - 6x)$$

$$= (2x^2 + 6x + 9)(2x^2 - 6x + 9)$$

Q3 $a^4 + a^2b^2 + b^4$

Solution

$$a^4 + a^2b^2 + b^4$$

$$= a^4 + b^4 + a^2b^2$$

$$= (a^2)^2 + (b^2)^2 + a^2b^2$$

$$\text{Add and Subtract } 2(a^2)(b^2)$$

$$= (a^2)^2 + (b^2)^2 + 2(a^2)(b^2) - 2(a^2)(b^2) + a^2b^2$$

$$\text{Using Formula } a^2 + b^2 + 2ab = (a + b)^2$$

$$= (a^2 + b^2)^2 - 2a^2b^2 + a^2b^2$$

$$= (a^2 + b^2)^2 - a^2b^2$$

$$= (a^2 + b^2)^2 - (ab)^2$$

$$\text{As } a^2 - b^2 = (a + b)(a - b)$$

$$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$$

Q4 $x^4 + x^2 + 1$

Solution

$$x^4 + x^2 + 1$$

$$= x^4 + 1 + x^2$$

$$= (x^2)^2 + (1)^2 + x^2$$

$$\text{Add and Subtract } 2(x^2)(1)$$

$$= (x^2)^2 + (1)^2 + 2(x^2)(1) - 2(x^2)(1) + x^2$$

$$\text{Using Formula } a^2 + b^2 + 2ab = (a + b)^2$$

$$= (x^2 + 1)^2 - 2x^2 + x^2$$

$$= (x^2 + 1)^2 - x^2$$

$$\text{As } a^2 - b^2 = (a + b)(a - b)$$

$$= (x^2 + 1 + x)(x^2 + 1 - x)$$

$$= (x^2 + x + 1)(x^2 - x + 1)$$

Q5 $x^8 + x^4 + 1$

Solution

$$x^8 + x^4 + 1$$

$$= x^8 + 1 + x^4$$

$$= (x^4)^2 + (1)^2 + x^4$$

$$\text{Add and Subtract } 2(x^4)(1)$$

$$(x^4)^2 + (1)^2 + 2(x^4)(1) - 2(x^4)(1) + x^4$$

$$\text{Using Formula } a^2 + b^2 + 2ab = (a + b)^2$$

$$(x^4 + 1)^2 - 2x^4 + x^4$$

$$(x^4 + 1)^2 - x^4$$

$$(x^4 + 1)^2 - (x^2)^2$$

$$\text{As } a^2 - b^2 = (a + b)(a - b)$$

$$(x^4 + 1 + x^2)(x^4 + 1 - x^2)$$

$$[(x^2)^2 + (1)^2 + x^2](x^4 + 1 - x^2)$$

$$[(x^2)^2 + (1)^2 + 2(x^2)(1) - 2(x^2)(1) + x^2](x^4 + 1 - x^2)$$

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Using Formula $a^2 + b^2 + 2ab = (a + b)^2$
 $= [(x^2 + 1)^2 - 2x^2 + x^2](x^4 + 1 - x^2)$
 $= [(x^2 + 1)^2 - x^2](x^4 + 1 - x^2)$
As $a^2 - b^2 = (a + b)(a - b)$
 $= [(x^2 + 1 + x)(x^2 + 1 - x)](x^4 + 1 - x^2)$
 $= [(x^2 + x + 1)(x^2 - x + 1)](x^4 - x^2 + 1)$

Q6 $x^4 + \frac{1}{x^4} - 7$

Solution
 $x^4 + \frac{1}{x^4} - 7$
 $= (x^2)^2 + \left(\frac{1}{x^2}\right)^2 - 7$

Add and Subtract $2(x^2)\left(\frac{1}{x^2}\right)$
 $= (x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x^2)\left(\frac{1}{x^2}\right) - 2(x^2)\left(\frac{1}{x^2}\right) - 7$
Using Formula $a^2 + b^2 + 2ab = (a + b)^2$
 $= \left(x^2 + \frac{1}{x^2}\right)^2 - 2 - 7$
 $= \left(x^2 + \frac{1}{x^2}\right)^2 - 9$
 $= \left(x^2 + \frac{1}{x^2}\right)^2 - (3)^2$
As $a^2 - b^2 = (a + b)(a - b)$
 $= \left(x^2 + \frac{1}{x^2} + 3\right)\left(x^2 + \frac{1}{x^2} - 3\right)$

Q7 $81x^4 + \frac{1}{81x^4} - 14$

Solution
 $81x^4 + \frac{1}{81x^4} - 14$
 $= (9x^2)^2 + \left(\frac{1}{9x^2}\right)^2 - 14$
Add and Subtract $2(9x^2)\left(\frac{1}{9x^2}\right)$
 $= (9x^2)^2 + \left(\frac{1}{9x^2}\right)^2 + 2(9x^2)\left(\frac{1}{9x^2}\right) - 2(9x^2)\left(\frac{1}{9x^2}\right) - 14$
Using Formula $a^2 + b^2 + 2ab = (a + b)^2$
 $= \left(9x^2 + \frac{1}{9x^2}\right)^2 - 2 - 14$

$$\begin{aligned}
&= \left(9x^2 + \frac{1}{9x^2}\right)^2 - 16 \\
&= \left(9x^2 + \frac{1}{9x^2}\right)^2 - (4)^2 \\
\text{As } a^2 - b^2 &= (a + b)(a - b) \\
&= \left(9x^2 + \frac{1}{9x^2} + 4\right)\left(9x^2 + \frac{1}{9x^2} - 4\right)
\end{aligned}$$

Q8 $4x^4 - 4x^2y^2 + 64y^4$

Solution
 $4x^4 - 4x^2y^2 + 64y^4$
 $= 4(x^4 - x^2y^2 + 16y^2)$
 $= 4(x^4 + 16y^4 - x^2y^2)$
 $= 4[(x^2)^2 + (4y^2)^2 - x^2y^2]$
Add and Subtract $2(x^2)(4y^2)$
 $= 4[(x^2)^2 + (4y^2)^2 + 2(x^2)(4y^2) - 2(x^2)(4y^2) - x^2y^2]$
Using Formula $a^2 + b^2 + 2ab = (a + b)^2$
 $= 4[(x^2 + 4y^2)^2 - 8x^2y^2 - x^2y^2]$
 $= 4[(x^2 + 4y^2)^2 - 9x^2y^2]$
 $= 4[(x^2 + 4y^2)^2 - (3xy)^2]$
As $a^2 - b^2 = (a + b)(a - b)$
 $= 4(x^2 + 4y^2 + 3xy)(x^2 + 4y^2 - 3xy)$
 $= 4(x^2 + 3xy + 4y^2)(x^2 - 3xy + 4y^2)$

Q9 $16m^4 + 4m^2n^2 + n^4$

Solution
 $16m^4 + 4m^2n^2 + n^4$
 $= 16m^4 + n^4 + 4m^2n^2$
 $= (4m^2)^2 + (n^2)^2 + 4m^2n^2$
Add and Subtract $2(4m^2)(n^2)$
 $= (4m^2)^2 + (n^2)^2 + 2(4m^2)(n^2) - 2(4m^2)(n^2) + 4m^2n^2$
Using Formula $a^2 + b^2 + 2ab = (a + b)^2$
 $= (4m^2 + n^2)^2 - 8m^2n^2 + 4m^2n^2$
 $= (4m^2 + n^2)^2 - 4m^2n^2$
 $= (4m^2 + n^2)^2 - (2mn)^2$
As $a^2 - b^2 = (a + b)(a - b)$
 $= (4m^2 + n^2 + 2mn)(4m^2 + n^2 - 2mn)$
 $= (4m^2 + 2mn + n^2)(4m^2 - 2mn + n^2)$

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Q10 $4x^5y + 11x^3y^3 + 9xy^5$

Solution

$$\begin{aligned}
 & 4x^5y + 11x^3y^3 + 9xy^5 \\
 &= xy(4x^4 + 11x^2y^2 + 9y^4) \\
 &= xy(4x^4 + 9y^4 + 11x^2y^2) \\
 &= xy[(2x^2)^2 + (3y^2)^2 + 11x^2y^2] \\
 &\text{Add and Subtract } 2(2x^2)(3y^2) \\
 &= xy[(2x^2)^2 + (3y^2)^2 + 2(2x^2)(3y^2) - 2(2x^2)(3y^2) + 11x^2y^2] \\
 &\text{Using Formula } a^2 + b^2 + 2ab = (a + b)^2 \\
 &= xy[(2x^2 + 3y^2)^2 - 12x^2y^2 + 11x^2y^2] \\
 &= xy[(2x^2 + 3y^2)^2 - x^2y^2] \\
 &= xy[(2x^2 + 3y^2)^2 - (xy)^2] \\
 &\text{As } a^2 - b^2 = (a + b)(a - b) \\
 &= xy(2x^2 + 3y^2 + xy)(2x^2 + 3y^2 - xy) \\
 &= xy(2x^2 + xy + 3y^2)(2x^2 - xy + 3y^2)
 \end{aligned}$$

Exercise# 5.3

Q1 $x^2 - 7x + 12$

Solution

$$\begin{aligned}
 & x^2 - 7x + 12 \\
 &= x^2 - 3x - 4x + 12 \\
 &= x(x - 3) - 4(x - 3) \\
 &= (x - 3)(x - 4)
 \end{aligned}$$

$(x^2)(12) = 12x^2$

Add	Multiply
-3x	-3x
-4x	-4x
-7x	12x ²

Q2 $x^2 + x - 12$

Solution

$$\begin{aligned}
 & x^2 + x - 12 \\
 &= x^2 - 3x + 4x - 12 \\
 &= x(x - 3) + 4(x - 3) \\
 &= (x - 3)(x + 4)
 \end{aligned}$$

$(x^2)(-12) = -12x^2$

Add	Multiply
-3x	-3x
+4x	+4x
x	-12x ²

Q3 $20 - x - x^2$

Solution

$$\begin{aligned}
 & 20 - x - x^2 \\
 &= 20 + 4x - 5x - x^2 \\
 &= 4(5 + x) - x(5 + x) \\
 &= (5 + x)(4 - x)
 \end{aligned}$$

$(20)(-x^2) = -20x^2$

Add	Multiply
+4x	+4x
-5x	-5x
-x	-20x ²

Q4 $2y^2 - 7y + 3$

Solution

$$\begin{aligned}
 & 2y^2 - 1y - 6y + 3 \\
 &= y(2y - 1) - 3(2y - 1) \\
 &= (2y - 1)(y - 3)
 \end{aligned}$$

$(2y^2)(3) = 6y^2$

Add	Multiply
-1y	-1y
-6y	-6y
-7y	6y ²

Q5 $4x^2 + 8x + 3$

Solution

$$\begin{aligned}
 & 4x^2 + 8x + 3 \\
 &= 4x^2 + 2x + 6x + 3 \\
 &= 2x(2x + 1) + 3(2x + 1) \\
 &= (2x + 1)(2x + 3)
 \end{aligned}$$

$(4x^2)(3) = 12x^2$

Add	Multiply
+2x	+2x
+6x	+6x
8x	12x ²

Q6 $10y^2 - 3y - 1$

Solution

$$\begin{aligned}
 & 10y^2 - 3y - 1 \\
 &= 10y^2 + 2y - 5y - 1 \\
 &= 2y(5y + 1) - 1(5y + 1) \\
 &= (5y + 1)(2y - 1)
 \end{aligned}$$

$(10y^2)(-1) = -10y^2$

Add	Multiply
+2y	+2y
-5y	-5y
-3y	-10y ²

Q7 $6x^3 - 15x^2 - 9x$

Solution

$$\begin{aligned}
 & 6x^3 - 15x^2 - 9x \\
 &= 3x(2x^2 - 5x - 3) \\
 &= 3x(2x^2 + 1x - 6x - 3) \\
 &= 3x[x(2x + 1) - 3(2x + 1)] \\
 &= 3x(2x + 1)(x - 3)
 \end{aligned}$$

$(2x^2)(-3) = -6x^2$

Add	Multiply
+1x	+1x
-6x	-6x
-5x	-6x ²

Q8 $2xy^2 + 8xy - 24x$

Solution

$$\begin{aligned}
 & 2xy^2 + 8xy - 24x \\
 &= 2x(y^2 + 4y - 12) \\
 &= 2x(y^2 - 2y + 6y - 12) \\
 &= 2x[y(y - 2) + 6(y - 2)] \\
 &= 2x(y - 2)(y + 6)
 \end{aligned}$$

$(y^2)(-12) = -12y^2$

Add	Multiply
-2y	-2y
+6y	+6y
+4y	-12y ²

Q10 $-16x^3y - 20x^2y^2 - 6xy^3$

Solution

$$\begin{aligned}
 & -16x^3y - 20x^2y^2 - 6xy^3 \\
 &= -2xy(8x^2 + 10xy + 3y^2) \\
 &= -2xy(8x^2 + 4xy + 6xy + 3y^2)
 \end{aligned}$$

Add	Multiply
+4xy	+4xy
+6xy	+6xy
+10xy	24x ² y ²

Q11 $(x + 1)^2 + 3(x + 1) + 2$

Solution

$$\begin{aligned}
 & (x + 1)^2 + 3(x + 1) + 2 \\
 &= x^2 + (1)^2 + 2(x)(1) + 3x + 3 + 2 \\
 &= x^2 + 1 + 2x + 3x + 5 \\
 &= x^2 + 5x + 6 \\
 &= x^2 + 2x + 3x + 6 \\
 &= x(x + 2) + 3(x + 2) \\
 &= (x + 2)(x + 3)
 \end{aligned}$$

$(x^2)(6) = 6x^2$

Add	Multiply
+2x	+2x
+3x	+3x
5x	6x ²

Chapter # 5

<p>Q12</p> $4x^8y^{10} - 40x^5y^7 + 84x^2y^4$ <p>Solution</p> $\begin{array}{ c c c } \hline & (x^6y^6)(21) = 21x^6y^6 & \\ \hline \end{array}$ $4x^8y^{10} - 40x^5y^7 + 84x^2y^4$ $= 4x^2y^4(x^6y^6 - 10x^3y^3 + 21)$ $= 4x^2y^4(x^6y^6 - 3x^3y^3 - 7x^3y^3 + 21)$ $= 4x^2y^4[x^3y^3(x^3y^3 - 3) - 7(x^3y^3 - 3)]$ $= 4x^2y^4(x^3y^3 - 3)(x^3y^3 - 7)$	<p>Q1</p> $(x^6y^6)(21) = 21x^6y^6$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 2px;">Add</td> <td style="padding: 2px;">Multiply</td> </tr> <tr> <td style="padding: 2px;">-3x³y³</td> <td style="padding: 2px;">-3x³y³</td> </tr> <tr> <td style="padding: 2px;">-7x³y³</td> <td style="padding: 2px;">-7x³y³</td> </tr> <tr> <td style="padding: 2px;">-10x³y³</td> <td style="padding: 2px;">21x⁶y⁶</td> </tr> </table> <p>Q13</p> <p>Find an expression for the perimeter of a rectangle with area given by $x^2 + 24x - 81$</p> <p>Given</p> <p><i>Area of rectangle = $x^2 + 24x - 81$</i></p> <p>To find</p> <p>Perimeter of rectangle = ?</p> $(x^2)(-81) = -81x^2$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 2px;">Add</td> <td style="padding: 2px;">Multiply</td> </tr> <tr> <td style="padding: 2px;">-3x</td> <td style="padding: 2px;">-3x</td> </tr> <tr> <td style="padding: 2px;">+27x</td> <td style="padding: 2px;">+27x</td> </tr> <tr> <td style="padding: 2px;">24x</td> <td style="padding: 2px;">-81x²</td> </tr> </table> <p>As <i>Area = l × w</i></p> <p>And <i>Perimeter = 2l + 2w</i></p> <p>Now</p> $x^2 + 24x - 81$ $= x^2 - 3x + 27x - 81$ $= x(x - 3) + 27(x - 3)$ $= (x - 3)(x + 27)$ <div style="text-align: center; margin-top: 10px;"> x - 3 x + 27 </div> <p>Now <i>l = (x + 27)</i> and <i>w = (x - 3)</i></p> <p>As</p> <p><i>Perimeter = 2l + 2w</i></p> <p><i>Perimeter = 2(x + 27) + 2(x - 3)</i></p> <p><i>Perimeter = 2x + 54 + 2x - 6</i></p> <p><i>Perimeter = 4x + 48</i></p> <p>Q9</p> <p>2 + 5t - 12t²</p> <p>Solution</p> $2 + 5t - 12t^2$ $-12t^2 + 5t + 2$ $-(12t^2 - 5t - 2)$ $-(12t^2 + 3t - 8t - 2)$ $-[3t(4t + 1) - 2(4t + 1)]$ $-(4t + 1)(3t - 2)$ <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 2px;">(12t²)(-2) = -24t²</td> <td style="padding: 2px;"></td> </tr> <tr> <td style="padding: 2px;">Add</td> <td style="padding: 2px;">Multiply</td> </tr> <tr> <td style="padding: 2px;">+3t</td> <td style="padding: 2px;">+3t</td> </tr> <tr> <td style="padding: 2px;">-8t</td> <td style="padding: 2px;">-8t</td> </tr> <tr> <td style="padding: 2px;">-5t</td> <td style="padding: 2px;">-24t²</td> </tr> </table>	Add	Multiply	-3x ³ y ³	-3x ³ y ³	-7x ³ y ³	-7x ³ y ³	-10x ³ y ³	21x ⁶ y ⁶	Add	Multiply	-3x	-3x	+27x	+27x	24x	-81x ²	(12t ²)(-2) = -24t ²		Add	Multiply	+3t	+3t	-8t	-8t	-5t	-24t ²
Add	Multiply																										
-3x ³ y ³	-3x ³ y ³																										
-7x ³ y ³	-7x ³ y ³																										
-10x ³ y ³	21x ⁶ y ⁶																										
Add	Multiply																										
-3x	-3x																										
+27x	+27x																										
24x	-81x ²																										
(12t ²)(-2) = -24t ²																											
Add	Multiply																										
+3t	+3t																										
-8t	-8t																										
-5t	-24t ²																										

<p>Exercise# 5.4</p> $(4x^2 - 16x + 7)(4x^2 - 16x + 15) + 16$ <p>Solution</p> $(4x^2 - 16x + 7)(4x^2 - 16x + 15) + 16$ $\text{Let } 4x^2 - 16x = y$ $= (y + 7)(y + 15) + 16$ $= y^2 + 15y + 7y + 105 + 16$ $= y^2 + 22y + 121$ $= y^2 + 11y + 11y + 121$ $= y(y + 11) + 11(y + 11)$ $= (y + 11)(y + 11)$ $\text{But } y = 4x^2 - 16x$ $= (4x^2 - 16x + 11)(4x^2 - 16x + 11)$ $= (4x^2 - 16x + 11)^2$	<p>Q2</p> $(9x^2 + 9x - 4)(9x^2 + 9x - 10) - 72$ <p>Solution</p> $(9x^2 + 9x - 4)(9x^2 + 9x - 10) - 72$ $\text{Let } 9x^2 + 9x = y$ $= (y - 4)(y - 10) - 72$ $= y^2 - 10y - 4y - 40 - 72$ $= y^2 - 14y - 32$ $= y^2 + 2y - 16y - 32$ $= y(y + 2) - 16(y + 2)$ $= (y + 2)(y - 16)$ $\text{But } y = 9x^2 + 9x$ So $= (9x^2 + 9x + 2)(9x^2 + 9x - 16)$
<p>Q3</p> $(x + 2)(x + 4)(x + 6)(x + 8) - 9$ <p>Solution</p> $(x + 2)(x + 4)(x + 6)(x + 8) - 9$ <p>Rearranging accordingly 4+6=2+8</p> $= (x + 2)(x + 8)(x + 4)(x + 6) - 9$ $= (x^2 + 8x + 2x + 16)(x^2 + 6x + 4x + 24) - 9$ $= (x^2 + 10x + 16)(x^2 + 10x + 24) - 9$ $\text{Let } x^2 + 10x = y$ $= (y + 16)(y + 24) - 9$ $= y^2 + 24y + 16y + 384 - 9$ $= y^2 + 40y + 375$ $= y^2 + 15y + 25y + 375$ $= y(y + 15) + 25(y + 15)$ $= (y + 15)(y + 25)$	

Chapter # 5

<p>But $y = x^2 + 10x$ So $= (x^2 + 10x + 15)(x^2 + 10x + 25)$</p> <p>Q4 $x(x + 1)(x + 2)(x + 3) + 1$ Solution $x(x + 1)(x + 2)(x + 3) + 1$ Rearranging accordingly $0+3=1+2$ $= x(x + 3)(x + 1)(x + 2) + 1$ $= (x^2 + 3x)(x^2 + 2x + 1x + 2) + 1$ $= (x^2 + 3x)(x^2 + 3x + 2) + 1$ Let $x^2 + 3x = y$ $= (y)(y + 2) + 1$ $= y^2 + 2y + 1$ $= (y)^2 + (1)^2 + 2(y)(1)$ $= (y + 1)^2$ But $y = x^2 + 3x$ So $= (x^2 + 3x + 1)^2$</p> <p>Q5 $(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2$ Solution $(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2$ Rearranging accordingly $1 \times 6 = 2 \times 3$ $= (x + 1)(x + 6)(x + 2)(x + 3) - 3x^2$ $= (x^2 + 6x + 1x + 6)(x^2 + 3x + 2x + 6) - 3x^2$ $= (x^2 + 7x + 6)(x^2 + 5x + 6) - 3x^2$ $= (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2$ Let $x^2 + 6 = y$ $= (y + 7x)(y + 5x) - 3x^2$ $= y^2 + 5xy + 7xy + 35x^2 - 3x^2$ $= y^2 + 12xy + 32x^2$ $= y^2 + 4xy + 8xy + 32x^2$ $= y(y + 4x) + 8x(y + 4x)$ $= (y + 4x)(y + 8x)$ But $y = x^2 + 6$ $= (x^2 + 6 + 4x)(x^2 + 6 + 8x)$ $= \left(\frac{x(x^2 + 6 + 4x)}{x}\right)\left(\frac{x(x^2 + 6 + 8x)}{x}\right)$ $= x \cdot x \left(\frac{x^2}{x} + \frac{6}{x} + \frac{4x}{x}\right) \left(\frac{x^2}{x} + \frac{6}{x} + \frac{8x}{x}\right)$ $= x^2 \left(x + \frac{6}{x} + 4\right) \left(x + \frac{6}{x} + 8\right)$</p>	<p>Q6 $64x^3 - 144x^2y + 108xy^2 - 27y^3$ Solution $64x^3 - 144x^2y + 108xy^2 - 27y^3$ $= (4x)^3 - 3(4x)^2(3y) + 3(4x)(3y)^2 - (3y)^3$ As $a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$ $= (4x - 3y)^3$</p> <p>Q7 $\frac{a^3}{8} - \frac{1}{4}a^2b + \frac{1}{6}ab^2 - \frac{b^3}{27}$ Solution $\frac{a^3}{8} - \frac{1}{4}a^2b + \frac{1}{6}ab^2 - \frac{b^3}{27}$ $= \left(\frac{a}{2}\right)^3 - 3\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right) + 3\left(\frac{a}{2}\right)\left(\frac{b}{3}\right)^2 - \left(\frac{b}{3}\right)^3$ As $a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$ $= \left(\frac{a}{2} - \frac{b}{3}\right)^3$</p> <p>Q9 $\frac{x^3}{a^3} + \frac{3x}{a} + \frac{3a}{x} + \frac{a^3}{x^3}$ Solution $\frac{x^3}{a^3} + \frac{3x}{a} + \frac{3a}{x} + \frac{a^3}{x^3}$ $= \left(\frac{x}{a}\right)^3 + 3\left(\frac{x}{a}\right)^2\left(\frac{a}{x}\right) + 3\left(\frac{x}{a}\right)\left(\frac{a}{x}\right)^2 + \left(\frac{a}{x}\right)^3$ As $a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$ $= \left(\frac{x}{a} + \frac{a}{x}\right)^3$</p> <p>Q10 $27a^3 + 189a^2b + 441ab^2 + 343b^3$ Solution $27a^3 + 189a^2b + 441ab^2 + 343b^3$ $= (3a)^3 + 3(3a)^2(7b) + 3(3a)(7b)^2 + (7b)^3$ As $a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3$ $= (3a + 7b)^3$</p> <p>Q11 $8x^3 - 4x + \frac{2}{3x} - \frac{1}{27x^3}$ Solution $8x^3 - 4x + \frac{2}{3x} - \frac{1}{27x^3}$ $= (2x)^3 - 3(2x)^2\left(\frac{1}{3x}\right) + 3(2x)\left(\frac{1}{3x}\right)^2 - \left(\frac{1}{3x}\right)^3$ As $a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3$ $= \left(2x - \frac{1}{3x}\right)^3$</p>
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Chapter # 5

Exercise# 5.5

Q1 $a^3 - 27$

Solution

$$a^3 - 27$$

$$= (a)^3 - (3)^3$$

Using Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$= (a - 3)[(a)^2 + (a)(3) + (3)^2]$$

$$= (a - 3)(a^2 + 3a + 9)$$

Q2 $a^6 + b^6$

Solution

$$a^6 + b^6$$

$$= (a^2)^3 + (b^2)^3$$

Using Formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$= (a^2 + b^2)[(a^2)^2 - (a^2)(b^2) + (b^2)^2]$$

$$= (a^2 + b^2)(a^4 - a^2b^2 + b^4)$$

Q3 $24x^3 + 3$

Solution

$$24x^3 + 3$$

$$= 3(8x^3 + 1)$$

$$= 3[(2x)^3 + (1)^3]$$

Using Formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$= 3\{(2x + 1)[(2x)^2 - (2x)(1) + (1)^2]\}$$

$$= 3(2x + 1)(4x^2 + 2x + 1)$$

Q4 $1 - 27r^3$

Solution

$$1 - 27r^3$$

$$= (1)^3 - (3r)^3$$

Using Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$= (1 - 3r)[(1)^2 + (1)(3r) + (3r)^2]$$

$$= (1 - 3r)(1 + 3r + 9r^2)$$

Q5 $2x^3 - 128$

Solution

$$2x^3 - 128$$

$$2(x^3 - 64)$$

$$2[(x)^3 - (4)^3]$$

Using Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$2\{(x - 4)[(x)^2 + (x)(4) + (4)^2]\}$$

$$2(x - 4)(x^2 + 4x + 16)$$

Q6 $4x^5 - 256x^2$

Solution

$$4x^5 - 256x^2$$

$$= 4x^2(x^3 - 64)$$

$$= 4x^2[(x)^3 - (4)^3]$$

Using Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$= 4x^2\{(x - 4)[(x)^2 + (x)(4) + (4)^2]\}$$

$$= 4x^2(x - 4)(x^2 + 4x + 16)$$

Q7 $18(x - y)^3 - 144(a - b)^3$

Solution

$$18(x - y)^3 - 144(a - b)^3$$

$$= 18[(x - y)^3 - 8(a - b)^3]$$

$$= 18[(x - y)^3 - (2(a - b))^3]$$

Using Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$= 18[(x - y) - 2(a - b)][(x - y)^2 + (x - y)(2(a - b)) + (2(a - b))^2]$$

$$= 18(x - y - 2a + 2b)[(x - y)^2 + 2(x - y)(a - b) + 4(a - b)^2]$$

Q8 $x^9 + 1$

Solution

$$x^9 + 1$$

$$= (x^3)^3 + (1)^3$$

Using Formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$= (x^3 + 1)[(x^3)^2 - (x^3)(1) + (1)^2]$$

$$= (x + 1)[(x)^2 - (x)(1) + (1)^2](x^6 - x^3 + 1)$$

Using Formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$= (2x + 1)(x^2 - x + 1)(x^6 - x^3 + 1)$$

Q9 $a^3 + (c + d)^3$

Solution

$$a^3 + (c + d)^3$$

Using Formula: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$= [a + (c + d)][(a)^2 - (a)(c + d) + (c + d)^2]$$

$$= (a + c + d)[a^2 - a(c + d) + (c + d)^2]$$

Q10 $27x^3 - y^3$

Solution

$$27x^3 - y^3$$

$$= (3x)^3 - (y)^3$$

Using Formula: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$= (3x - y)[(3x)^2 + (3x)(y) + (y)^2]$$

$$= (3x - y)(9x^2 + 3xy + y^2)$$

Chapter # 5

$$\begin{array}{r}
 & \frac{x^2 - x - 6}{x - 1} \\
 & \overline{\quad | \quad x^3 - 2x^2 - 5x + 6} \\
 & \underline{+x^3 \mp x^2} \\
 & \quad -x^2 - 5x + 6 \\
 & \underline{\mp x^2 \pm x} \\
 & \quad -6x + 6 \\
 & \underline{\mp 6x \pm 6} \\
 & \quad x
 \end{array}$$

Here $Q(x) = (x^2 - x - 6)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$$\begin{aligned}
 x^3 - 2x^2 - 5x + 6 &= (x - 1)(x^2 - x - 6) \\
 &= (x - 1)(x^2 + 2x - 3x - 6) \\
 &= (x - 1)[x(x + 2) - 3(x + 2)] \\
 &= (x - 1)(x + 2)(x - 3)
 \end{aligned}$$

(ii) $x^3 + x^2 - 4x - 4$

Solution

$$P(x) = x^3 + x^2 - 4x - 4$$

Let $x = -1$

$$\begin{aligned}
 \text{So } P(-1) &= (-1)^3 + (-1)^2 - 4(-1) - 4 \\
 &= -1 + 1 + 4 - 4 \\
 &= 0
 \end{aligned}$$

Since $P(x) = 0$, So $x + 1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x + 1$

$$\begin{array}{r}
 & \frac{x^2 - 4}{x + 1} \\
 & \overline{\quad | \quad x^3 + x^2 - 4x - 4} \\
 & \underline{+x^3 \pm x^2} \\
 & \quad -4x - 4 \\
 & \underline{\mp 4x \mp 4} \\
 & \quad x
 \end{array}$$

Here $Q(x) = (x^2 - 4)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

$$\begin{aligned}
 \text{Hence} \\
 x^3 + x^2 - 4x - 4 &= (x + 1)(x^2 - 4) \\
 &= (x + 1)[(x)^2 - (2)^2] \\
 &= (x + 1)(x + 2)(x - 2)
 \end{aligned}$$

$x^3 - 7x + 6$

Solution

$$P(x) = x^3 - 7x + 6$$

Let $x = 1$

$$\begin{aligned}
 \text{So } P(1) &= (1)^3 - 7(1) + 6 \\
 &= 1 - 7 + 6 \\
 &= -6 + 6 \\
 &= 0
 \end{aligned}$$

Since $P(x) = 0$, So $x - 1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x - 1$

$$\begin{array}{r}
 & \frac{x^2 + x - 6}{x - 1} \\
 & \overline{\quad | \quad x^3 - 7x + 6} \\
 & \underline{+x^3 \mp x^2} \\
 & \quad x^2 - 7x + 6 \\
 & \underline{\pm x^2 \mp x} \\
 & \quad -6x + 6 \\
 & \underline{\mp 6x \pm 6} \\
 & \quad x
 \end{array}$$

Here $Q(x) = (x^2 + x - 6)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$$\begin{aligned}
 x^3 - 7x + 6 &= (x - 1)(x^2 + x - 6) \\
 &= (x - 1)(x^2 - 2x + 3x - 6) \\
 &= (x - 1)[x(x - 2) + 3(x - 2)] \\
 &= (x - 1)(x - 2)(x + 3)
 \end{aligned}$$

(iv) $x^3 - 9x^2 + 23x - 15$

Solution

$$P(x) = x^3 - 9x^2 + 23x - 15$$

Let $x = 1$

$$\begin{aligned}
 \text{So } P(1) &= (1)^3 - 9(1)^2 + 23(1) - 15 \\
 &= 1 - 9 + 23 - 15 \\
 &= 1 - 9 + 8 \\
 &= -8 + 8 \\
 &= 0
 \end{aligned}$$

Since $P(x) = 0$, So $x - 1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x - 1$

Chapter # 5

$$\begin{array}{r}
 x^2 - 8x + 15 \\
 \hline
 x - 1 \quad | \quad x^3 - 9x^2 + 23x - 15 \\
 \underline{-x^3 \pm x^2} \\
 \hline
 -8x^2 + 23x \\
 \underline{\mp 8x^2 \pm 8x} \\
 \hline
 15x - 15 \\
 \underline{\pm 15x \mp 15} \\
 \hline
 x
 \end{array}$$

Here $Q(x) = (x^2 - 8x + 15)$ and $R = 0$
As $P(x) = (x - r)Q(x) + R$

Hence

$$\begin{aligned}
 x^3 - 7x + 6 &= (x - 1)(x^2 - 8x + 15) \\
 &= (x - 1)(x^2 - 3x - 5x + 15) \\
 &= (x - 1)[x(x - 3) - 5(x - 3)] \\
 &= (x - 1)(x - 3)(x - 5)
 \end{aligned}$$

(v) $x^3 - 4x^2 - 3x + 18$

Solution

$$P(x) = x^3 - 4x^2 - 3x + 18$$

Let $x = -2$

$$\begin{aligned}
 \text{So } P(-2) &= (-2)^3 - 4(-2)^2 - 3(-2) + 18 \\
 &= -8 - 4(4) + 6 + 18 \\
 &= -8 - 16 + 24 \\
 &= -24 + 24 \\
 &= 0
 \end{aligned}$$

Since $P(x) = 0$, So $x + 2$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x + 2$

$$\begin{array}{r}
 x^2 - 6x + 9 \\
 \hline
 x + 2 \quad | \quad x^3 - 4x^2 - 3x + 18 \\
 \underline{-x^3 \pm 2x^2} \\
 \hline
 -6x^2 - 3x \\
 \underline{\mp 6x^2 \mp 12x} \\
 \hline
 9x + 18 \\
 \underline{\pm 9x \pm 18} \\
 \hline
 x
 \end{array}$$

Here $Q(x) = (x^2 - 6x + 9)$ and $R = 0$
As $P(x) = (x - r)Q(x) + R$

Hence

$$\begin{aligned}
 x^3 - 4x^2 - 3x + 18 &= (x + 2)(x^2 - 6x + 9) \\
 &= (x + 2)[(x)^2 - 2(x)(3) + (3)^2] \\
 &= (x + 2)(x - 3)^2
 \end{aligned}$$

(vi) $x^3 + 2x^2 - 19x - 20$

Solution

$$P(x) = x^3 + 2x^2 - 19x - 20$$

Let $x = -1$

$$\begin{aligned}
 \text{So } P(-1) &= (-1)^3 + 2(-1)^2 - 19(-1) - 20 \\
 &= -1 + 2(1) + 19 - 20 \\
 &= -1 + 2 - 1 \\
 &= 1 - 1 \\
 &= 0
 \end{aligned}$$

Since $P(x) = 0$, So $x + 1$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x + 1$

$$\begin{array}{r}
 x^2 + x - 20 \\
 \hline
 x + 1 \quad | \quad x^3 + 2x^2 - 19x - 20 \\
 \underline{-x^3 \pm x^2} \\
 \hline
 x^2 - 19x \\
 \underline{\pm x^2 \pm x} \\
 \hline
 -20x - 20 \\
 \underline{\mp 20x \mp 20} \\
 \hline
 x
 \end{array}$$

Here $Q(x) = (x^2 + x - 20)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$$\begin{aligned}
 x^3 + 2x^2 - 19x - 20 &= (x + 1)(x^2 + x - 20) \\
 &= (x + 1)(x^2 - 4x + 5x - 20) \\
 &= (x + 1)[x(x - 4) + 5(x - 4)] \\
 &= (x + 1)(x - 4)(x + 5)
 \end{aligned}$$

Chapter # 5

(vii) $x^3 - x^2 - 14x + 24$

Solution

$$P(x) = x^3 - x^2 - 14x + 24$$

Let $x = 2$

$$\begin{aligned} \text{So } P(-2) &= (2)^3 - (2)^2 - 14(2) + 24 \\ &= 8 - 4 - 28 + 24 \\ &= 4 - 4 \\ &= 0 \end{aligned}$$

Since $P(x) = 0$, So $x - 2$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x - 2$

$$\begin{array}{r} x^2 + x - 12 \\ \hline x - 2 \left| \begin{array}{r} x^3 - x^2 - 14x + 24 \\ \pm x^3 \mp 2x^2 \\ \hline x^2 - 14x \\ \pm x^2 \mp 2x \\ \hline -12x + 24 \\ \mp 12x \pm 24 \\ \hline \end{array} \right. \\ \qquad\qquad\qquad x \end{array}$$

Here $Q(x) = (x^2 + x - 12)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$$\begin{aligned} x^3 - x^2 - 14x + 24 &= (x - 2)(x^2 + x - 12) \\ &= (x - 2)(x^2 + 4x - 3x - 12) \\ &= (x - 2)[x(x + 4) - 3(x + 4)] \\ &= (x - 2)(x + 4)(x - 3) \end{aligned}$$

(viii) $x^3 - 6x^2 + 32$

Solution

$$P(x) = x^3 - 6x^2 + 32$$

Let $x = -2$

$$\begin{aligned} \text{So } P(-2) &= (-2)^3 - 6(-2)^2 + 32 \\ &= -8 - 6(4) + 32 \\ &= -8 - 24 + 32 \\ &= -32 + 32 \\ &= 0 \end{aligned}$$

Since $P(x) = 0$, So $x + 2$ is a factor of $P(x)$. To find other, divide $P(x)$ by $x + 2$

$$\begin{array}{r} x^2 - 8x + 16 \\ \hline x + 2 \left| \begin{array}{r} x^3 - 6x^2 + 32 \\ \pm x^3 \pm 2x^2 \\ \hline -8x^2 + 32 \\ \mp 8x^2 \quad \mp 16x \\ \hline 16x + 32 \\ \pm 16x \pm 32 \\ \hline \end{array} \right. \\ \qquad\qquad\qquad x \end{array}$$

Here $Q(x) = (x^2 - 8x + 16)$ and $R = 0$

As $P(x) = (x - r)Q(x) + R$

Hence

$$\begin{aligned} x^3 - 6x^2 + 32 &= (x + 2)(x^2 - 8x + 16) \\ &= (x + 2)[(x)^2 - 2(x)(4) + (4)^2] \\ &= (x + 2)(x - 4)^2 \end{aligned}$$

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Example # 12 Page # 133

Example # 17 Page # 136

Example # 22, 23, 24, 25 Page # 140, 141